An Enhanced Bearing Estimation Technique for DIFAR Sonobuoy Underwater Target Tracking

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***Abstract*—We consider the DIFAR sonobuoy bearing estimation problem for underwater acoustic sources. The standard arctangent based approach utilises the orthogonality between the observation noises for the different channels to form the bearing estimates, and ignores the correlation structure of the actual source signal. In this paper, we propose a new state space technique, which exploits the correlations structure in the source signal to achieve enhanced performance, particularly in low signal-to-noise (SNR) conditions, compared to the standard arctangent estimator. The analysis is supported by simulations using some realistic classes of signals.**

***Index Terms*—Bearing estimation, DIFAR sonobuoy, augmented complex Kalman filter, random-walk modeling, complex circularity, widely linear estimation**

I. Introduction

Bearing or direction-of-arrival (DOA) estimation is a problem encountered in a wide range of applications, including navigation, surveillance and communication systems. In underwater environments, the DIFAR sonobuoy, consisting of two crossed dipoles and an omni-directional hydrophone, is a typical arrangement used to provide three observations of a source signal (target), which together allow for the bearing (angle) of a source (target) to be estimated. In the ocean, however, there are many sources of background noise, such as environmental noise from wind, rain and waves, and biological noise from whales and other marine mammals. These all contribute to the total power spectrum (both broadband and narrowband) of the observed signals. Moreover, the propagation of acoustic signals in the ocean is generally not uniform or isotropic, which also contributes to the difficulty of the bearing problem in underwater environments [1] [2] [3].

The standard solutions for sonobuoy target detection and bearing estimation are based on spectral analysis of the observed signals using the discrete Fourier transform (DFT) [1] [4] [5] or using spectral modeling approaches, such as autoregressive moving average (ARMA). However, these techniques usually suffer from limited frequency resolution, which becomes especially pronounced for low signal to noise ratios (SNRs), leading to poor performance. Moreover, due to their block-processing nature, these techniques are not suited to rapidly moving targets, where the target bearing is nonstationary during the collection of the data block used for the DFT. Among the popular solutions for underwater sonobuoy bearing estimation is the DFT based 'arctangent' estimator [4] which utilises time-averaged products of the observation data blocks to form the bearing estimate.

In this paper, embarking upon the recently introduced augmented complex statistics and widely linear modeling, we propose an online sonobuoy target bearing estimation solution, based on widely linear (augmented) complex state space model [6]. The second order statistics of both the state and observation noises are estimated from the observation data, and their estimates are also updated online. It is shown that the state space model is inherently nonlinear, and we use the recently introduced augmented complex extended Kalman filter to address the problem [7] [8]. Simulations illustrate the robustness of the proposed technique, yielding enhanced performance compared to the standard arctangent estimator, especially in unfavourable signal-to-noise (SNR) conditions.

II. Background

A. Augmented Complex Statistics and Widely Linear Modeling

To introduce an optimal second order estimator for the generality of complex signals, consider first the real valued mean square estimator (MSE) of a random vector $\mathbf{y}$ in terms of a real observation $\mathbf{x}$, that is, $\hat{\mathbf{y}} = E\{\mathbf{y}|\mathbf{x}\}$. For zero-mean, jointly normal $\mathbf{y}$ and $\mathbf{x}$, the optimal estimator is strictly linear, that is [9] [10]

$$\hat{\mathbf{y}} = \mathbf{A}\mathbf{x} \tag{1}$$

where $\mathbf{A} = \mathbf{R}_{\mathbf{yx}}\mathbf{R}_{\mathbf{x}}^{-1}$ is a coefficient matrix, and $\mathbf{R}_{\mathbf{yx}} = E\{\mathbf{y}\mathbf{x}^T\}$. Standard, 'strictly linear' estimation in $\mathbb{C}$ assumes the same model but with complex valued $\mathbf{y}, \mathbf{x}$, and $\mathbf{A}$. Since both the real $\mathbf{y}_r$ and imaginary $\mathbf{y}_i$ parts of the vector $\mathbf{y}$ are real valued, we have

$$\hat{\mathbf{y}}_r = E\{\mathbf{y}_r|\mathbf{x}_r, \mathbf{x}_i\} \qquad \hat{\mathbf{y}}_i = E\{\mathbf{y}_i|\mathbf{x}_r, \mathbf{x}_i\} \tag{2}$$

Substituting in $\mathbf{x}_r = (\mathbf{x} + \mathbf{x}^*)/2$ and $\mathbf{x}_i = (\mathbf{x} - \mathbf{x}^*)/2\jmath$ yields [11]

$$\hat{\mathbf{y}}_r = E\{\mathbf{y}_r|\mathbf{x}, \mathbf{x}^*\} \qquad \hat{\mathbf{y}}_i = E\{\mathbf{y}_i|\mathbf{x}, \mathbf{x}^*\} \tag{3}$$

where $(\cdot)^*$ is the complex-conjugate operator. Hence, we obtain the *widely linear* complex estimator[1]

$$\hat{\mathbf{y}} = E\{\mathbf{y}|\mathbf{x}, \mathbf{x}^*\} = \mathbf{H}\mathbf{x} + \mathbf{G}\mathbf{x}^* = \mathbf{W}\mathbf{x}^a \tag{4}$$

[1]The 'widely linear' model is associated with the signal generating system, whereas "augmented statistics" describe statistical properties of measured signals. Both the terms 'widely linear' and 'augmented' are used to name the resulting algorithms - in our work we mostly use the term 'augmented'.

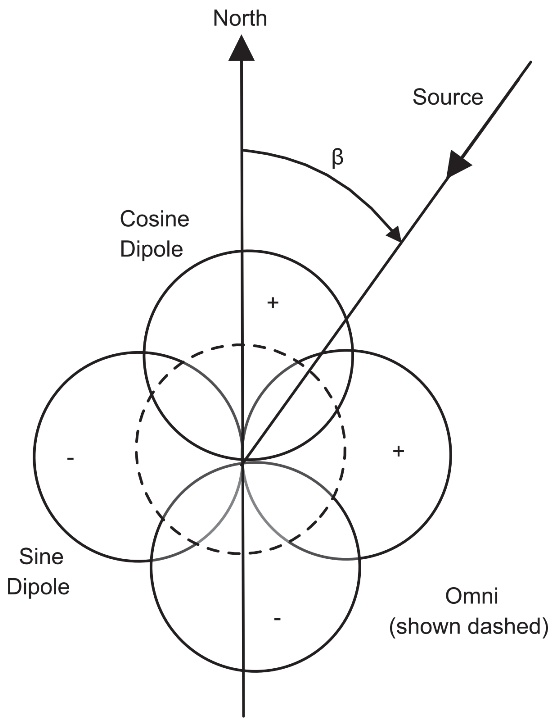

Fig. 1. A geometric view of the three sonobouy sensors (top view).

The matrix $\mathbf{W}$ comprises the coefficient matrices $\mathbf{H}$ and $\mathbf{G}$, and $\mathbf{x}^a = [\mathbf{x}^T, \mathbf{x}^H]^T$ is the augmented input vector, where $[\cdot]^T$ and $[\cdot]^H$ are the transpose and complex conjugate-transpose operators, respectively. The full second-order information is thus contained in the augmented covariance matrix

$$\mathbf{R}^a_{\mathbf{x}} = E\{\mathbf{x}^a\mathbf{x}^{aH}\} = \begin{bmatrix} \mathbf{R}_{\mathbf{x}} & \mathbf{P}_{\mathbf{x}} \\ \mathbf{P}^*_{\mathbf{x}} & \mathbf{R}^*_{\mathbf{x}} \end{bmatrix} \tag{5}$$

and as such, estimation based on $\mathbf{R}^a_{\mathbf{x}}$ incorporates both the covariance $\mathbf{R}_{\mathbf{x}} = E\{\mathbf{x}\mathbf{x}^H\}$ and pseudocovariance $\mathbf{P}_{\mathbf{x}} = E\{\mathbf{x}\mathbf{x}^T\}$, and provides the complete second order characterisation of complex signals [12] [6]. Complex signals with vanishing pseudocovariances, that is $\mathbf{P}_{\mathbf{x}} = \mathbf{0}$, are termed second order circular (proper), and are characterised by rotation invariant probability distributions, otherwise, the signals are noncircular (improper), and requires widely linear estimation for optimal performance.

III. New State Space Formulation

Figure 1 illustrates the arrangement of the sonobuoy sensors for a source at bearing β, the crossed-dipole sensor observes the following three waveforms [4]

$$y_{o,k} = s_k + v_{o,k} \tag{6a}$$
$$y_{c,k} = s_k \cos[\beta] + v_{c,k} \tag{6b}$$
$$y_{s,k} = s_k \sin[\beta] + v_{s,k} \tag{6c}$$

where the subscripts o, c and s denote the omni, cosine and sine channels respectively, while s_k is the signal emitted by the source (target) at time instant k, and $v_{o,k}$, $v_{c,k}$ and $v_{s,k}$ are the uncorrelated, zero-mean, observation noises. . In the standard arctangent bearing estimator, the discrete Fourier transform (DFT) of the observation signals are taken, and the frequency domain representation of the equations above assume the following forms [4]

$$Y_{o,\omega} = S_\omega + V_{o,\omega} \tag{7a}$$
$$Y_{c,\omega} = S_\omega \cos[\beta] + V_{c,\omega} \tag{7b}$$
$$Y_{s,\omega} = S_\omega \sin[\beta] + V_{s,\omega} \tag{7c}$$

where ω is the frequency argument. A number of data snapshots or observations (M), are collected before taking the DFT, and the source bearing β is inherently assumed to be constant over this observation period. In the standard arctangent estimator, the target bearing is estimated as

$$\hat{\beta} = \arctan[\hat{s}/\hat{c}] \tag{8}$$

where the variables $\hat{c}$ and $\hat{s}$ are computed using the M observations, that is

$$\hat{c} = \Re\Big\{ \sum_{m=1}^{M} Y^{(m)}_{c,\omega} Y^{(m),*}_{o,\omega} \Big\} \tag{9}$$

$$\hat{s} = \Re\Big\{ \sum_{m=1}^{M} Y^{(m)}_{s,\omega} Y^{(m),*}_{o,\omega} \Big\} \tag{10}$$

The superscript m is in the range $1 \leq m \leq M$ and denotes the mth Fourier bin, while $\Re\{\cdot\}$ is the real part of a complex quantity. Observe that the variables $\hat{c}$ and $\hat{s}$ may alternatively be estimated in the time domain (without taking Fourier transforms) as shown in [1].

The arctangent estimator is essentially based on the time (or frequency) averaged products (correlations) of the omni directional sensor $y_{o,k}$ with the outputs from the sine and cosine sensors, $y_{s,k}$ and $y_{c,k}$. It does not attempt to cater for the dynamics of the source signal s_k, and deals with the individual observations (or frequency bins) independently of each other.

However, it is possible to model or exploit possible transitional (correlation) properties in the source signal s_k, which can be inferred from the M available observations, and updated online. For this purpose, we here propose utilising a random-walk (first order Markov) modeling of the signal s_k, that is

$$s_k = s_{k-1} + w_k \tag{11}$$

where w_k is the driving noise, together with an augmented complex state space formulation to address the bearing estimation problem, which takes on the following form

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{w}_k \tag{12}$$
$$\mathbf{y}_k = \mathbf{h}[\mathbf{x}_k] + \mathbf{v}_k \tag{13}$$

where $\mathbf{x}_k$ is the state vector to be estimated, $\mathbf{y}_k$ the noisy observation, $\mathbf{h}[\cdot]$ the nonlinear observation function, while $\mathbf{w}_k$

and $\mathbf{v}_k$ are respectively the state and observation noises with covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$ [13]. The state equation (12) can be explicitly expressed as

$$\underbrace{\begin{bmatrix} s_k \\ z_k \\ z_k^* \end{bmatrix}}_{\mathbf{x}_k} = \underbrace{\begin{bmatrix} s_{k-1} \\ z_{k-1} \\ z_{k-1}^* \end{bmatrix}}_{\mathbf{x}_{k-1}} + \underbrace{\begin{bmatrix} w_k \\ e_k \\ e_k^* \end{bmatrix}}_{\mathbf{w}_k} \tag{14}$$

where $z_k = \cos[\beta] + j\sin[\beta] = e^{j\beta}$, and e_k is the state noise used to model nonstationary bearings β. Similarly, the observation equation in (13) takes the form

$$\underbrace{\begin{bmatrix} y_{o,k} \\ u_k \\ u_k^* \end{bmatrix}}_{\mathbf{y}_k} = \underbrace{\begin{bmatrix} s_k \\ s_k z_k \\ s_k z_k^* \end{bmatrix}}_{\mathbf{h}[\mathbf{x}_k]} + \underbrace{\begin{bmatrix} v_{o,k} \\ n_k \\ n_k^* \end{bmatrix}}_{\mathbf{v}_k}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & s_k & 1 \\ 0 & 0 & s_k \end{bmatrix} \begin{bmatrix} s_k \\ z_k \\ z_k^* \end{bmatrix} + \begin{bmatrix} v_{o,k} \\ n_k \\ n_k^* \end{bmatrix} \tag{15}$$

where $u_k = y_{c,k} + jy_{s,k}$ is the complex representation of the sine and cosine observations channels from (6a), and $n_k = v_{c,k} + jv_{s,k}$ is the corresponding noise.

The augmented (widely linear) state space model in (12) and (13) is nonlinear, and can be used in conjunction with a number of algorithms to estimate the source bearing, including the augmented complex extended and unscented Kalman filters as well as the augmented complex particle filter, [6].

A. Noise Statistics

In state space estimation we need to specify the second order statistics of the state and observation noises. To that end, given the observation noise variance of the omni channel, that is, $E\{v_{o,k}v_{o,k}^*\}$, the variances of the other two observation noises, $v_{c,k}$ and $v_{s,k}$, are given by

$$E\{v_{c,k}v_{c,k}^*\} = E\{v_{s,k}v_{s,k}^*\} = \frac{1}{\gamma}E\{v_{o,k}v_{o,k}^*\} \tag{16}$$

where γ is the noise gain of either dipole, whereby $\gamma = 1/2$ or $\gamma = 1/3$ for 2D-isotropic or 3D-isotropic noise respectively [1], while the variance of the complex observation noise is $E\{n_k n_k^*\} = E\{v_{c,k}v_{c,k}^*\} + E\{v_{s,k}v_{s,k}^*\}$.

Therefore, the noise statistics to be computed are $E\{v_{o,k}v_{o,k}^*\}$ and $E\{w_k w_k^*\}$, and can be estimated online as follows. We start by forming a new variable defined as the difference between two consecutive omni channel observations, that is

$$r_k = y_{o,k} - y_{o,k-1} \tag{17}$$

then assuming that both noise processes, w_k and $v_{o,k}$, are white, it is can be shown that

$$E\{r_k r_k^*\} = 2E\{v_{o,k}v_{o,k}^*\} + E\{w_k w_k^*\} \tag{18}$$

and that the correlation between r_k and r_{k-1} becomes

$$E\{r_k r_{k-1}^*\} = -E\{w_k w_k^*\} \tag{19}$$

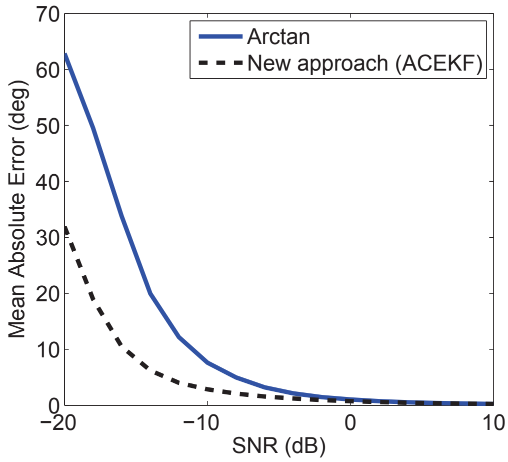

Fig. 2. Performance comparison between the proposed augmented complex state space approach and the arctan estimator for the case where the target source signal is a sinusoid.

therefore, from (18) and (19), we obtain

$$E\{w_k w_k^*\} = -E\{r_k r_{k-1}^*\} \tag{20}$$

$$E\{v_{o,k}v_{o,k}^*\} = \frac{E\{r_k r_k^*\} - E\{w_k w_k^*\}}{2} \tag{21}$$

Hence, the state and observation noise statistics of the state space model described by (14) and (15), can be estimated and tracked online based on the observation data.

Remark #1: The state space formulation of the problem enables tracking of the source (target) bearing in real-time, that is, the bearing estimate can be updated with every new observation.

Remark #2: The random-walk model in conjunction with the preprocessing of the observation data (when computing the noise variances), allows for some of the correlation structure of the source signal to be incorporated into the state space model, even when the true source signal does not follow a random-walk model.

IV. Simulations

To illustrate the potential of our augmented complex state space based solution for sonobuoy bearing estimation, we considered examples where the source signal s_k is modeled as a sinusoid (as in [4]) and as a first order autoregressive process. The augmented complex extended Kalman filter (ACEKF) is used to implement the approach described above and is compared with the standard arctangent (arctan) bearing estimator. In all the simulations, both the arctan and ACEKF algorithms utilise $M = 1024$ observations to estimate the bearing.

A. Signal Model: Sinusoid

Consider the case where the signal is a sinusoid, that is

$$s_k = \cos[2\pi fTk] + n_k \tag{22}$$

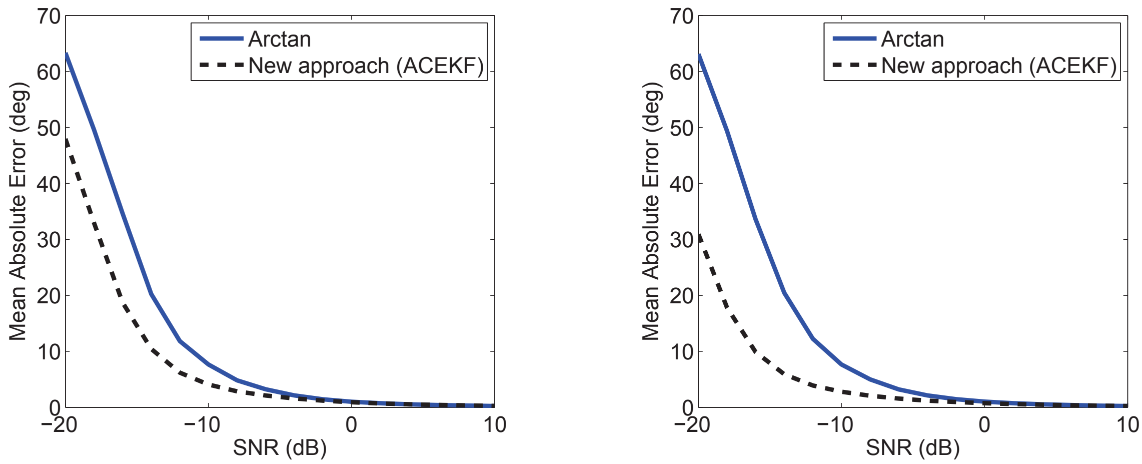

Fig. 3. Performance comparison between the proposed augmented complex state space approach and the arctan estimator for the case where source signal is an autoregressive process with (a) a Gaussian; (b) a uniform driving noise.

with a frequency of $f = 50Hz$, sampled at a rate of $f_s = \frac{1}{T} = 10kHz$.

Figure 2 shows the superior performances of the proposed ACEKF based solution compared with the arctan estimator, for the case where the source signal is a pure sinusoid, illustrated in Figure 2a. The results show that the proposed technique was able to outperform the arctan algorithm for low signal to noise (SNR) levels, while the two algorithms had similar performances for SNRs greater than $0dB$.

B. Signal Model: Autoregressive

We next modelled the source signal as a first order autoregressive process, that is

$$s_k = 0.9s_{k-1} + n_k$$

where n_k is either a white Gaussian or uniform driving noise.

The results are shown in Figure 3, where again the new ACEKF based algorithm achieved a lower bearing estimation error than the arctan estimator, for both Gaussian and uniform driving noises. Observe that the performance of the arctan estimator was similar in all the simulations, while the performance of the proposed approach was superior because fully exploits the correlation structure of the signals.

V. Conclusion

In this paper, we have proposed a new augmented (widely linear) complex state space solution for the DIFAR sonobuoy bearing estimation problem, with the aim of catering for the correlations in target source signals. This was archived through random-walk modelling of the source signal. It has been shown that the second order statistics of the state and observation noises can be estimated and updated online using the observation data; this together with the augmented state space model nature of our solution enables online tracking of target bearings. The superiority of the proposed approach over the standard arctan bearing estimator has been illustrated for the cases where the source signals are sinusoidal or autoregressive processes.

References

[1] S. Davies, “Bearing accuracies for arctan processing of crossed dipole arrays,” in *Proc. Oceans Conference*, pp. 351 –356, 1987.

[2] P. K. Tam and K. T. Wong, “Cramer-Rao bounds for direction finding by an acoustic vector sensor under nonideal gain-phase responses, noncollocation, or nonorthogonal orientation,” *IEEE Sensors Journal*, vol. 9, pp. 969 –982, Aug. 2009.

[3] J. Georgy, A. Noureldin, and G. R. Mellema, “Clustered mixture particle filter for underwater multitarget tracking in multistatic active sonobuoy systems,” *IEEE Transactions on Systems, Man, and Cybernetics, Part C Applications and Reviews*, vol. 42, pp. 547 –560, July 2012.

[4] B. H. Maranda, “The statistical accuracy of an arctangent bearing estimator,” in *Proc. Oceans Conference*, vol. 4, pp. 2127 –2132, 2003.

[5] A. R. Runnalls, “Likelihood function for a simple cardioid sonobuoy,” *IEE Proceedings Radar, Sonar and Navigation*, vol. 153, pp. 417 –426, Oct. 2006.

[6] D. H. Dini and D. P. Mandic, “A class of widely linear complex Kalman filters,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, pp. 775 –786, May 2012.

[7] D. H. Dini and D. P. Mandic, “Widely linear complex extended Kalman filters,” in *Sensor Signal Processing for Defence Conference*, 2011.

[8] D. P. Mandic and V. S. L. Goh, *Complex Valued Nonlinear Adaptive Filters: Noncircularity, Widely Linear and Neural Models*. John Wiley and Sons Ltd, 2009.

[9] A. van den Bos, “The multivariate complex normal distribution-a generalization,” *IEEE Transactions on Information Theory*, vol. 41, pp. 537 –539, Mar 1995.

[10] B. Picinbono, “Second-Order Complex Random Vectors and Normal Distributions,” *IEEE Transactions on Signal Processing*, vol. 44, no. 10, pp. 2637–2640, 1996.

[11] D. P. Mandic, S. Still, and S. C. Douglas, “Duality between widely linear and dual channel adaptive filtering,” in *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, pp. 1745 –1748, 2009.

[12] B. Picinbono and P. Bondon, “Second-order Statistics of Complex Signals,” *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 411–420, 1997.

[13] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall International, Inc, 1993.