Power Efficiency of Decode-And-Forward Cooperative Relaying

Zhengguo Sheng*, Bong Jun Ko†, Ananthram Swami‡, Kang-Won Lee†, and Kin K Leung*
*Department of Electrical and Electronic Engineering, Imperial College, UK
†IBM T. J. Watson Research Center, NY, USA
‡U.S. Army Research Laboratory (ARL), Adelphi, MD, USA
{zhengguo.sheng06, kin.leung}@imperial.ac.uk, {bongjun_ko, kangwon}@us.ibm.com, ananthram.swami@us.army.mil

***Abstract*—We investigate fundamental characteristics of cooperative transmission in terms of power efficiency. By introducing the concept of "cooperative region", we evaluate the average power efficiency which is defined as the ratio of total consumed transmit power with cooperation to that of direct transmission and show how the average performance depends upon the QoS requirement, distance between source and destination and on node density. Further, we propose a dynamic cooperation scheme that combines both cooperative and direct transmission. Analytical results are supplemented by simulation results to demonstrate the energy saving of cooperation transmission.**

I. Introduction

Cooperative communication mechanisms [1]–[5] have been proposed as an effective way of exploiting spatial diversity to improve the quality of wireless links. The key idea is to have multiple wireless devices in different locations cooperatively share their antenna resources and aid each other's wireless transmission, forming virtual and distributed antenna arrays, and as a result, the overall quality of the wireless transmission, in terms of the reception reliability [2], [6], [7], the communication range [8], and power consumption [9], [10], can be improved significantly.

We explore a fundamental aspect of cooperative communications (CC): *power consumption*. Since the participation of a wireless device in other devices' transmissions is critical in cooperative communication, it is of fundamental importance to understand how much energy each participant is required to consume in order to achieve the full benefit of CC. *Our focus is on the energy saving aspect of CC*, and as such, we want to answer the following fundamental questions: *whether CC can save energy, and if so, under what conditions, and how much*, given a desired quality of the wireless link. The Decode-And-Forward (DAF) cooperative protocol considered in this paper is similar to that in [11], [12], where at least one relay is always employed. In contrast, we consider an adaptive version of DAF, which reverts back to direct transmission if the relay cannot decode successfully. More specifically, we investigate power consumption, using at most one relay node as shown in Figure 1. It is shown in [2] that such a cooperative protocol can achieve full second-order diversity and therefore provide significant improvement to reception reliability. As our interest is solely in the power consumption aspects, we assume that solutions to other practical issues in realizing CC are in place (for example, medium access [13], channel state estimation [14]), which are outside the scope of this paper.

This research was sponsored by US Army Research laboratory and the UK Ministry of Defence and was accomplished under Agreement Number W911NF-06-3-0001. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the US Army Research Laboratory, the U.S. Government, the UK Ministry of Defense, or the UK Government. The US and UK Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

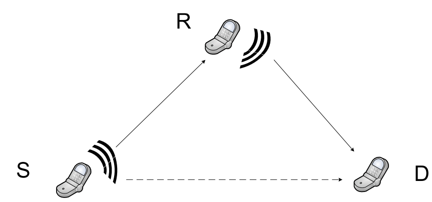


Fig. 1. An example of a wireless cooperative link

Our focus is on characterizing the power consumption of DAF over direct transmission, subject to given QoS requirements. Our contribution is three-fold: first, we provide a closed form solution for the minimum total transmission power required for DAF in a Rayleigh fading channel, subject to given QoS requirements. Second, we analyze the condition under which CC is preferable to direct transmission, and characterize the geometric constraints (which we call the *cooperative region*) on the location of the relay (relative to those of the source and the destination) that lead to lower power consumption. Using the concept of the cooperative region, we provide a probabilistic analysis of the expected energy saving obtained by CC. This is expressed as a function of the node distances, the QoS parameters, and the density of the relays, where the potential relays are assumed to be Poisson distributed. Third, with a better understanding of CC, we propose an dynamic cooperation scheme, in which relay selection is based on the availability of relays and the quality of the links.

II. System Model and Power Consumption

We consider a cooperative network as shown in Figure 1: source node (S), destination node (D), and a relay node (R), which overhears S's transmission to D, retransmits or *relays* the received signal to D, improving the reception quality of the (combined) signal at D. We assume that all nodes use the same transmit power. Our scheme employs two transmission slots: In the first time slot, the source broadcasts its data to the relay and the destination. In the second time slot, the relay transmits the signal it received in the previous time slot, if the SNR exceeds a threshold; otherwise, the source retransmits the signal. Thus an ACK from relay to source is assumed. Two time slots are used to transmit and relay a given data signal to avoid RF capture effects when simultaneously transmitting and receiving in the same frequency band. As a result, the destination receives two independent copies of the same packets transmitted through different wireless channels. Diversity gain can be achieved by combining the data copies using one of a variety of combining techniques, e.g., Maximum Ratio Combining (MRC) where the received signals are weighted with respect to their SNR and then summed together.

Our channel model incorporates path loss and Rayleigh fading. The received signal at node j is modeled as

$$y_j = \mathsf{a}_{ij} x_i + n_j \tag{1}$$

where x_i is the signal transmitted by node i and n_j is additive white Gaussian noise, with variance σ_n^2, at the receiver. The channel gain a_{ij} between the nodes i and j is modelled as $\mathsf{a}_{ij} = h_{ij}/d_{ij}^{\alpha/2}$, where d_{ij} is the distance between the nodes i and j, α is the path-loss exponent. The channel fading parameter h_{ij} is assumed to be complex Gaussian with zero mean and unit variance, and independent and identically distributed (i.i.d.) across time slots and across links.

We derive the total power consumption of the source and the relay, required to satisfy given quality of service (QoS) requirement of the link. The QoS requirement is expressed by a tuple (R, p^{out}), where R is the desired data rate in bits/s/Hz and p^{out} is the outage probability defined by the probability of channel capacity being smaller than the rate R.

A. Direct transmission

To establish baseline performance, we first consider direct transmission. The channel capacity between a source S and a destination D is

$$I_D = \log(1 + P|\mathsf{a}_{s,d}|^2) \tag{2}$$

where $P = E_b/N_0$ is defined as the transmission power normalized by noise power. Since for Rayleigh fading, $|\mathsf{a}_{s,d}|^2$ is exponentially distributed with parameter $d_{s,d}^{\alpha}$, the outage probability satisfies

$$\begin{aligned} p_D^{out} = \Pr[I_D < R] &= 1 - \exp\left(-\frac{(2^R-1)d_{s,d}^{\alpha}}{P}\right) \\ &\approx d_{s,d}^{\alpha}\left(\frac{2^R-1}{P}\right) \end{aligned} \tag{3}$$

for large P. Here R is the desired data rate in bit/s/Hz, which is defined by the quality of service (QoS) requirement. We then have the normalized transmission power for direct transmission

$$P_D = d_{s,d}^{\alpha}\left(\frac{2^R-1}{p_D^{out}}\right). \tag{4}$$

B. DAF cooperative transmission

Let $d_{s,d}$, $d_{s,r}$ and $d_{r,d}$ be the distances among the source, relay and destination. During the first time slot, the destination receives $y_{d,1} = \frac{h_{s,d}}{d_{s,d}^{\alpha/2}} x_s + n_d$ from the source. During the second time slot, the destination node receives

$$y_{d,2} = \begin{cases} \frac{h_{s,d}}{d_{s,d}^{\alpha/2}} x_s + n_d, & \text{if } |\frac{h_{s,r}}{d_{s,r}^{\alpha/2}}|^2 < f(P) \\ \frac{h_{r,d}}{d_{r,d}^{\alpha/2}} x_r + n_d, & \text{if } |\frac{h_{s,r}}{d_{s,r}^{\alpha/2}}|^2 \geq f(P) \end{cases} \tag{5}$$

where $f(P) = (2^{2R} - 1)/P$ can be derived analogous to (3). In this protocol, the relay transmits only if the SNR exceeds a threshold; otherwise, the source retransmits in the second time slot. We thus implicitly assume a mini-slot at the beginning of the second slot during which ACKs are sent error-free from relay to source.

Assuming that the relay node can perform perfect decoding when the received SNR exceeds a threshold, the channel capacity of this cooperative link can be shown to be

$$I_C = \begin{cases} \frac{1}{2}\log(1 + 2P|\mathsf{a}_{s,d}|^2), & |\mathsf{a}_{s,r}|^2 < f(P) \\ \frac{1}{2}\log(1 + P|\mathsf{a}_{s,d}|^2 + P|\mathsf{a}_{r,d}|^2), & |\mathsf{a}_{s,r}|^2 \geq f(P) \end{cases} \tag{6}$$

Note that the same noise variance is assumed at both relay and destination. Therefore, the outage probability of cooperative transmission becomes

$$\begin{aligned} & p_C^{out} = \Pr[I_C < R] \\ & = \Pr[|\mathsf{a}_{s,r}|^2 < f(P)]\Pr[2|\mathsf{a}_{s,d}|^2 < f(P)] \\ & + \Pr[|\mathsf{a}_{s,r}|^2 \geq f(P)]\Pr[|\mathsf{a}_{s,d}|^2 + |\mathsf{a}_{r,d}|^2 < f(P)] \end{aligned} \tag{7}$$

By computing the high SNR limit, we obtain from (7)

$$\begin{aligned} \frac{1}{f^2} p_C^{out} = & \underbrace{\frac{1}{f}\Pr[|\mathsf{a}_{s,r}|^2 < f]}_{\mathbf{T1}} \underbrace{\frac{1}{f}\Pr[2|\mathsf{a}_{s,d}|^2 < f]}_{\mathbf{T2}} \\ & + \underbrace{\Pr[|\mathsf{a}_{s,r}|^2 \geq f]}_{\mathbf{T3}} \underbrace{\frac{1}{f^2}\Pr[|\mathsf{a}_{s,d}|^2 + |\mathsf{a}_{r,d}|^2 < f]}_{\mathbf{T4}} \end{aligned} \tag{8}$$

where $f = f(P)$, $\mathbf{T1} \longrightarrow d_{s,r}^{\alpha}$, $\mathbf{T2} \longrightarrow d_{s,d}^{\alpha}/2$, $\mathbf{T3} \longrightarrow 1$, $\mathbf{T4} \longrightarrow d_{s,d}^{\alpha} d_{r,d}^{\alpha}/2$. Because $f(P) = (2^{2R} - 1)/P$, we obtain a closed-form expression for the outage probability between the source and the destination

$$p_C^{out} = \frac{1}{2} d_{s,d}^{\alpha}(d_{s,r}^{\alpha} + d_{r,d}^{\alpha}) \frac{(2^{2R}-1)^2}{P^2} \tag{9}$$

Hence the total normalized power consumption for DAF cooperation is

$$P_{\text{DAF}} = 2P_C = 2\sqrt{\frac{1}{2} d_{s,d}^{\alpha}(d_{s,r}^{\alpha} + d_{r,d}^{\alpha}) \frac{(2^{2R}-1)^2}{p_C^{out}}}. \tag{10}$$

It is worth noting that for a fair comparison with direct transmission using only one time slot, cooperative transmission actually employs twice the date rate at $2R$ during two consecutive time slots, so that both schemes have the same effective data rate.

III. Cooperative Region for DAF

In this section, we establish the conditions under which our cooperative transmission scheme performs better than direct transmission in terms of the energy efficiency and analyze the geometric properties of the conditions with respect to various parameters.

Given the locations of the source and the destination, we define the *cooperative region* as the geometric region of the location of the relay within which the ratio $\beta = \frac{P_{\text{DAF}}}{P_D}$ is smaller than 1. We will often refer to β as an efficiency factor, so one should keep in mind that small values of β are preferable. According to (4) and (10), the cooperative region is defined by

$$\beta = \frac{P_{\text{DAF}}}{P_D} = \frac{\sqrt{d_{s,r}^{\alpha} + d_{r,d}^{\alpha}}(2^R+1)\sqrt{2p^{out}}}{\sqrt{d_{s,d}^{\alpha}}} < 1 \qquad (11)$$

Further defining a QoS factor $K = \frac{1}{(2^R+1)\sqrt{2p^{out}}}$, the boundary of the cooperative region is defined by

$$d_{s,r}^{\alpha} + d_{r,d}^{\alpha} = K^2 d_{s,d}^{\alpha} \qquad (12)$$

Consider the Cartesian Coordinate sysem shown in Figure 2, with relay at (x, y), source at $(-\frac{d_{s,d}}{2}, 0)$ and destination at $(\frac{d_{s,d}}{2}, 0)$. Then (12) yields

$$[(x+\frac{d_{s,d}}{2})^2 + y^2]^{\frac{\alpha}{2}} + [(x-\frac{d_{s,d}}{2})^2 + y^2]^{\frac{\alpha}{2}} = K^2 d_{s,d}^{\alpha} \qquad (13)$$

Note that the cooperative region is determined by the QoS factor K, the source-destination distance $d_{s,d}$, and the path loss exponent α. In what follows, we analyze the characteristics of the cooperative region w.r.t. these parameters, starting with the special cases of $\alpha = 1$ and $\alpha = 2$.

A. Path loss exponent $\alpha = 1$

It is possible to have a path loss exponent smaller than 2 when there is a waveguide effect, such as in underwater acoustic communications or beamforming. Consider an extreme case $\alpha = 1$ for which the boundary of the cooperative region is

$$d_{s,r} + d_{r,d} = K^2 d_{s,d} \qquad (14)$$

Thus, the cooperative region is an ellipse in canonical form with foci located at the source and destination and can be described through the canonical equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (15)$$

where $a = \frac{K^2 d_{s,d}}{2}$ and $b = \frac{\sqrt{K^4-1}d_{s,d}}{2}$. The area of the cooperative region is $\mathcal{A} = \pi ab$.

B. Path loss exponent $\alpha = 2$

According to (12), we have

$$d_{s,r}^2 + d_{r,d}^2 = K^2 d_{s,d}^2 \qquad (16)$$

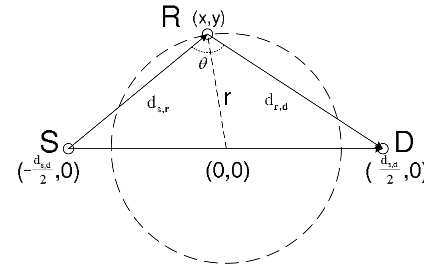


Fig. 2. Geometric analysis for path loss $\alpha = 2$

The cooperative region is a circle and the foci coincide with the origin. With r denoting the distance of the relay from the origin, we have using (13)

$$x^2 + y^2 = r^2, \quad d_{s,r}^2 + d_{r,d}^2 = 2r^2 + \frac{d_{s,d}^2}{2} = K^2 d_{s,d}^2 \qquad (17)$$

Hence, the radius of the cooperative region satisfies

$$2\hat{r}^2 + \frac{d_{s,d}^2}{2} = K^2 d_{s,d}^2 \quad \Rightarrow \quad \hat{r} = d_{s,d}\sqrt{\frac{1}{2}(K^2 - \frac{1}{2})} \qquad (18)$$

and the area of the cooperative region is $\mathcal{A} = \pi \hat{r}^2$.

C. General path loss exponents

For other path loss exponents, e.g., $\alpha = 3$ or 4, we can use numerical analysis to characterize the shape of the cooperative region. Motivated by the case of $\alpha = 1$ or 2, it is natural to assume the cooperative region is a general ellipse which can be determined by minor and major radius, a and b. Setting $x = 0$, $y = b$ in (13), we can obtain parameter b explicitly as:

$$b = d_{s,d}\sqrt{\left(\frac{K^2}{2}\right)^{\frac{2}{\alpha}} - \frac{1}{4}}\ . \qquad (19)$$

Setting $y = 0$, $x = a$ in (13), we can obtain parameter a implicitly via

$$\left|a + \frac{d_{s,d}}{2}\right|^{\alpha} + \left|a - \frac{d_{s,d}}{2}\right|^{\alpha} = K^2 d_{s,d}^{\alpha} \qquad (20)$$

which can be solved numerically. Then the cooperative region can be defined, approximately, by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (21)$$

Figure 3 illustrates the curves obtained from (21) and simulation result, for $\alpha = 3$ and 4 when the data rate $R = 2$ bps/Hz, $p^{\text{out}} = 0.01$ and the source and destination are located at (10m, 0) and (-10m, 0), respectively, the two curves are seen to overlap exactly. Moreover, we observe the same in the numerical results for different α, R, and p^{out}, indicating that the approximation of the cooperative region by an ellipse is very accurate.

From the above analysis as well as the simulation results in Figure 5 (a), we see that the cooperative region, which is a circle for $\alpha = 2$, gets elongated along the x-axis for $\alpha < 2$ and along the y-axis for $\alpha > 2$. Even within the cooperative region, different relays could have different power ratios and we have the following result on the best relay location.

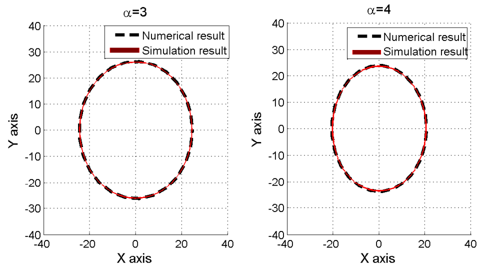


Fig. 3. Cooperative region specified by (12) and its ellipse approximation, $\alpha = 3$ and 4

Result 1: For $\alpha > 1$, the best relay location for DAF cooperation is midway between source and destination.

Proof: The best power efficiency can be achieved when the left hand side of (13) is minimum, and for any x-coordinate of the relay location, $d_{s,r}$ and $d_{r,d}$ is minimum at $y = 0$. Setting $y = 0$, we can obtain

$$f(x) = |x + \frac{d_{s,d}}{2}|^\alpha + |x - \frac{d_{s,d}}{2}|^\alpha \tag{22}$$

Obtaining the first order derivative $f'(x) = \alpha(x + \frac{d_{s,d}}{2})^{\alpha-1} - \alpha(\frac{d_{s,d}}{2} - x)^{\alpha-1}$ for $-\frac{d_{s,d}}{2} < x < \frac{d_{s,d}}{2}$, we have $f'(0) = 0$. Moreover, it is not difficult to observe that $f'(x) > 0$ for $x > 0$, and due to symmetry of $f(x)$, we have the similar result $f'(x) < 0$ for $x < 0$. This shows that $f(x)$ monotonically decreases for $x < 0$ and monotonically increases for $x > 0$, and hence $f(x)$ is minimum at $x = 0$. $\square$

Notice that for $\alpha = 1$, $f(x)$ in (22) is constant over $-\frac{d_{s,d}}{2} < x < \frac{d_{s,d}}{2}$. The first order derivative of $f(x)$ is 0, and all points on the line segment between source and destination can achieve the minimum value.

Result 2: The minimum K (QoS factor) to guarantee the existence of the cooperation region is $\sqrt{2^{1-\alpha}}$, i.e., $p^{out} < 1/[(2^R + 1)^2 2^{2-\alpha}]$.

Proof: From Result 1, the left hand side of (13) gives the minimum when $x = 0$ and $y = 0$, then we have the right hand side of (13) satisfying $K^2 d_{s,d}^\alpha \geq 2(\frac{d_{s,d}}{2})^\alpha$. Therefore, we can obtain $K \geq \sqrt{2^{1-\alpha}}$. $\square$

Thus, DAF is useful when low outage is required.

Result 3: The area of the cooperative region depends on the QoS factor $K = ((2^R + 1)\sqrt{2p^{out}})^{-1}$, the path loss exponent α and transmission distance $d_{s,d}$, and is bounded[1] by

$$\pi \left[\left(\frac{K^2}{2} \right)^{\frac{1}{\alpha}} - \frac{1}{2} \right]^2 d_{s,d}^2 < \mathcal{A}(\alpha) < \pi \left(\frac{K^2}{2} \right)^{\frac{2}{\alpha}} d_{s,d}^2 \tag{23}$$

Proof: From (19), we obtain $b < d_{s,d}(\frac{K^2}{2})^{\frac{1}{\alpha}}$. From (20), we obtain $a > d_{s,d}(\frac{K^2}{2})^{\frac{1}{\alpha}} - \frac{d_{s,d}}{2}$. Note that the lower and upper bound are given by a circle with radii a and b, respectively.

The area of the cooperative region given by the ellipse with radii a and b is bounded by $\pi a^2 < \mathcal{A}(\alpha) < \pi b^2$. $\square$

Figure 4 shows the area of the cooperative region, obtained via numerical evaluation of (12) vs. $K^{4/\alpha}$. The linear relationship seen in the curve verifies the theoretical result in Result 3 and confirms that the elliptical approximation is very accurate.

[1]The lower bound is only valid when $\alpha > 2$.

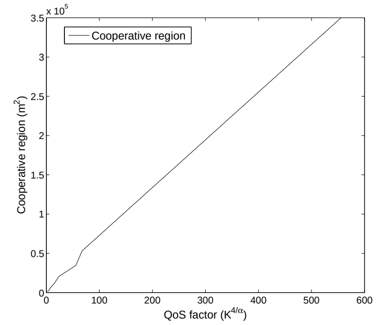


Fig. 4. Area of cooperative region versus QoS factor

In essence, the size of the cooperative region increases as the path loss exponent, targeted data rate or outage probability decreases. Moreover, a longer transmission distance between the source and destination also indicates an extended opportunity for benefiting from the cooperation when the link condition between the source and the destination is poor.

IV. Average Power Efficiency of DAF

In this section, we further investigate how much transmission power can be saved by using cooperative transmission and propose a dynamic cooperation scheme. We assume that relay candidates are randomly located in space according to a Poisson point process with density λ. A source-destination pair will choose the best relay node to achieve the minimum total transmission power among all available relay candidates, where the best relay is the one that results in the best efficiency factor provided in (11). A network with a higher density of relay nodes can provide better choices for relay selection.

A. Average Power Efficiency for $\alpha = 2$

When the path loss exponent $\alpha = 2$, the selected relay to achieve the minimum β will be as close as possible to the origin (0, 0). We let r^* be a random variable of the selected relay distance to the destination and r denote the distance between the closest relay and the destination. The probability distribution function of r is given by

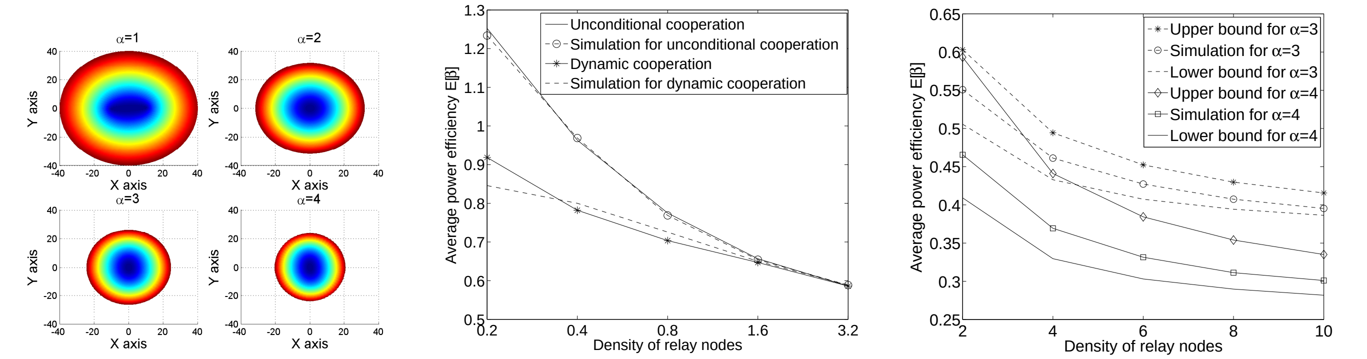
$$\begin{aligned} \Pr[r^* < r] &= 1 - \Pr[r^* \geq r] \\ &= 1 - \Pr[N_r = 0] = 1 - e^{-\lambda \pi r^2} \end{aligned} \tag{24}$$

where N_r is the number of relays within distance r from the origin. The probability density function (pdf) of the selected relay distance is

$$f(r) = 2\lambda\pi r e^{-\lambda\pi r^2}, \quad r \geq 0 \tag{25}$$

According to (11) and (17), the expected value of the power efficiency is

$$\mathrm{E}[\beta] = \mathrm{E}\left[\sqrt{\frac{d_{s,r}^\alpha + d_{r,d}^\alpha}{d_{s,d}^\alpha K^2}} \right] = \mathrm{E}\left[\frac{\sqrt{2}}{K} \sqrt{\frac{1}{4} + \frac{r^2}{d_{s,d}^2}} \right] \tag{26}$$



(a) Cooperative regions for different path loss exponents

(b) Average power efficiency for $\alpha = 2$

(c) Average power efficiency of unconditional cooperation for $\alpha = 3$ and $\alpha = 4$

Fig. 5. Simulation results

where the pdf of the random variable r is given by (25); hence

$$\mathrm{E}\left[\beta\right] = \frac{\sqrt{2}e^{\frac{\lambda\pi d_{s,d}^2}{4}}}{K\sqrt{\lambda\pi d_{s,d}^2}}\Gamma\left(\frac{3}{2}, \frac{\lambda\pi d_{s,d}^2}{4}\right). \tag{27}$$

where $\Gamma(\alpha, x) = \int_x^\infty e^{-t}t^{\alpha-1}dt$ is the incomplete gamma function. Details are provided in Appendix A. Notice that the parameter $\rho := \pi\lambda d_{sd}^2/4$ has a nice interpretation as the expected number of relays in a circle with diameter d_{sd}, the source-destination distance.

Result 4: The average power efficiency of DAF cooperation relative to direct transmission for $\alpha = 2$ is

$$\frac{1}{\sqrt{2}K}\sqrt{\frac{\pi}{4\rho}} < \mathrm{E}[\beta] < \frac{1}{\sqrt{2}K}\left(\sqrt{\frac{\pi}{4\rho}} + 1\right) \tag{28}$$

where λ is the density of the relay nodes and $d_{s,d}$ is the source-destination distance. Details are provided in Appendix B. It is worth noting that targeting a smaller outage probability or a longer distance can lead to better power efficiency, which means that cooperative transmission can better combat a harsher network environment.

B. General Path Loss Exponent

The average power efficiency for the general case is

$$\mathrm{E}\left[\beta\right] = \mathrm{E}\left[\frac{\sqrt{d_{s,r}^\alpha + d_{r,d}^\alpha}}{d_{s,d}^{\frac{\alpha}{2}}K}\right] \tag{29}$$

1) Geometric Lower Bound: [2]

$$\mathrm{E_L}\left[\beta\right] = \frac{\sqrt{2}e^{\frac{\lambda\pi d_{s,d}^2}{4}}}{d_{s,d}^{\frac{\alpha}{2}}K(\lambda\pi)^{\frac{\alpha}{4}}}\Gamma\left(\frac{\alpha+4}{4}, \frac{\lambda\pi d_{s,d}^2}{4}\right) \tag{30}$$

[2]Due to space limitations, we do not include the proof which may be found at http://www.commsp.ee.ic.ac.uk/~zs206/images/proof-milcom.pdf

2) Geometric Upper Bound:

$$\mathrm{E_U}\left[\beta\right] = \frac{\sqrt{\mathrm{E}[(\frac{d_{s,d}}{2}+r)^\alpha + (\frac{d_{s,d}}{2}-r)^\alpha]}}{d_{s,k}^{\frac{\alpha}{2}}K} \tag{31}$$

where $\mathrm{E}[(\frac{d_{s,d}}{2}+r)^\alpha] = 2\lambda\pi\int_0^\infty(\frac{d_{s,d}}{2}+r)^\alpha re^{-\lambda\pi r^2}dr$ and $\mathrm{E}[(\frac{d_{s,d}}{2}-r)^\alpha] = 2\lambda\pi\int_0^\infty(\frac{d_{s,d}}{2}-r)^\alpha re^{-\lambda\pi r^2}dr$.

C. Dynamic Cooperation scheme

We propose a dynamic cooperation scheme where cooperative transmission is only used if a relay is available within the cooperative region, otherwise, direct transmission is adopted. We compare its performance with unconditional cooperation where cooperative transmission is always adopted regardless of the location of the relay. Let $r_c := d_{s,d}\sqrt{\frac{1}{2}(K^2 - \frac{1}{2})}$. We can derive an expression for the mean power efficiency for the dynamic cooperation scheme

$$\mathrm{E}\left[\beta\right] = \hat{\mathrm{E}}\left[\beta\right]\Pr[N_{r_c} > 0] + 1\cdot\Pr[N_{r_c} = 0] \tag{32}$$

From (24) we have $\Pr[N_{r_c} = 0] = e^{-\pi\lambda r_c^2} = e^{-\delta}$, where $\delta := \pi\lambda r_c^2$. The expression for $\hat{\mathrm{E}}[\beta]$ is given in Appendix C. Note that the scheme requires knowledge of the relay locations.

V. Simulation Results

In this section, we provide simulation results to further illustrate the performance of cooperative transmission.

Figure 5 (a) shows the cooperative regions for different path loss exponents. We assume the data rate $R = 2$ bps/Hz, $p^{\mathrm{out}} = 0.01$ and the source and destination are located at (10m, 0) and (-10m, 0), respectively. The darker (blue) the colour is, the better the power efficiency (lower values of β) can be achieved. It is also clear that as the path loss exponent increases, the cooperative region becomes smaller.

Figure 5 (b) shows the performance of the dynamic cooperation scheme. The dynamic cooperation scheme can always guarantee better performance even when the node density is low. Moreover, theoretical results are seen to be very close to the simulation results. Figure 5 (c) shows the average power efficiency for other path loss exponents; it tells that the theoretical bounds in (30) and (31) well define the behavior of β for general path loss cases and furthermore we can observe that a larger path loss exponent can lead to better power efficiency.

VI. Conclusions

We have investigated some fundamental characteristics of cooperative transmission. We defined the notion of a cooperative region and analyzed the average power efficiency of DAF cooperative transmission. The cooperative region is an ellipse when the path loss exponent α is unity, a circle for $\alpha = 2$, and can be well approximated by an ellipse for $\alpha > 2$. The major radius can be obtained in closed-form, and the minor radius as the root of a non-linear equation. As may be expected, the best relay location for $\alpha \geq 1$ is midway between source and destination. We showed that cooperation can lead to energy savings only if the QoS parameter is larger than a threshold which depends upon α. Opportunities for cooperation increase in harsher environments: as the source-destination distance $d_{s,d}$, the data rate or the path loss exponent increase, or as the desired outage probability decreases. We established bounds on the average power efficiency due to cooperation in terms of the QoS parameter and $d_{s,d}$. Moreover, we will consider more realistic channel models, e.g., shadowing, in our future work to further explore the characteristic of CC.

Appendix A

According to (25), we have

$$\mu := \mathrm{E}\left[\sqrt{\frac{1}{4}+\frac{r^2}{d_{s,d}^2}}\right] = 2\lambda\pi\int_0^\infty \sqrt{\frac{1}{4}+\frac{r^2}{d_{s,d}^2}}re^{-\lambda\pi r^2}dr$$

Let $y = \frac{1}{4}+\frac{r^2}{d_{s,d}^2}$, then $\frac{2r}{d_{s,d}^2}dr = dy$, $rdr = \frac{d_{s,d}^2}{2}dy$ and $r^2 = d_{s,d}^2(y-\frac{1}{4})$, so that

$$\mu = \lambda\pi d_{s,d}^2 e^{\frac{\lambda\pi d_{s,d}^2}{4}}\int_{\frac{1}{4}}^\infty y^{\frac{1}{2}}e^{-\lambda\pi d_{s,d}^2 y}dy$$

Further let $\gamma = \lambda\pi d_{s,d}^2$, and $\gamma y = t$; then recalling the definition of the incomplete upper gamma function

$$\Gamma(u,x) := \int_x^\infty e^{-t}t^{u-1}dt$$

where $u > 0$, we have

$$\mu = \frac{e^{\frac{\lambda\pi d_{s,d}^2}{4}}}{\sqrt{\lambda\pi d_{s,d}^2}}\Gamma\left(\frac{3}{2},\frac{\lambda\pi d_{s,d}^2}{4}\right)$$

which establishes (27).

Appendix B

Let $\rho := \pi\lambda d_{sd}^2/4$. From the definition of the incomplete gamma function, we have

$$g := e^\rho\Gamma\left(\frac{3}{2},\rho\right) = \int_\rho^\infty t^{\frac{1}{2}}e^{\rho-t}dt$$

1. Upper bound:

$$g = \int_0^\infty(\rho+s)^{\frac{1}{2}}e^{-s}ds < \int_0^\infty(\rho^{\frac{1}{2}}+s^{\frac{1}{2}})e^{-s}ds = \rho^{\frac{1}{2}}+\Gamma(\frac{3}{2})$$

2. Lower bound:

$$g > \int_0^\infty t^{\frac{1}{2}}e^{-t}dt = \Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$$

Using the two bounds in (27) leads to (28).

Appendix C

Using (18), the expected power efficiency when the relay is available within the cooperative region can be derived similar to (27) as

$$\begin{aligned}\hat{\mathrm{E}}[\beta] &= \frac{2\sqrt{2}\lambda\pi}{K}\int_0^{r_c}\sqrt{\frac{1}{4}+\frac{r^2}{d_{s,d}^2}}re^{-\lambda\pi r^2}dr \\ &= \frac{1}{\sqrt{2}K}\frac{e^{\rho+\delta}}{\sqrt{\rho+\delta}}\Gamma(\frac{3}{2},\rho+\delta)-\frac{1}{\sqrt{2}K}\frac{e^\rho}{\sqrt{\rho}}\Gamma(\frac{3}{2},\rho)\end{aligned} \quad (33)$$

where r_c is defined in Section IV-C, $\rho = \pi\lambda d_{sd}^2/4$ was defined earlier, and $\delta := \pi\lambda r_c^2$.

References

[1] A. Scaglione, D. Goeckel, and J. N. Laneman, “Cooperative communications in mobile ad-hoc networks: Rethinking the link abstraction,” *IEEE Signal Processing Magazine*, vol. 23, pp. 18–29.

[2] J. N. Laneman, D. N. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. On Information Theory*, vol. 50(12), Dec. 2004.

[3] I. M. Gerhard Kramer and R. D. Yates, *Cooperative Communications*. Now Publishers Inc., Hanover, USA, 2006.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. part i. system description,” *IEEE Trans. on Communications*, vol. 51, no. 11, pp. 1927 – 1938, nov. 2003.

[5] A. Nosratinia, T. Hunter, and A. Hedayat, “Cooperative communication in wireless networks,” *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74 – 80, oct. 2004.

[6] J. N. Laneman and G. W. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” *IEEE Trans. on Information Theory*, vol. 49, pp. 2415–2425, Oct. 2003.

[7] A. Ribeiro, X. Cai, and G. Giannakis, “Symbol error probabilities for general cooperative links,” *IEEE Trans. on Wireless Communications*, vol. 4, no. 3, pp. 1264 – 1273, may 2005.

[8] D. Goeckel, B. Liu, D. Towsley, L. Wang, and C. Westphal, “Asymptotic connectivity properties of cooperative wireless ad hoc networks,” in *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, september 2009, pp. 1226 –1237.

[9] K. W. F. Li and A. Lippman, “Energy-efficient cooperative routing in multi-hop wireless ad hoc networks,” in *IEEE IPCCC*, 2006.

[10] A. S. Ibrahim, Z. Han, and K. J. R. Liu, “Distributed energy- efficient cooperative routing in wireless networks,” in *IEEE Global Telecommunication Conference, Globecom*, 2007.

[11] Z. Yi and I.-M. Kim, “Diversity order analysis of the decode-and-forward cooperative networks with relay selection,” *IEEE Trans. on Wireless Communications*, vol. 7, no. 5, pp. 1792 –1799, may 2008.

[12] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A. Haimovich, “Decode-and-forward cooperative diversity with power allocation in wireless networks,” *IEEE Trans. on Wireless Communications*, vol. 6, no. 3, pp. 793 –799, march 2007.

[13] J. Zheng and M. Ma, “Qos-aware cooperative medium access control for mimo ad-hoc networks,” *IEEE Communications Letters*, vol. 14, no. 1, pp. 48 –50, january 2010.

[14] A. Ibrahim and K. Liu, “Mitigating channel estimation error with timing synchronization tradeoff in cooperative communications,” *IEEE Trans. on Signal Processing*, vol. 58, no. 1, pp. 337 –348, jan. 2010.