

# Power Efficient Decode-and-Forward Cooperative Relaying

Zhengguo Sheng, Bong Jun Ko, and Kin K. Leung

**Abstract**—Cooperative communications have been proven to be effective in enhancing performance of wireless networks. In this letter, we introduce and propose adaptive relay-selection rules to offer a tradeoff between fairness and energy consumption at each node for a network with multiple, mutually cooperative nodes. Our performance analysis is supplemented by simulation results to illustrate the significant energy savings of the proposed optimal power allocation and the relay-selection rules.

**Index Terms**—Relaying, power efficient, fairness.

## I. INTRODUCTION

COOPERATIVE communication mechanisms [1]–[3] have been proposed as an effective way of exploiting spatial diversity to improve the quality of wireless links. The key idea is to have multiple wireless devices in different locations cooperatively share their antenna resources and aid each other’s wireless transmission effectively to form virtual and distributed antenna arrays. The previous work in the literature shows that relay selection can significantly improve the overall quality of the wireless transmission, in terms of power consumption [4], the QoS performance [5], [6], the spectral efficiency [7].

In cooperative communication, the term “cooperation” refers to a node’s willingness to sacrifice its own resources (e.g., energy, transmission opportunity) for the benefit of other nodes. It is thus of fundamental importance to understand how much resources must be consumed to reap the benefits of the cooperative communication. Putting it in another way, does cooperative communication requires more (or less) overall resources than conventional, non-cooperative communication to achieve the same level of wireless link quality? How can we best achieve the resource saving when employing cooperative communication. This letter is our answers to these questions, with particular focus on the energy consumption issues in cooperative communication.

More specifically, we explore the energy consumption aspect of Decode-And-Forward (DAF) cooperative communications (CC) from various angles, yet in a unified manner. First, we introduce the DAF cooperation scheme in terms of outage probability and power consumption. Second, we propose adaptive cooperation mechanism that will help select appropriate relays for the maximal energy saving of each node in a multi-node environment, and show that the proposed relay selection can benefit individual nodes from participating in

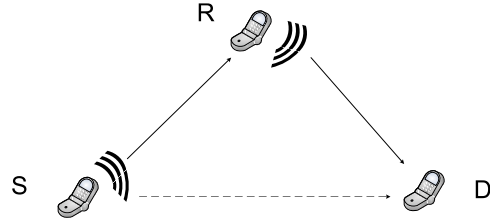


Fig. 1. An example of a wireless cooperative link.

CC. We also study the trade-off between fairness in energy saving and total energy consumption.

## II. SYSTEM MODEL AND TRANSMISSION POWER CONSUMPTION

We consider a cooperative network in Figure 1: source node (S), destination node (D) and a relay node (R). Our scheme employs two transmission slots: In the first time slot, the source broadcasts its data to the relay and the destination. In the second time slot, the relay retransmits the received signal, if the SNR exceeds a threshold; otherwise, the source retransmits the signal. As a result, the destination receives two independent copies of the same packets transmitted through different wireless channels. Diversity gain can be achieved by combining the data copies using the Maximum Ratio Combining (MRC) where the received signals are weighted with respect to their SNR and then summed together.

### A. Direct transmission

We start by deriving the power required in the direct transmission (without relay) and assume the channel model incorporating path loss and Rayleigh fading. The received signal at a destination  $d$  is modeled as  $y_d[n] = a_{s,d}x_s[n] + n_d[n]$ , where  $x_s[n]$  is the signal transmitted by a source  $s$ ,  $n \in [1, \dots, N]$  is the index of the transmitting packet and  $n_d[n]$  is additive white Gaussian noise, with variance  $\sigma_n^2$ , at the receiver. The channel gain  $a_{s,d}$  between the nodes  $s$  and  $d$  is modelled as  $a_{s,d} = h_{s,d}/d_{s,d}^{\alpha/2}$ , where  $d_{s,d}$  is the distance between the nodes  $s$  and  $d$ ,  $\alpha$  is the path-loss exponent and  $h_{s,d}$  captures the channel fading characteristics. The channel fading parameter  $h_{s,d}$  is assumed to be complex Gaussian with zero mean and unit variance, and independent and identically distributed (i.i.d.) across times slots, packets and across links.

The channel capacity between the source  $s$  and the destination  $d$  is  $I_{s,d} = \log(1 + p|a_{s,d}|^2)$ , where  $p = E_b/N_0$  is defined as the normalized transmission power. Since for Rayleigh fading,  $|a_{s,d}|^2$  is exponentially distributed with parameter  $d_{s,d}^\alpha$ . The outage probability satisfies

$$\epsilon^{out} = \Pr[I_{s,d} < b] = 1 - \exp\left(-\frac{(2^b - 1)d_{s,d}^\alpha}{p}\right) \approx d_{s,d}^\alpha \left(\frac{2^b - 1}{p}\right)$$

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for large  $p$ . Here  $b$  is the desired data rate in bit/s/Hz, which is defined by QoS requirement. We then have the normalized transmission power for direct transmission

$$p_D = d_{s,d}^{\alpha} \left( \frac{2^b - 1}{\epsilon^{out}} \right). \quad (1)$$

### B. DAF cooperative transmission

Let  $d_{s,d}$ ,  $d_{s,r}$  and  $d_{r,d}$  be the respective distances among the source, relay and destination node. During the first time slot, the destination and relay receive  $y_{d,1}[n] = \frac{h_{s,d}}{d_{s,d}^{\alpha/2}} x_s[n] + n_d[n]$  from the source node, where  $x_s[n]$  is the information transmitted by the source and  $n_d[n]$  is white noise. During the second time slot, the destination node receives

$$y_{d,2}[n] = \begin{cases} \frac{h_{s,d}}{d_{s,d}^{\alpha/2}} x_s[n] + n_d[n], & \text{if } \left| \frac{h_{s,r}}{d_{s,r}^{\alpha/2}} \right|^2 < f(p) \\ \frac{h_{r,d}}{d_{r,d}^{\alpha/2}} x_r[n] + n_d[n], & \text{if } \left| \frac{h_{s,r}}{d_{s,r}^{\alpha/2}} \right|^2 \geq f(p) \end{cases} \quad (2)$$

where  $f(p) = (2^{2b} - 1)/p$  can be derived from direct transmission and is analogous to (1). In this protocol, the relay transmits only if the SNR exceeds a threshold; otherwise, the source retransmits in the second time slot.

Assuming that the relay node can perform perfect decoding when the received SNR exceeds a threshold, the channel capacity of this cooperative link can be shown as

$$I_{s,d} = \begin{cases} \frac{1}{2} \log(1 + 2p|a_{s,d}|^2), & |a_{s,r}|^2 < f(p) \\ \frac{1}{2} \log(1 + p|a_{s,d}|^2 + q|a_{r,d}|^2), & |a_{s,r}|^2 \geq f(p) \end{cases} \quad (3)$$

where  $p$  and  $q$  are the normalized source and relay power, respectively. It is worth noting that the same noise variance is assumed at both relay and destination. Therefore, the outage event is given by  $I_{s,d} < b$  and the outage probability becomes

$$\begin{aligned} \epsilon^{out} &= \Pr[I_{s,d} < b] \\ &= \Pr[|a_{s,r}|^2 < f(p)] \Pr[2|a_{s,d}|^2 < f(p)] \\ &\quad + \Pr[|a_{s,r}|^2 \geq f(p)] \Pr[|a_{s,d}|^2 + \left| \sqrt{\frac{q}{p}} a_{r,d} \right|^2 < f(p)]. \end{aligned} \quad (4)$$

By computing the limit, we obtain a closed-form expression for the outage probability between the source and the destination using cooperative transmission

$$\epsilon_C^{out} = \frac{1}{2} d_{s,d}^{\alpha} \left( d_{s,r}^{\alpha} + \frac{p}{q} d_{r,d}^{\alpha} \right) \frac{(2^{2b} - 1)^2}{p^2}. \quad (5)$$

It is worth noting that for a fair comparison with direct transmission using only one time slot, cooperative transmission actually employs twice the data rate at  $2b$  during two consecutive time slots, so that both schemes have the same effective data rate.

## III. ENERGY-EFFICIENT RELAY SELECTION FOR DAF

Our interest in this section is to find a set of rules that determine which node to select as the relay for the maximal power efficiency of each node in this multi-node environment by using the cooperative scheme.

More specifically, our setup consists of a set of nodes  $N = \{1, \dots, n\}$ , where each node  $i \in N$  transmits a number of packets over time, each time with some arbitrary destination

node in the network. For simplicity, we assume all packets have the same constant length with the same QoS constraints, though it is straight-forward to derive relay selection rules in a more general setup. We also assume that time is divided into discrete time slots, and that TDMA is used to provide collision-free transmissions from the sources and the relays.

We denote by  $p_{i,j}(t)$  and  $q_{i,j}(t)$  the transmit power of a source node  $i$  and a relay node  $j$ , respectively, when  $i$  would use cooperative transmission with  $j$  as the relay to some destination at time  $t$ . When node  $i$  uses direct transmission at time  $t$ , we denote its transmit power as  $p^D(t)$ . The *energy consumption* of a node  $E_i(t_1 : t_2)$  during a time interval  $[t_1 : t_2]$  is the sum of node  $i$ 's transmit power either as a source or a relay over all  $t \in [t_1, t_2]$  (we assume a node consumes zero-power at  $t$  if it is neither a source or a relay).

To represent how much energy saving the cooperative transmission can yield in comparison to direct transmission, we begin by introducing the notion of the ‘‘payoffs’’ of the nodes.

### A. Payoff function

The *payoff function*,  $u_i(t)$ , of node  $i$  at time  $t$  is defined as:

$$u_i(t) = \begin{cases} p_i^D(t) - p_{i,j}(t) & \text{if } \exists j \text{ s.t., } r_i(t) = j \\ -q_{j,i}(t) & \text{if } i = r_j(t) \text{ for some source } j \\ 0 & \text{otherwise.} \end{cases}$$

The above represents how much energy a node  $i$  *locally* saves (or loses) compared to direct transmission at time  $t$ , where  $p_i^D(t) - p_{i,j}(t)$  denotes the power saved from  $i$ 's cooperative transmission using some relay  $j$  at time  $t$ , and  $-q_{j,i}(t)$  the power spent in  $i$ 's transmission as a relay for some other node  $j$  at time  $t$ . In all other cases, the payoff is 0. We assume the initial  $u_i(t) = 0$  for all  $i \in N$ . Then the *cumulative payoff* over a time interval  $[t_1 : t_2]$  is defined as  $u_i(t_1 : t_2) = \sum_{\tau=t_1}^{t_2} u_i(\tau)$ , which represents the overall energy savings of a node during the time interval.

### B. Relay selection by minimum power

Our first relay-selection rule makes use of the result in previous section in a straightforward manner:

**Min-Total-Power Relay Selection:** A relay is selected for source  $i$  at time  $t$  such that

$$r_i(t) = \arg \min_{j \in \mathcal{R}_i(t)} \{p_{i,j}(t) + q_{i,j}(t)\}.$$

where  $\mathcal{R}_i(t)$  denotes relay candidates of source  $i$ , i.e.,  $\mathcal{R}_i(t) = \{j \in N - \{i\} | p_{i,j}(t) + q_{i,j}(t) < p^D(t)\}$ .

In other words, for each packet from node  $i$ , a relay  $j$  is selected which minimizes  $p_{i,j}(t) + q_{i,j}(t)$  among those in  $i$ 's cooperative region at  $t$ . If  $\mathcal{R}_i(t) = \emptyset$ ,  $r_i(t) = \text{null}$ .

Now, the optimization problem is to minimize the total transmission power consumption of a cooperative link given that a relay and a target Quality-of-service (QoS) is satisfied and can be formulated as

$$\begin{aligned} \min \quad & p_{i,j}(t) + q_{i,j}(t) \\ \text{s.t.} \quad & \epsilon_C^{out}(p_{i,j}(t), q_{i,j}(t)) \leq \eta \end{aligned} \quad (6)$$

where  $p_{i,j}(t)$  and  $q_{i,j}(t)$  denote the source and relay power, respectively, and  $\epsilon_C^{\text{out}}(p_{i,j}(t), q_{i,j}(t))$  is the outage probability defined by (5).

**Theorem 1:** The optimal transmission power to minimize the total power consumption of DAF cooperation given that a target QoS is satisfied, is given by

$$p_{i,j}(t) = \sqrt{\frac{a+2b}{2} + \frac{\sqrt{a^2+8ab}}{2}}, \quad q_{i,j}(t) = \frac{ap_{i,j}(t)}{p_{i,j}(t)^2 - b} \quad (7)$$

where  $a = \frac{\mu d_{s,d}^\alpha d_{r,d}^\alpha}{2\eta}$ ,  $b = \frac{\mu d_{s,d}^\alpha d_{s,r}^\alpha}{2\eta}$ ,  $\mu = (2^{2b} - 1)^2$  and  $\eta$  is the outage constraint.

*Proof:* See Appendix A.  $\square$

It is easy to see that, though simple, the Min-Total-Power rule is optimal (among all relay selection rules) in the sense that it minimizes the total energy consumption of the network,  $\sum_{i \in N} E_i(t_1 : t_2)$  for any time interval  $[t_1, t_2]$ , and hence maximizes the aggregate cumulative payoffs  $\sum_{i \in N} u_i(t_1 : t_2)$  of all nodes.

From the individual nodes' perspective, however, the relay selection by Min-Total-Power rule can lead to the situation that some nodes end up with higher energy consumption than would be the case when all nodes employ direct transmission. This is especially true if some unfortunate nodes are heavily selected as relays and hence consume more energy in relaying than that saved from its own transmission as a source. We now consider how to handle such unfairness issue in CC.

### C. Adaptive relay selections

The main idea of the adaptive relay selection is to let each node act as a relay only when it has saved more energy than that it has lost from cooperative transmission in the past. For this, a binary decision variable  $C_i(t)$  is maintained for each node  $i$  and updated at each time  $t$  (hence the term "adaptive") such that

$$C_i(t) = \begin{cases} 1 & \text{if } u_i(0 : t-1) \geq 0 \\ 0 & \text{if } u_i(0 : t-1) < 0. \end{cases}$$

This  $C_i(t)$  value is used in the decision as to whether node  $i$  can act as a relay for other nodes (when  $C_i(t) = 1$ , i.e., in "cooperative" mode) or  $i$  should not be selected as relay for any other node (when  $C_i(t) = 0$ ).

**Adaptive Relay Selection:** A relay is selected for source  $i$  at time  $t$  such that

$$r_i(t) = \arg \min_{j \in \mathcal{R}_i(t), C_j(t)=1} \{p_{i,j}(t) + q_{i,j}(t)\}.$$

In other words, a relay  $j$  is selected for the  $i$ 's transmission at time  $t$  that minimizes  $p_{i,j}(t) + q_{i,j}(t)$  among the nodes whose cumulative payoffs are positive or zero.<sup>1</sup>

Recognizing that some nodes may benefit more from the larger cooperative transmission opportunities than the others due to difference in the amount of data and to potentially unfair medium access protocol, we can generalize the rule

<sup>1</sup>We set  $C_i(t) = 1$  if  $u_i(0 : t-1) = 0$  in order to enable the initial cooperative condition when all nodes's payoffs are zero. If  $C_i(0) = 0$  for all  $i$ , no node would cooperate to other nodes.

even more to bring the balance (or "fairness") of the amount of payoffs that individual nodes collect:

**Weighted Adaptive Relay Selection:** A relay is selected for source  $i$  at time  $t$  such that

$$r_i(t) = \arg \min_{j \in \mathcal{R}_i(t), C_j(t)=1} \{w(u_j(0 : t-1)) (p_{i,j}(t) + q_{i,j}(t))\},$$

where  $w(u)$  is a non-increasing function of the payoff value  $u$ . Here, along with the power consumption factor  $(p_{i,j}(t) + q_{i,j}(t))$ , the weight function  $w(u_j(0 : t-1))$  is introduced in the relay selection criteria, such that the nodes with larger payoffs (i.e., smaller weight) will have a higher chance to get selected as the relay for each packet transmission.

How much importance will be given to the weight term reflecting the payoff and how much to the power consumption term depends on how fast the function  $w(u)$  decays as the payoff value  $u$  increases. In our simulation study, we find that  $w(u) = u^{-6}$  strikes a good balance.

### D. On distributed implementation of relay selection

We close this section by discussing how our relay selection rules can be realized in a distributed manner. Specifically, we use a distributed protocol proposed in [8], which employs a 4-way handshake of messages to control medium accesses for cooperative communication. Our relay selection rule can be readily implemented by innovatively using the above distributed protocol. Note that the relay selection rule requires the knowledge of (i) the estimates of the channel state information, and (ii) the current cumulative payoffs of the potential relays at the time of the packet transmission.

## IV. SIMULATION RESULTS

In this section, we provide the numerical and simulation results obtained using MATLAB. We evaluate the performance of our relay selection schemes via simulation, in which we place  $N$  (varied between 5 and 25) nodes at uniformly random locations in a 100m  $\times$  100m region (the edges of the region are wrapped (toroid) to eliminate edge effects). Throughout the simulation, we set the path-loss exponent  $\alpha = 3$ , the data rate  $b = 1$  bps/Hz and the targeted  $\epsilon^{\text{out}} = 0.01$ . A total of  $200 \times N$  packets are transmitted, and at each time  $t$ , a packet is transmitted by a randomly selected source and a randomly selected destination. The simulation result is averaged over 1000 times for each  $N$ .

Figure 2 shows the average energy consumption of each node, normalized by the minimum value in the data set (i.e., Minimum total energy selection with 25 nodes) for different relay selection methods. Overall, our relay selection schemes perform far better than the direct transmission or that when a random relay is selected for each packet, and the adaptive relay-selection performs close to the minimum power selection<sup>2</sup>, which is the optimal one in terms of average (or total) energy consumption. The weighted adaptive relay selection performs a bit worse (this is compensated by fairness results below). Furthermore, as the number of nodes increases,

<sup>2</sup>It is noted that there is a clear gap when node number is 5, this is simply because when the number of node is small, there is a high probability that some unlucky nodes are extensively selected as relays in the optimal solution.

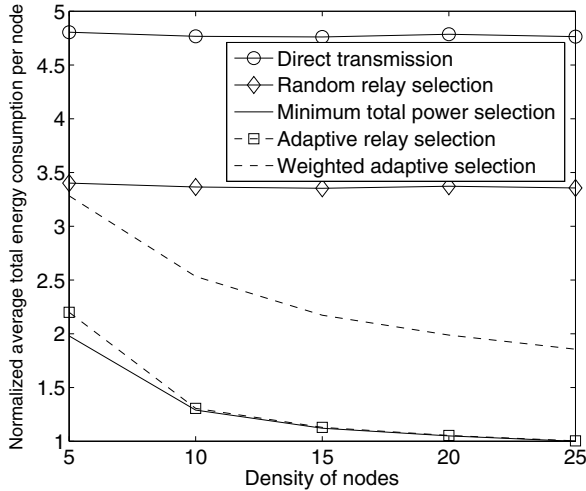


Fig. 2. Average total energy consumption per node.

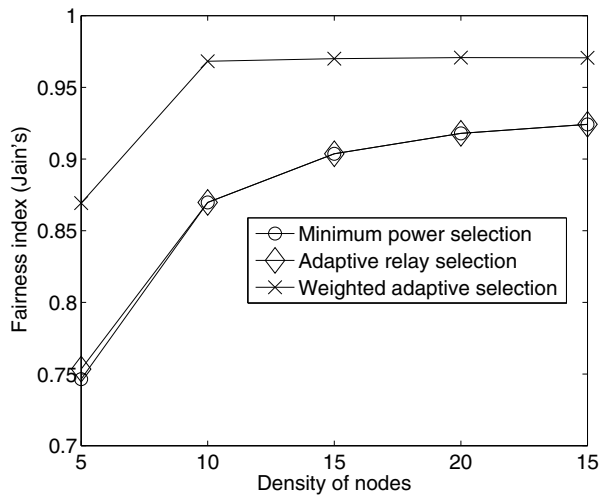


Fig. 3. Jain's fairness on payoff function.

the average energy consumption of our relay selection schemes decreases; this is because it is easier to find a well-positioned relay and thus save more power in a dense network.

## V. CONCLUSIONS

We have shown in this letter that it is advantageous to employ cooperative transmission, which can significantly reduce the total power consumption while maintaining a given level of quality of service (QoS). To develop further robust cooperative schemes to cope with new demands in our future wireless networks, we plan to propose additional robust relay-selection mechanisms, which can account for other parameters of importance as the fairness measures, such as the difference in traffic loads and the remaining energy of each nodes.

Figure 3 shows the fairness in terms of how much energy is saved for individual nodes using each relay selection methods, where the  $y$ -axis represent Jain's fairness index of nodes' cumulative payoffs.<sup>3</sup> It is clear that the weighted adaptive

relay-selection scheme achieves the best fairness compared with other two schemes.

## APPENDIX A

In order to simplify the notation, we use  $p$  and  $q$  to denote  $p_{i,j}(t)$  and  $q_{i,j}(t)$ , respectively. According to the Kuhn-Tucker condition (p.244: KKT conditions for convex problems [9]), the inequality constraint in (6) can be converted to the equality constraint and have the target outage probability

$$\frac{1}{2}d_{s,d}^{\alpha}(d_{s,r}^{\alpha} + \frac{p}{q}d_{r,d}^{\alpha})\frac{(2^{2b}-1)^2}{p^2} = \eta.$$

Then we obtain

$$q = f(p) = \frac{ap}{p^2 - b} \quad (8)$$

where  $a = \mu d_{s,d}^{\alpha} d_{r,d}^{\alpha} / 2\eta$ ,  $b = \mu d_{s,d}^{\alpha} d_{s,r}^{\alpha} / 2\eta$ ,  $\mu = (2^{2b} - 1)^2$  and  $\eta$  is the outage constraint.

Substituting (8) into  $p + q$ , and minimizing wrt  $p$ , we have the solution  $p^2 = \frac{a+2b}{2} \pm \frac{\sqrt{a^2+8ab}}{2}$ .

To be a valid solution for  $q$ , the solution must satisfy  $p^2 > b$ . So, we have a unique solution given by  $p^* = \sqrt{\frac{a+2b}{2} + \frac{\sqrt{a^2+8ab}}{2}}$ , using this result in (8) leads to (7).

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<sup>3</sup>Jain's fairness index is defined by  $(\sum u_i)^2 / (N \sum u_i^2)$ . The result ranges from  $\frac{1}{N}$  (worst case) to 1 (best case). The larger the index is, the better fairness that we can achieve.