Physical Layer Network Coding and Precoding for the Two-Way Relay Channel in Cellular Systems

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Abstract

In this paper, we study the application of physical layer network coding to the joint design of uplink and downlink transmissions, where the base station and the relay have multiple antennas, and all M mobile stations only have a single antenna. A new network coding transmission protocol is proposed, where $2M$ uplink and downlink transmissions can be accomplished within two time slots. Since each single antenna user has poor receive capability, precoding at the base station and relay has been carefully designed to ensure that co-channel interference can be removed completely. Explicit analytic results have been developed to demonstrate that the multiplexing gain achieved by the proposed transmission protocol is M, much better than existing time sharing schemes. To further increase the achievable diversity gain, two variations of the proposed transmission protocols have also been proposed when there are multiple relays and the number of the antennas at the base station and relay is increased. Monte-Carlo simulation results have also been provided to demonstrate the performance of the proposed network coded transmission protocol.

I. Introduction

In mobile communication systems, it is challenging to provide high-speed high-quality service due to the scarce bandwidth resource and harsh radio propagation environments [1]. Many sophisticated transmission technologies have been developed to improve the robustness and throughput of mobile systems. For example, the use of multiple antennas has been shown to increase the capacity and reliability of mobile communications [2]. As a low-cost alternative to multiple-input multiple-output systems, cooperative diversity has been developed to combat multi-path fading which is the main factor causing the unreliability of wireless transmission [3], [4]. By encouraging single-antenna nodes to cooperate with each other, a virtual antenna array can be formed accordingly, however, the overall system throughput may not be increased significantly by only using cooperative transmission.

Network coding has recently emerged as a promising transmission technology to improve spectral efficiency and system throughput [5]. The key idea of network coding is to ask an intermediate node to

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mix the messages it received and forward the mixture to several destinations simultaneously. Compared to time sharing based schemes where destinations are served in turn, the use of network coding can increase the overall throughput dramatically. Originally designed in the context of wireline communications, there have been a lot of papers in which network coding was applied to wireless communications. Actually the broadcast nature of wireless transmission is perfect for the application of network coding. For example, when there are multiple simultaneous transmissions to a single intermediate node, the multiple messages will be superimposed at the receiver. Similarly one relay transmission can also be overheard by multiple destinations because of the broadcast nature of wireless medium.

The first wireless communication scenario where the network coding was applied to is two way relaying channel, where two source nodes exchange information with the help of a relay (sometimes referred as physical layer network coding or analogue network coding) [6]–[8]. In [6], [9] the authors assumed the messages transmitted by the two sources arrive at the relay without any distortion, and exclusive-or has been proposed to mix the two messages at the relay. Because of the effects of multi-path fading, it is not practical to assume that there is no channel distortion of the transmitted messages, which is the motivation of the works in [7], [10]. As proposed in [7], [10], the relay does not have to perform demodulation/modulation or exclusive-or, but just forwards the mixture which is the superposition of two source messages with channel distortion. Such a transmission strategy can reduce the computational complexity at the relay and also yield a performance gain in terms of both robustness and throughput simultaneously provided that there are sufficient relays. In [11], [12], the use of network coding has been proposed to wireless uplink transmission and in [13] network coding has been applied to wireless broadcasting transmission. The impact of two way-communications on the transmission capacity of wireless ad hoc networks was studied in [14]. In [15], [16], the use of network coding for two way relaying channels with multiple antennas has been studied. The scenario of multi-way relaying channel has been studied in [17], where a new transmission protocol has been developed with the number of transmission phases being the same as the number of the sources.

In this paper, we focus on a scenario similar to two-way relaying channel where the base station and the relay have M antennas, but each of the M users is equipped with a single antenna due to size constraints. Such a scenario is important because the base station typically has better capability than mobile stations which are constrained by the small size of handsets and limited battery life. The contributions of this paper are three-fold. *Firstly*, new network coding based protocols have been developed, where $2M$ uplink and downlink transmissions can be accomplished within two time slots. The most challenging problem for the addressed communication scenario is how to handle the co-channel interference, where the capability of mobile users is poor due to the fact that each user is only equipped with a single antenna. Inspired by the concept of interference alignment [18], the key idea for the proposed network coding protocol is to ensure that the two messages delivered to and from the same mobile user fall in the same spatial direction at the relay. Sophisticated precoding and beamforming techniques have been designed to ensure that signals to and from the same user can be paired together and co-channel interference can be avoided. As a result, the original multi-user channels can be decomposed into multiple two-way relaying channels without co-channel interference.

Secondly, explicit analytic results, such as the outage probability and diversity-multiplexing tradeoff, have been developed to facilitate performance evaluation for the proposed network coding transmission protocols. We first study the outage performance for the M messages sent through the uplink as well as the M messages delivered through the downlink, which demonstrates that co-channel interference has been removed successfully. Then based on the outage performance of individual messages, the performance for the sum rate is studied, where we show that the multiplexing gain for the sum rate can be up to M. Recall that existing network coding schemes can be applied to the addressed scenario by using time sharing approaches, which supports the multiplexing gain less than M. *Thirdly*, two variations of the proposed network coding transmission protocol are developed to further increase the diversity gain achievable for the proposed protocol. Specifically, provided that there are L relays, we demonstrate that the proposed transmission protocol can achieve a diversity gain L without reducing the achievable multiplexing gain. Similarly, when the number of the antennas at the base station and the relay is increased, the proposed protocol can still be applied. Analytical results have been developed to demonstrate the impact of the number of antennas at the relay and base station on the outage performance and achievable diversity gains.

This paper is organized as follows. The proposed network coding transmission strategy is described in Section II. And then in Section III, the performance achieved by the proposed transmission protocol is analyzed by using information theoretic metrics, such as outage probability and diversity-multiplexing tradeoff. Then in Section IV two approaches to increase the diversity gain for the proposed protocol are described and analyzed. Monte-Carlo simulation results are provided in Section V. Finally, concluding remarks are given in Section VI.

II. Description for the Proposed Network Coding Protocol

Consider a scenario with M mobile users, one base station and a single relay. Both the relay and the base station are equipped with M antennas, as shown in Fig. 1. Each of the M mobile users only has a single antenna, which could be due to the constraints of small handset size or limited processing power. Different choices of the number of antennas at the relay and base station will be discussed in the next section. We assumed quasi-static independent and identically Rayleigh fading for all channels and there is no direct link between the base station and mobile users as in [4], [6], [7]. The time division duplexing mode has been used for its simplicity and the half-duplex constraint is applied to all nodes. Due to the symmetry of time division duplex systems, the uplink channels and the downlink channels are assumed to be reciprocal. Since precoding is required at the base station and relay, it is assumed in this paper that the base station and relay have global channel state information prior to transmission. It is important to point out that the base station does not have to know the precoding matrix at the relay since these precoding matrices can be obtained from the channel information directly. At the mobile user side, only the CSI at the receiver is required. Note that it is straightforward for the relay and the users to obtain the required CSI by applying traditional training based channel estimation approaches and utilizing the feature of reciprocal TDD systems. The base station can obtain the channel information between it and the relay similarly. The accuracy of channel estimation can be further enhanced by exploring the redundant information of network coding transmissions. For example, the base station has some priori information about the mixture broadcasted by the relay since this information was generated by the base station. Such priori information can be utilized and the so-called first order statistics based channel estimation approaches can be applied [19]. Other channel estimation methods, such as in [20], can also be applied. In order for the base station to obtain the CSI between the relay and the users, it is assumed that there is a reliable feedback channel between the base station and the users. Note that the fact that each user only has a single antenna is helpful to reduce the system overhead. Alternatively we can ask the relay to forward the relay-user channel information to the base station. Note that the channels between the base station and the relay are MIMO links and therefore the relay can communicate with the base station in a high transfer data rate.

The base station needs to deliver M messages to the M mobile users respectively, where we denote s_m as the message to the m-th user. At the same time, each of the M users needs to send information to the base station, where u_m is used to denote the message from the m-th user. A symmetrical system is considered in this paper, where the targeted data rate between the base station and each user is the same, denoted as R. The physical layer network coding proposed in [6], [10] can be applied to the addressed scenario by using time sharing approaches. Each mobile user is paired with the base station, and information exchange can be accomplished with two time slots for each pair with the help of the relay. A straightforward application of network coding requires $2M$ time slots in total, which means the number of time slots required will be proportional to the number of mobile users. In the following we will propose a new network coding scheme which only requires 2 time slots conditioned on that the base station and relay have M antennas, no matter how many mobile users we have.

During the first time slot, the base station transmits the precoded version of the information bearing symbols, $\mathbf{P}\mathbf{s}$, where $\mathbf{s} = \begin{bmatrix} s_1 & \cdots & s_M \end{bmatrix}^T$ and $\mathbf{P}$ is a $M \times M$ precoding matrix at the base station. It is important to ensure that the total transmission power at the base station is constrained. In this paper, we assume that the transmission power at each antenna at the base station or the multiple users is 1. Hence the precoding matrix should satisfy trace$\{\mathbf{P}\mathbf{P}^H\} \leq M$, where trace$(\cdot)$ denotes the trace. The design of the precoding matrix will be introduced in detail later. At the same time, each of the M users send its own message u_i, for $i \in \{1, \ldots, M\}$, to the base station.

Hence at the end of the first time slot, the relay observes

$$\mathbf{r} = \mathbf{G}\mathbf{P}\mathbf{s} + \sum_{m=1}^{M} \mathbf{h}_{mR} u_m + \mathbf{n}_R, \tag{1}$$

where $\mathbf{G}$ is the $M \times M$ channel matrix between the base station and the relay, $\mathbf{h}_{mR}$ denotes the $M \times 1$ channel vector between the relay and the m-th mobile user, $\mathbf{n}_R$ denotes the $M \times 1$ additive white Gaussian noise vector.

During the second time slot, the relay transmits a precoded version of its observation during the previous time slot. Denote $\mathbf{W}$ as the precoding matrix at the relay. The relay will transmit $(\mathbf{W}\mathbf{r})^*$, where the conjugate operation is applied to simplify the signal model. Again the transmission power constraint should be satisfying trace $\left\{\mathbf{W}\mathbf{r}\mathbf{r}^H\mathbf{W}^H\right\} \leq M$ and the design of the precoding matrix at the relay will be discussed further in the next section. Hence during the second time slot, the observations at the base station can be expressed as

$$\mathbf{y}_{BS} \;=\; \mathbf{G}^H\mathbf{W}\left(\mathbf{G}\mathbf{P}\mathbf{s} + \sum_{m=1}^{M} \mathbf{h}_{mR} u_m + \mathbf{n}_R\right) + \mathbf{n}_{BS} \tag{2}$$

and the observation at the m-th user can be expressed as

$$y_m = \mathbf{h}_{mR}^H\mathbf{W}\left(\mathbf{G}\mathbf{P}\mathbf{s} + \sum_{m=1}^{M} \mathbf{h}_{mR} u_m + \mathbf{n}_R\right) + n_m, \tag{3}$$

where n_m and $\mathbf{n}_{BS}$ are defined similarly to $\mathbf{n}_R$.

As can be observed from (3), it could be difficult for each of the single-antenna users to achieve correct detection due to the existence of co-channel interference. For example, s_i and u_i could cause strong interference to the j-th mobile receiver, for $i \neq j$, and such interference will severely degrade the performance of the single-antenna receiver. Hence great care should be taken to ensure each mobile user does not observe the information transmitted from or destined to other users. On the other hand, it is interesting to observe that co-channel interference can be simply handled at the base station. Specifically at the base station, the messages known to the base station can be removed, and the signal model at (2) becomes similar to the traditional $M \times M$ MIMO scheme, where the classical detection mechanisms, such as zero forcing or minimum mean square error (MMSE) filtering, can be applied to achieve detection. This observation is the key for the proposed network coding strategy, where we only need to focus on how to cope with co-channel interference at the multiple mobiles and ensure that the m-th mobile user only observes s_m without interference.

A. The Design of Precoding Matrices $\mathbf{P}$ *at the Base Station*

The design of the precoders at the base station and the relay shall satisfy two conditions. One is that the transmission power at the base station and the relay should be constrained, and secondly each mobile user should not receive any information for other users. Inspired by the concept of interference alignment

[18], the key idea of the proposed network coding protocol is that the relay tries to group the messages from and to the same mobile user, i.e. s_i and u_i together. This can be facilitated by defining the precoding matrix $\mathbf{P}$ at the base station as the following

$$\mathbf{P} = \mathbf{G}^{-1}\mathbf{H}\mathbf{D}_s. \tag{4}$$

where $\mathbf{H} = \begin{bmatrix} \mathbf{h}_{1R} & \cdots & \mathbf{h}_{MR} \end{bmatrix}$ and $\mathbf{D}_s$ is a diagonal matrix which is to ensure the transmission power at the base station is constrained. By using such a precoding matrix $\mathbf{P}$, the relay can group the messages from and to the same user as the following

$$\mathbf{r} = \mathbf{H}(\mathbf{D}_s\mathbf{s} + \mathbf{u}) + \mathbf{n}_R, \tag{5}$$

where $\mathbf{u} = \begin{bmatrix} u_1 & \cdots & u_M \end{bmatrix}^T$. It is interesting to observe that the two messages sent from and to the same mobile user have been aligned and grouped together. Similar to physical layer network coding (PNC) [6] or analogue network coding (ANC) [9], the relay is not going to separate the two messages for the same user, but just broadcast the mixture to the users directly.

To find an appropriate power normalization matrix $\mathbf{D}_s$, we first express the total transmission power at the base station with the use of $\mathbf{P}$ as

$$\text{trace}\left\{\mathbf{P}\mathbf{P}^H\right\} = \text{trace}\left\{\mathbf{G}^{-1}\mathbf{H}\mathbf{D}_s^2\left(\mathbf{G}^{-1}\mathbf{H}\right)^H\right\} = \text{trace}\left\{\left(\mathbf{G}^{-1}\mathbf{H}\right)^H\mathbf{G}^{-1}\mathbf{H}\mathbf{D}_s^2\right\}. \tag{6}$$

In this paper, we assume that each transmit antenna has the transmission power constraint 1. To satisfy such a power constraint, we propose the following power normalization matrix

$$\mathbf{D}_s = \begin{bmatrix} \frac{1}{\sqrt{\mathbf{h}_{1R}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{1R}}} & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & \frac{1}{\sqrt{\mathbf{h}_{MR}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{MR}}} \end{bmatrix}. \tag{7}$$

By using such a normalization matrix, the total transmission power of the base station can be shown as

$$\text{trace}\left\{\mathbf{P}\mathbf{P}^H\right\} = \text{trace}\left\{\left(\mathbf{G}^{-1}\mathbf{H}\right)^H\mathbf{G}^{-1}\mathbf{H}\mathbf{D}_s^2\right\} = M, \tag{8}$$

which is exactly the same as the transmission power constraint assumed in this paper.

B. The Design of Precoding Matrices $\mathbf{W}$ *at the Relay*

Recall that in order to ensure that each user does not receive any information for other users, we apply an $M \times M$ precoding matrix $\mathbf{W}$ to the observations $\mathbf{r}$ prior to transmission. By applying the proposed precoding matrix $\mathbf{P}$ at the base station, the messages transmitted by the relay can be expressed as

$$(\mathbf{W}\mathbf{r})^* = \left(\mathbf{W}\left[\mathbf{H}(\mathbf{D}_s\mathbf{s} + \mathbf{u}) + \mathbf{n}_R\right]\right)^*. \tag{9}$$

Note that the reason to have this conjugate operation is to simplify the notation in the following equations. As discussed before, during the second time slot, the relay transmits this precoded version of its observations received during the previous time slot. The signal model at each mobile user can now be written as

$$y_m = \mathbf{h}_{mR}^H \mathbf{W}\left(\mathbf{H}(\mathbf{D}_s\mathbf{s} + \mathbf{u}) + \mathbf{n}_R\right) + n_m. \tag{10}$$

Recall that one of the two goals of the precoding design is to ensure that each user does not receive any information for other users, which means the precoding matrix $\mathbf{W}$ at the relay should satisfy the following criterion

$$\mathbf{H}^H\mathbf{W}\mathbf{H} = \text{diag}\{\xi_1, \cdots, \xi_M\}, \tag{11}$$

where the value of ξ_m is dependent on the choice of the precoding matrix. One simple choice of the precoding matrix is $\mathbf{W} = (\mathbf{H}^H)^{-1}\mathbf{H}^{-1}$, which means that $\xi_1 = \cdots = \xi_M = 1$. However such a choice of precoding can violate the transmission power constraint since the total transmission power at the relay based on such a simple choice of precoding gives

$$\begin{aligned}\mathcal{E}\left\{\text{trace}\{\mathbf{W}\mathbf{H}(\mathbf{D}_s\mathbf{D}_s^H + \mathbf{I}_M)\mathbf{H}^H\mathbf{W}^H + \mathbf{W}\mathbf{W}^H/\rho\}\right\} &\geq \mathcal{E}\left\{\text{trace}\left\{(\mathbf{H}^H)^{-1}\mathbf{H}^{-1}\mathbf{H}\mathbf{H}^H\left(\mathbf{H}^{-1}\right)^H(\mathbf{H})^{-1}\right\}\right\} \\ &= \mathcal{E}\left\{\text{trace}\left\{(\mathbf{H}\mathbf{H}^H)^{-1}\right\}\right\} \rightarrow \infty\end{aligned}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation, ρ denotes the transmit signal-to-noise ratio (SNR), the last equation follows from the fact that $\mathbf{H}$ is a square random matrix and hence the expectation of the trace of the inverse Wishart matrix $(\mathbf{H}\mathbf{H}^H)^{-1}$ is not bounded [21].

Therefore in order to avoid such unstable transmission power, we proposed the following form for the precoding matrix

$$\mathbf{W} = (\mathbf{H}^H)^{-1}\mathbf{D}_r\mathbf{H}^{-1}, \tag{12}$$

where $\mathbf{D}_r$ is a diagonal matrix to meet the power constraint. To decide $\mathbf{D}_r$, recall that by using the precoding matrix $\mathbf{W}$ proposed in (12), the total transmission power at the relay can be expressed as

$$\begin{aligned}P_{ow} &= \text{trace}\left\{\mathbf{W}\mathbf{H}(\mathbf{D}_s\mathbf{D}_s^H + \mathbf{I}_M)\mathbf{H}^H\mathbf{W}^H + \frac{1}{\rho}\mathbf{W}(\mathbf{D}\mathbf{D}^H + \mathbf{I}_M)\mathbf{W}^H\right\} \\ &\approx \text{trace}\left\{(\mathbf{H}^H)^{-1}\mathbf{D}_r(\mathbf{D}_s^2 + \mathbf{I}_M)\mathbf{D}_r^H\mathbf{H}^{-1}\right\},\end{aligned} \tag{13}$$

where the last approximation is obtained due to the high SNR assumption. Furthermore, we utilize the property of the trace and obtain

$$P_{ow} \approx \text{trace}\left\{\mathbf{H}^{-1}(\mathbf{H}^H)^{-1}\mathbf{D}_r\mathbf{D}_r^H(\mathbf{D}_s^2 + \mathbf{I}_M)\right\} = \text{trace}\left\{(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{D}_r^2(\mathbf{D}_s^2 + \mathbf{I}_M)\right\}. \tag{14}$$

As assumed previously, we set the transmission power constraint at each antenna to be 1, which means $P_{ow} \leq M$. To ensure the overall transmission power constraint is met, we propose the following power normalization matrix as

$$\mathbf{D}_r = \begin{bmatrix} \sqrt{\left([\mathbf{H}^H\mathbf{H}]^{-1}_{1,1}\right)^{-1} \Big/ 2} & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & \sqrt{\left([\mathbf{H}^H\mathbf{H}]^{-1}_{M,M}\right)^{-1} \Big/ 2} \end{bmatrix}.$$

By using such a choice of precoding, the expectation of the total transmission power at the relay can be expressed as

$$\begin{aligned} \mathcal{E}\{P_{ow}\} &\approx \mathcal{E}\left\{\text{trace}\left\{(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{D}_r^2(\mathbf{D}_s^2+\mathbf{I}_M)\right\}\right\} \\ &= \frac{1}{2}\mathcal{E}\left\{\text{trace}\left\{(\mathbf{I}_M+\mathbf{D}_s^2)\right\}\right\} = \frac{M}{2}\left(1+\mathcal{E}\left\{\frac{1}{\mathbf{h}_{1R}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{1R}}\right\}\right), \end{aligned} \tag{15}$$

where $[\mathbf{A}]_{m,m}$ means the m-th element on the diagonal of the matrix $\mathbf{A}$. As shown in Table I, $\mathcal{E}\left\{\frac{1}{\mathbf{h}_{1R}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{1R}}\right\}$ is always less than or very close to one, which means that the power constraint at the relay will be satisfied with the use of the proposed precoder, i.e., $\mathcal{E}\{P_{ow}\} \leq M$.

By using such a precoding matrix, the signal model at each mobile user can be written as

$$\begin{aligned} y_m &= \mathbf{h}_{mR}^H\left[\mathbf{W}\mathbf{H}(\mathbf{D}_s\mathbf{s}+\mathbf{u})+\mathbf{W}\mathbf{n}_R\right]+n_m \\ &= \mathbf{h}_{mR}^H(\mathbf{H}^H)^{-1}\mathbf{D}_r\left[((\mathbf{D}_s\mathbf{s}+\mathbf{u})+(\mathbf{H})^{-1}\mathbf{n}_R\right]+n_m = (d_{sm}s_m+u_m)+\tilde{\mathbf{h}}_m\mathbf{n}_R+d_{rm}^{-1}n_m, \end{aligned} \tag{16}$$

where d_{sm} and d_{rm} are the m-th elements at the diagonal of the matrices $\mathbf{D}_s$ and $\mathbf{D}_r$ and $\tilde{\mathbf{h}}_m$ is the m-th row vector of $\mathbf{H}^{-1}$. As can be observed from (16), the m-th mobile user only observes the information s_m and u_m where the information for the other users, s_j and u_j with $j \neq m$, has been removed because of the application of the proposed precoding matrices. In this paper, we assume that the nodes can perfectly cancel their own information from the observations as in [6], [7], [9], [10], [22]. At the base station, the signal model can now be written as

$$\begin{aligned} \mathbf{y}_{BS} &= \mathbf{G}^H\mathbf{W}\left(\mathbf{H}(\mathbf{D}_s\mathbf{s}+\mathbf{u})+\mathbf{n}_R\right)+\mathbf{n}_{BS} = \mathbf{G}^H\left(\mathbf{H}^H\right)^{-1}\mathbf{D}_r\mathbf{H}^{-1}\left(\mathbf{H}(\mathbf{D}_s\mathbf{s}+\mathbf{u})+\mathbf{n}_R\right)+\mathbf{n}_{BS} \\ &= \mathbf{G}^H\left(\left(\mathbf{H}^H\right)^{-1}\mathbf{D}_r(\mathbf{D}_s\mathbf{s}+\mathbf{u})+\left(\mathbf{H}^H\right)^{-1}\mathbf{D}_r\mathbf{H}^{-1}\mathbf{n}_R\right)+\mathbf{n}_{BS}. \end{aligned} \tag{17}$$

Evidently the use of the two precoding matrices has complicated the signal model at the base station, however, we will show that the diversity order achieved by the proposed network coding scheme is still one, exactly the same as the single-input single-output (SISO) scheme. In Section IV, we will introduce several strategies to increase the diversity gain without any loss of multiplexing gain.

III. Performance Analysis for the Proposed Network Coding Protocol

Given the signal models shown in (16) and (17), different detection approaches can be applied, but the zero forcing approach will be applied in this paper because of its simplicity [23]. Recall that the zero forcing approaches can achieve the same performance as the MMSE based detection algorithm at high

SNR. As can be observed from (16) and (17), the signal models at the base station and the mobile users are different, which will cause some difference for the development of analytical results. Therefore in the following two subsections, the receive performance at the base station and the mobile users will be analyzed separately.

A. Performance analysis for the receiver reliability at the mobile users

Subtracting its own information u_m from y_m, the m-th mobile user can achieve the detection of s_m, where the signal-to-noise ratio can be expressed as following

$$SNR_{U_m} = \frac{\rho}{\mathbf{h}_{mR}^H \left(\mathbf{G}^{-1}\right)^H \mathbf{G}^{-1}\mathbf{h}_{mR}\left[\tilde{\mathbf{h}}_m\tilde{\mathbf{h}}_m^H + \frac{[\mathbf{H}^H\mathbf{H}]^{-1}_{M,M}}{2}\right]} = \frac{\rho}{\frac{3}{2}\mathbf{h}_{mR}^H\left(\mathbf{G}^{-1}\right)^H\mathbf{G}^{-1}\mathbf{h}_{mR}\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{m,m}}. \quad (18)$$

In the above equation, we have used the fact that $\tilde{\mathbf{h}}_m\tilde{\mathbf{h}}_m^H$ is the same as $\left[\mathbf{H}^H\mathbf{H}\right]^{-1}_{M,M}$. It has been shown in [24] that the element $\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{m,m}$ can be expressed as follows

$$\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{m,m} = \frac{\det(\tilde{\mathbf{H}}_m^H\tilde{\mathbf{H}}_m)}{\det(\mathbf{H}^H\mathbf{H})} = \frac{1}{\mathbf{h}_{mR}^H\left(\mathbf{I}_M - \tilde{\mathbf{P}}_m\right)\mathbf{h}_{mR}},$$

where $\tilde{\mathbf{H}}_m = \begin{bmatrix}\mathbf{h}_{1R} & \cdots & \mathbf{h}_{(m-1)R} & \mathbf{h}_{(m+1)R} & \cdots & \mathbf{h}_{MR}\end{bmatrix}$ and $\tilde{\mathbf{P}}_m = \tilde{\mathbf{H}}_m(\tilde{\mathbf{H}}_m^H\tilde{\mathbf{H}}_m)^{-1}\tilde{\mathbf{H}}_m^H$. Furthermore, by using the facts that $\tilde{\mathbf{P}}_m$ is an idempotent matrix and it only has one non-zero eigenvalue, we can express the inverse matrix in the SNR expression as follows

$$\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{m,m} = \frac{1}{\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}}, \quad (19)$$

where $\mathbf{u}_m$ is the eigenvector of $\left(\mathbf{I}_M - \tilde{\mathbf{P}}_m\right)$ corresponding to the eigenvalue 1. As a result, the data rate supportable at the m-th mobile user can be expressed as $\mathcal{I}_{U_m} = \log(1 + SNR_{U_m})$.

To obtain a better understanding for the overall system performance, the information theoretic metrics, the outage probability and the diversity-multiplexing tradeoff, will be used. As in [25], the diversity gain is defined as $d \triangleq -\lim_{\rho\to\infty}\frac{\log[P_e(\rho)]}{\log\rho}$, and $r \triangleq \lim_{\rho\to\infty}\frac{R(\rho)}{\log\rho}$, where P_e is the ML probability of detection error. As discussed in [25], [26], the outage probability can tightly bound the ML error probability at high SNR.

By using the simplified expression of the SNR, now the outage probability for the m-th mobile user can be expressed as

$$P\left(\mathcal{I}_{U_m} < 2R\right) = P\left(\frac{\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}}{\mathbf{h}_{mR}^H\left(\mathbf{G}^{-1}\right)^H\mathbf{G}^{-1}\mathbf{h}_{mR}} < \frac{3(2^{2R}-1)}{2\rho}\right), \quad (20)$$

Note that the constant in front of R is 2 is due to the fact that 2 time slots have been used for the network coding transmissions. The following theorem is provided to show the outage probability at the m-th mobile receiver achieved by the proposed network coding protocol.

Theorem *1:* Through the downlink channels, at the m-th mobile user, the achievable outage probability for the proposed network coding transmission protocol can be approximated as

$$P\left(\mathcal{I}_{U_m} < 2R\right) \leq -\frac{3M(M-1)}{4}\left(\frac{2^{2R}-1}{\rho}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right), \tag{21}$$

when $\rho \to \infty$. The achievable diversity-multiplexing tradeoff for the m-th downlink transmission can be expressed as

$$d_{U_m}(r) = 1 - 2r,$$

for the multiplexing gains $0 \leq r \leq \frac{1}{2}$.

Proof: Please refer to the appendix. ■

Theorem 1 demonstrates that the use of the proposed network coding protocol can ensure all users experience the same outage performance through the downlink channels and the diversity gain for all users will be one, exactly the same as the single-input single-output direct transmission scheme without co-channel interference. Note that traditional MIMO transmission schemes will need at least 4 time slots. Specifically during the first time slot, the base station uses the MIMO transmission techniques and delivers M messages to the relay, and during the second time slot, the relay forwards the messages to the mobile users. Another two time slots are required to deliver messages from the mobile users to the base station. Apparently the use of the proposed protocol can decrease the system overhead significantly.

B. Performance analysis for the receiver reliability at the base station

At the base station, the signal model is more complicated than the ones at the mobile users. Recall that during the second time slot, the base station receives

$$\mathbf{y}_{BS} = \mathbf{G}^H\left(\left(\mathbf{H}^H\right)^{-1}\mathbf{D}_r(\mathbf{s}+\mathbf{u}) + \left(\mathbf{H}^H\right)^{-1}\mathbf{D}_r\mathbf{H}^{-1}\mathbf{n}_R\right) + \mathbf{n}_{BS}. \tag{22}$$

Again applying zero-forcing approaches, removing the information known at the base station and after some algebraic manipulations, we can obtain

$$\mathbf{D}_r^{-1}\mathbf{H}^H(\mathbf{G}^H)^{-1}\mathbf{y}_{BS} = \mathbf{u} + \mathbf{H}^{-1}\mathbf{n}_R + \mathbf{D}_r^{-1}\mathbf{H}^H(\mathbf{G}^H)^{-1}\mathbf{n}_{BS}. \tag{23}$$

Hence the SNR for the m-th user's information, u_i, at the base station can be expressed as

$$SNR_{BS_m} = \frac{\rho}{[\mathbf{H}^{-1}(\mathbf{H}^H)^{-1} + \mathbf{D}_r^{-1}\mathbf{H}^H(\mathbf{G}^H)^{-1}\mathbf{G}^{-1}\mathbf{H}\mathbf{D}_r^{-1}]_{m,m}}. \tag{24}$$

Using the similar steps to the previous section, we obtain

$$\begin{aligned} SNR_{BS_m} &= \frac{\rho}{[(\mathbf{H}^H\mathbf{H})^{-1}]_{m,m} + [\mathbf{D}_r^{-1}\mathbf{H}^H\mathbf{U}\Lambda^{-1}\mathbf{U}^H\mathbf{H}\mathbf{D}_r^{-1}]_{m,m}} \\ &= \frac{\rho\mathbf{h}_{mR}^H\mathbf{v}\mathbf{v}^H\mathbf{h}_{mR}}{1 + \frac{1}{2}\mathbf{h}_{mR}^H\mathbf{U}\Lambda^{-1}\mathbf{U}^H\mathbf{h}_{mR}}. \end{aligned} \tag{25}$$

As a result, the mutual information achievable for the m-th user's information at the base station is $\mathcal{I}_{BS_m} = \log(1 + SNR_{BS_m})$. The following theorem provides the outage probability for the i-th user's information at the base station.

Theorem *2:* Through uplink channels, at the base station, the achievable outage probability for the i-th user's information by using the proposed network coding transmission protocol can be approximated as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq -\frac{M(M-1)}{2}\left(\frac{2^{2R}-1}{\rho}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right), \tag{26}$$

when $\rho \to \infty$. And the achievable diversity-multiplexing tradeoff for the m-th uplink transmission can be expressed as

$$d_{BS_m}(r) = 1 - 2r,$$

for the multiplexing gains $0 \leq r \leq \frac{1}{2}$.

Proof: Please refer to the appendix. ■

Compared Theorem 1 to Theorem 2, we can easily find out that the receive performance at the mobile users and the base station is quite similar, where the outage probabilities of all M uplink and M donwlink transmissions are proportional to $\frac{2^{2R}-1}{\rho}$.

In the above, we have studied the outage performance of the M downlink and M uplink transmissions separately. To obtain a better understanding of the impact of the proposed network coding transmission protocol on the overall system performance, the sum rate and the worst performance among the $2M$ transmissions will be studied in the following. The following corollary about the overall diversity-multiplexing tradeoff can be obtained by applying the two theorems.

Corollary *3:* The overall diversity-multiplexing tradeoff for the sum rate achieved by the proposed network coding protocol can be shown as follows

$$d(r) = 1 - \frac{1}{M}r, \tag{27}$$

for $0 \leq r \leq M$. The worst outage performance among the M uplink and M downlink transmissions is

$$P(\mathcal{I}_{min} < 2R) \leq -2M^2(M-1)\left(\frac{2^{2R}-1}{\rho}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right),$$

where $\mathcal{I}_{min} = \min\{\mathcal{I}_{BS_1}, \cdots, \mathcal{I}_{BS_M}, \mathcal{I}_{U_1}, \cdots, \mathcal{I}_{U_M}\}$ and the high SNR assumption has been used.

Proof: The sum rate achieved by the proposed network coding scheme can be expressed as

$$\mathcal{I} = \sum_{m=1}^{M}(\mathcal{I}_{BS_m} + \mathcal{I}_{U_m})$$

where $\mathcal{I}_{BS_m} = \log(1 + SNR_{BS_m})$ and $\mathcal{I}_{U_m} = \log(1 + SNR_{U_m})$. The overall outage probability based on the sum rate can be expressed as

$$\begin{aligned} P\left(\frac{\mathcal{I}}{2} \leq r\log\rho\right) &= P\left(\sum_{m=1}^{M}(\mathcal{I}_{BS_m} + \mathcal{I}_{U_m}) \leq 2r\log\rho\right) \\ &\leq P\left(M\mathcal{I}_{BS,min} + M\mathcal{I}_{U,min} \leq 2r\log\rho\right) \end{aligned} \tag{28}$$

where $\mathcal{I}_{BS,min} = \min\{\mathcal{I}_{BS_1}, \cdots, \mathcal{I}_{BS_M}\}$ and $\mathcal{I}_{U,min}$ is defined similarly. The above outage probability can be further upper bounded as

$$\begin{aligned} P\left(\frac{\mathcal{I}}{2} \leq r\log\rho\right) &\leq P\left(\min\{\mathcal{I}_{BS,min}, \mathcal{I}_{U,min}\} \leq \frac{r}{M}\log\rho\right) \\ &\leq P\left(\mathcal{I}_{BS,min} \leq \frac{r}{M}\log\rho\right) + P\left(\mathcal{I}_{U,min} \leq \frac{r}{M}\log\rho\right) \end{aligned} \tag{29}$$

Now we can apply the two theorems and the outage probability can be obtained as

$$\begin{aligned} P\left(\frac{\mathcal{I}}{2} \leq r\log\rho\right) &\leq \sum_{m=1}^{M}\left(P\left(\mathcal{I}_{BS_m} \leq \frac{r}{M}\log\rho\right) + P\left(\mathcal{I}_{U_m} \leq \frac{r}{M}\log\rho\right)\right) \\ &\doteq \rho^{-\left(1-\frac{r}{M}\right)} \end{aligned} \tag{30}$$

where $f(\rho)$ is said to be exponentially equal to ρ^d, denoted as $f(\rho) \doteq \rho^d$, when $\lim\limits_{\rho\rightarrow\infty} \frac{\log[f(\rho)]}{\log\rho} = d$. The worst performance among the $2M$ uplink and downlink transmissions can be obtained similarly. ■

Note that traditional network coding schemes, such as the ones in [6], [10], can be applied to the addressed communication scenario by applying time sharing approaches among the multiple users. However, such a straightforward application of the existing network coding scheme can only support the multiplexing gain one. As indicated by Corollary 3, the multiplexing gain achieved by the proposed network coding scheme is M, much larger than the existing network coding schemes. Apparently the diversity gain achieved by the proposed scheme is still only one, and we will study how to improve the diversity gain the next section.

IV. Approaches to Improve Reception Reliability

As can be seen from the previously developed analytical results, the use of the proposed network coding scheme can ensure information exchange between the base station and the M single-antenna users within two time slots, where co-channel interference can be effectively handled without degrading the reception reliability. Compared to the single user network coding scheme, the proposed multiuser scheme achieved exactly the same diversity order. In the this section, we study how to improve the reception reliability of the addressed communication system by increasing the number of relays and the number of antennas at the relay and the base station.

A. When the number of relays is larger than one $L \geq 1$

In this section, we focus on the scenario that the base station has M antennas, each mobile is equipped with a single antenna, and there are L relays each of which is equipped with M antennas. When there are multiple relays, different approaches can be applied to use the available relays. One option is to apply distributed beamforming which provides the superior performance; however, the coordination among multiple relay transmissions can result in huge system overhead. For example, distributed beamforming invites all relays to transmit, which requires tight phase synchronization among multiple transmitters.

Note that huge system overhead will be consumed to achieve such rigorous coordination among the transmitters. On the other hand, the use of relay selection only requires one transmitter, which causes less system overhead compared to distributed beamforming. In addition, relay selection can be realized in a distributed way, which can avoid the use of the global CSI assumption and hence further reduce system overhead. As shown in [27], each relay individually calculates its backoff period inversely proportional to its channel condition, so the relay with the best channel condition can get the control of the channel. In such a way, there is no need for a super-node which has the access to the global CSI. Therefore in this section, we only focus on the use of a single best relay.

Provided that only the best relay will be used, the network coding protocol proposed in the previous section can be easily applied to the addressed scenario. The key questions are what the criterion for relay selection is, and what kind of outage performance can be achieved. Provided that the i-th relay, R_i, is used, the SNR at each mobile user can be written as

$$SNR_{U_m,i} = \rho \frac{2\mathbf{h}_{mR_i}^H \mathbf{u}_{m,i} \mathbf{u}_{m,i}^H \mathbf{h}_{mR_i}}{3\mathbf{h}_{mR_i}^H \mathbf{G}^{-1} \mathbf{G}^{-1} \mathbf{h}_{mR_i}} \tag{31}$$

and the SNR at the base station for the i-th user can be

$$SNR_{BS_m,i} = \rho \frac{\mathbf{h}_{mR_i}^H \mathbf{u}_{m,i} \mathbf{u}_{m,i}^H \mathbf{h}_{mR_i}}{1 + \frac{1}{2}\mathbf{h}_{mR_i}^H \mathbf{G}^{-1} \mathbf{G}^{-1} \mathbf{h}_{mR_i}}, \tag{32}$$

where $\mathbf{h}_{mR_i}$ is the channel between the m-th user and the i-th relay and $\mathbf{u}_{m,i}$ is defined similarly.

Since the user with the worst performance dominates the overall system performance, our goal for the relay selection is to maximize the reliability for the worst user, which can be formulated as the following

$$\begin{aligned} &\arg\min_i && P\left(\min\left\{\mathcal{I}_{BS,i}, \mathcal{I}_{U,i}\right\} < 2R\right). \\ &s.t. && \mathcal{I}_{BSm,i} = \log\left(1 + SNR_{BS_m,i}\right), \quad \mathcal{I}_{Um,i} = \log\left(1 + SNR_{U_m,i}\right) \\ &s.t. && \mathcal{I}_{BS,i} = \min\left\{\mathcal{I}_{BS_1,i}, \cdots, \mathcal{I}_{BS_M,i}\right\}, \quad \mathcal{I}_{U,i} = \min\left\{\mathcal{I}_{U_1,i}, \cdots, \mathcal{I}_{U_M,i}\right\}. \end{aligned} \tag{33}$$

The following lemma provides the achievable outage probability for the strategy of the relay selection.

Lemma *4:* Provided that there are L relays, the worst outage performance among the $2M$ uplink and downlink transmissions achieved by the proposed network coding with relay selection can be upper bounded as

$$P_{out,worst,L} \leq \left[-2M(M-1)\left(\frac{2^{2R}-1}{\rho}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right)\right]^L, \tag{34}$$

for $\rho \to \infty$ and the corresponding diversity-multiplexing tradeoff can be expressed as

$$d_{worst,L}(r) = L(1-2r),$$

for $0 \leq r \leq \frac{1}{2}$.

Proof: Define i^* as the index for the relay which is selected by the above optimization problem. By using such a notation, the overall outage probability for the proposed network coding scheme with relay selection can be expressed as

$$\begin{aligned} P\left(\min\left\{\mathcal{I}_{BS,i^*},\mathcal{I}_{U,i^*}\right\}<2R\right) &= P\left(\max\left\{\min\{\mathcal{I}_{BS,1},\mathcal{I}_{U,1}\},\cdots,\min\{\mathcal{I}_{BS,L},\mathcal{I}_{U,L}\}\right\}<2R\right) \\ &= \left[P\left(\min\{\mathcal{I}_{BS,i},\mathcal{I}_{U,i}\}<2R\right)\right]^L \end{aligned} \tag{35}$$

where the second equation follows from the fact that the use of different relays can ensure that $SNR_{BS_m,i}$ and $SNR_{BS_m,j}$ are independent. By applying Corollary 3, the lemma can be easily obtained. ∎

B. When the number of the antennas at the relay and the base station is larger than M

In this section, we focus on the scenario where the base station has Q antennas, the single relay has N antennas, and each of the M mobile users is equipped with a single antenna, $Q > N > M$. The motivation to study such a scenario is that the base station typically has the best capability in its cell, and therefore it is reasonable to assume that the base station has the largest number of antennas, where some idle users' handsets, acting as relays, are more capable than the others. For such a scenario, the question of interest is what the order of the achievable diversity gain will be, which will be focused in the following.

Apparently when the number of the relay and base station antennas is larger than M, the fact that the channel matrices, $\mathbf{H}$ and $\mathbf{G}$, are no longer square implies that the pseudo-inverse should be used in place of the inverse in (4) and (12). Without too much modifications to the proposed network coding protocol, we use the following simple form for the precoding matrix at the base station

$$\mathbf{P} = \sqrt{\theta_1}\mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{H}, \tag{36}$$

where the factor θ_1 is to ensure the transmission power at the base station is normalized

$$\frac{1}{\theta_1} = \frac{\mathcal{E}\{\text{trace}(\mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{H}\mathbf{H}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G})\}}{Q} = \frac{\mathcal{E}\{\text{trace}(\mathbf{H}\mathbf{H}^H(\mathbf{G}\mathbf{G}^H)^{-1})\}}{Q}.$$

where the factor Q is due to that the base station has Q antennas. As discussed in Section II it is important for power conservation that $\mathcal{E}\{\text{trace}(\mathbf{H}\mathbf{H}^H(\mathbf{G}\mathbf{G}^H)^{-1})\}$ is bounded. Actually this is indeed the case as shown in the appendix.

By using such a precoding matrix, during the second time slot, the observations at the base station can be expressed as

$$\mathbf{y}_{BS} = \mathbf{G}^H\mathbf{W}\left(\mathbf{H}\sqrt{\theta_1}\mathbf{s}+\mathbf{H}\mathbf{u}+\mathbf{n}_R\right)+\mathbf{n}_{BS} \tag{37}$$

and the observation at the m-th user can be expressed as

$$y_m = \mathbf{h}_{mR}^H\mathbf{W}\left(\mathbf{H}\sqrt{\theta_1}\mathbf{s}+\mathbf{H}\mathbf{u}+\mathbf{n}_R\right)+n_m. \tag{38}$$

To remove co-channel interference at the mobile stations, we use the following precoding matrix

$$\mathbf{W} = \sqrt{\theta_2}\mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H, \tag{39}$$

where θ_2 is the power normalization which can be obtained as follows [21]

$$\frac{1}{\theta_2} = \frac{\mathcal{E}\{\text{trace}(\mathbf{W}((1+\theta_1)\mathbf{H}\mathbf{H}^H + \rho^{-1}\mathbf{I}_N)\mathbf{W}^H)\}}{N} \approx \frac{(1+\theta_1)\mathcal{E}\{\text{trace}(\left(\mathbf{H}^H\mathbf{H}\right)^{-1})\}}{N} = \frac{(1+\theta_1)M}{N(N-M)}.$$

By using this precoding matrix, the SNR at the m-th mobile user can be written as

$$SNR_{U_m} = \frac{\theta_2\theta_1\rho}{1+\theta_2\left[(\mathbf{H}^H\mathbf{H})^{-1}\right]_{m,m}}, \tag{40}$$

and the corresponding mutual information is $\mathcal{I}_{U_m} = \log(1+SNR_{U_m})$. The SNR for the m-th user's information at the base station can be written as

$$SNR_{BS_m} = \frac{\theta_2\rho}{\theta_2\left[(\mathbf{H}^H\mathbf{H})^{-1}\right]_{m,m} + \mathbf{h}_{iR}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{h}_{iR}} \tag{41}$$

the corresponding mutual information is $\mathcal{I}_{BS_m} = \log(1+SNR_{BS_m})$. The following lemma provides the outage probability achievable for the proposed network coding scheme.

Lemma *5:* Consider that the base station has Q antennas, the relay has N antennas and all of the M users are equipped with a single antenna, $Q > N > M$. Through the downlink channels, at the m-th mobile user, the achievable outage probability for the proposed network coding transmission protocol can be approximated at high SNR as

$$P\left(\mathcal{I}_{U_m} < 2R\right) \approx \frac{1}{(N-M+1)!}\frac{1}{\theta_1^{N-M+1}}\left(\frac{2^{2R}-1}{\rho}\right)^{N-M+1}. \tag{42}$$

Through the uplink channels, at the base station, the outage probability for the m-th user's information achieved by the proposed network coding transmission protocol can be expressed as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq \max\left\{\frac{\Gamma(Q)}{\Gamma(N)}\frac{\phi\left(\frac{1}{\theta_2(1-\frac{1}{c})}\right)^{Q-N}}{(Q-N)}\left(\frac{2^{2R}-1}{\rho}\right)^{Q-N}, \frac{c^{N-M+1}}{(N-M+1)!}\left(\frac{2^{2R}-1}{\rho}\right)^{N-M+1}\right\},$$

where the constants, c and ϕ, are defined in the proof.

Proof: Please refer to the appendix. ■

As can be seen from the lemma, the expression of the outage performance for uplink transmissions is more complicated than that for the downlink transmissions. Note that for the special case where $Q = N = M$, the upper bound given in (84) is quite loose and λ_{min} becomes exponentially distributed. Substituting such a distribution into (85) we can find that the outage performance for the uplink transmissions as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq \left(\frac{2^{2R}-1}{\rho}\right)\cdot\max\left\{\phi\left(\frac{N}{\theta_2(1-\frac{1}{c})}\right), \frac{c^{N-M+1}}{(N-M+1)!}\right\}, \tag{43}$$

which still provides the same diversity-multiplexing tradeoff as the scheme proposed in Section II. Note that the exact expressions of the outage probabilities achieved by the protocol in this section and the one in Section II are not the same since different precoding matrix has been used in this section to simplify the analytic development as shown in (36).

V. Numerical Results

In this section, the performance of the proposed network coding transmission protocol will be evaluated by using Monte-Carlo simulations. The scheme it is compared to is based on the time sharing physical layer network coding scheme [6], [10]. Specifically, each user takes turns to be paired with the base station and the information exchange between the user and the base station can be accomplished within two time slots by using physical layer network coding. For simplicity, both the base station and the relay will only use a single antenna selected by the optimal antenna selection strategy. The elements of the channel and noise matrices are zero-mean, circular complex Gaussian random variables, where the variances of the channel and noise are set according to the signal-to-noise ratio. A symmetric system is considered here where all pairs of sources and destinations have the same target data rate R.

In Fig. 2, the target data rate has been set as $R = 1$ bit per channel use (BPCU), where the outage performance of the proposed and time sharing network coding schemes are compared with different choices of M. Note that the outage performance shown in Figs. 2, 3, 5 represents the worst user performance, i.e., $P(\min\{\mathcal{I}_{U_1}, \cdots, \mathcal{I}_{U_M}, \mathcal{I}_{BS_1}, \cdots, \mathcal{I}_{BS_M}\} < R)$. As can be seen from the figure, the proposed network coding scheme can achieve better outage performance than the time sharing one, particularly when the number of the users is larger. Such a performance gain is due to the fact that the proposed transmission scheme only requires two time slots no matter how many users are involved, whereas the time sharing scheme needs $2M$ time slots. As a result, when the number of the users is larger, the performance degradation of the time sharing network coding scheme is much more significant than the proposed protocol. Or in other words, the proposed network coding scheme is not as sensitive to the changes of the user number as the time sharing approach. Another observation from Fig. 2 is that the time sharing scheme can achieve larger diversity gain than the proposed protocol.

In Fig. 3, we fixed the parameter of the number of users, $M = 3$, but used different values for the target data rate. In general, increasing the target data rate will decrease the performance of both schemes since the outage event is more likely to happen for a larger value of R. However, the proposed network coding scheme can achieve better outage performance than the time sharing protocol in general, and the performance gap between the two network coding schemes can be further increased by increasing the target data rate. Such a performance gain is due to the fact that the proposed scheme can achieve a multiplexing gain up to M, whereas the time sharing scheme can only achieve a multiplexing gain up to one. This performance gain can also be explained by using Fig. 4.

In Fig. 4, the averaged sum rate has been used as the criterion for the performance evaluation. As can be observed from the figure, the proposed network coding protocol can yield a significant capacity improvement compared to the time sharing protocol. When the number of the users is increased, it is interesting to observe that the performance of the comparable approach does not increase significantly, which is due to the use of the time sharing approach. However for the proposed network coding scheme, the more users participate in cooperation, the larger the sum rate can be. Such a performance gain is due to careful coordination among the base station and relay transmissions, where all $2M$ uplink and downlink transmissions can be accomplished within two time slots. Obviously the more users are involved, the more antennas are required at the base station and the relay, which could cause extra system complexity.

As stated in Theorem 1 and 2, the diversity gain achieved by the proposed scheme is only one, which can also be confirmed from Figs. 2 and 3. Hence in Fig. 5, we study the impact of the relay selection strategy on the outage performance. Again the number of the users is fixed at $M = 3$, and we used different choices of the number of relays. As can be observed from the figure, the curves of the outage performance become steeper when the number of the relays is larger, which implies that the diversity gain achieved by the proposed scheme is proportional to the number of relays.

Finally in Fig. 6 we study the performance of the proposed scheme in the scenario that there is only one relay, but the number of the relay and base station antennas is larger than M. As can be seen from the figure, increasing the number of antennas can improve the outage performance of the proposed network coding schemes. It can be observed that the performance for the worst downlink can be better than the worst uplink. This is due to the fact that the performance of the receivers at the single antenna mobile users has been put as the top priority when the proposed network coding scheme was designed. Such an asymmetrical configuration is important to mobile broadband service which requires higher data rate for downlink than uplink.

VI. Conclusion

In this paper, we first focused the scenario where the base station and the relay have M antennas, and all M mobile stations only have a single antenna. A new network coding transmission protocol has been proposed, where $2M$ uplink and downlink transmissions can be accomplished within two time slots. The key step to avoid co-channel interference is to carefully design the precoding matrices at the base station and relay by pairing messages to and from the same mobile users. Explicit analytic results have been developed and demonstrated that the multiplexing gain achieved by the proposed transmission protocol is M, much better than existing time sharing schemes. To further increase the achievable diversity gain, two transmission protocols have also been proposed when there are multiple relays and the number of the antennas at the base station and relay is increased. Numerical results have been provided to demonstrate the performance of the proposed network coded transmission protocol with the comparison to the time sharing based network coding protocol.

Appendix

Proof for Theorem 1: Define the following eigenvalue decomposition $\mathbf{G}\mathbf{G}^H = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where λ_{min} is the smallest eigenvalue of $\mathbf{G}\mathbf{G}^H$. Therefore the eigenvalue decomposition of $\left(\mathbf{G}\mathbf{G}^H\right)^{-1}$ can be expressed as $\left(\mathbf{G}\mathbf{G}^H\right)^{-1}=\mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^H$, where λ_{min} becomes the largest eigenvalue of $\left(\mathbf{G}\mathbf{G}^H\right)^{-1}$. By using such an eigenvalue, an upper bound of the outage probability can be obtained as follows

$$\begin{aligned} P\left(SNR_{U_m}<2^{2R}-1\right) &= P\left(\frac{\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}}{\mathbf{h}_{mR}^H\mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^H\mathbf{h}_{mR}}<\frac{3(2^{2R}-1)}{2\rho}\right) \\ &\leq P\left(\frac{\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}}{\lambda_{min}^{-1}\mathbf{h}_{mR}^H\mathbf{h}_{mR}}<\frac{3(2^{2R}-1)}{2\rho}\right), \end{aligned} \tag{44}$$

Apparently $\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}$ is correlated to $\mathbf{h}_{mR}^H\mathbf{h}_{mR}$, which complicates the development. Therefore we further simplify the expression of the upper bound as

$$\begin{aligned} P\left(SNR_{U_m}<2^{2R}-1\right) &\leq P\left(\mathbf{h}_{mR}^H\mathbf{u}_m\mathbf{u}_m^H\mathbf{h}_{mR}<\theta\lambda_{min}^{-1}\mathbf{h}_{mR}^H\mathbf{h}_{mR}\right) \\ &= P\left(\mathbf{h}_{mR}^H\mathbf{U}_m\left(\mathbf{D}_1-\theta\lambda_{min}^{-1}\mathbf{I}_M\right)\mathbf{U}_m^H\mathbf{h}_{mR}<0\right), \end{aligned} \tag{45}$$

where $\mathbf{D}_1=\text{diag}\{1,0,\cdots 0\}$, $\mathbf{U}_m$ collects all eigenvectors of $(\mathbf{I}_M-\tilde{\mathbf{P}}_m)$, and $\theta=\frac{3(2^{2R}-1)}{2\rho}$. Given the facts that $\mathbf{U}_m$ is an unitary matrix and independent of $\mathbf{h}_{mR}$, the statistical properties of $\mathbf{U}_m\mathbf{h}_{mR}$ will be the same as $\mathbf{h}_{mR}$. As a result, define $\tilde{\mathbf{h}}_m=\mathbf{U}_m\mathbf{h}_{mR}$ where each element from this vector is independent and identically complex Gaussian distributed. By using such a vector, the upper bound of the outage probability can be now expressed as

$$\begin{aligned} P\left(SNR_{U_m}<2^{2R}-1\right) &\leq P\left(\tilde{\mathbf{h}}_{mR}^H\left(\mathbf{D}_1-\theta\lambda_{min}^{-1}\mathbf{I}_M\right)\tilde{\mathbf{h}}_{mR}<0\right) \\ &= P\left((1-\theta\lambda_{min}^{-1})|\tilde{h}_{mR,1}|^2-\theta\lambda_{min}^{-1}\sum_{n=2}^{M}|\tilde{h}_{mR,n}|^2<0\right). \end{aligned} \tag{46}$$

To simplify the notation, define $x=|\tilde{h}_{mR,1}|^2$, $y=\sum_{n=2}^{M}|\tilde{h}_{mR,n}|^2$, $a=1-\theta\lambda_{min}^{-1}$ and $b=\theta\lambda_{min}^{-1}$. Because the virtual channels are still independent and identically complex Gaussian distributed, x will be exponentially distributed with unit variance. y will be Chi-square distributed with degree of freedom $(M-1)$, so its pdf will be $f_y(f)=\frac{y^{M-2}}{\Gamma(M-1)}e^{-y}$. Note that x, y and λ_{min} are independent distributed. Using these variables, the upper bound of the outage probability can be expressed as

$$P\left(SNR_{U_m}<2^{2R}-1\right) \leq P\left(ax-by<0\right), \tag{47}$$

Based on the value of λ_{min}, the upper bound of the outage probability can be expressed as

$$\begin{aligned} P\left(\mathcal{I}_{U_m}<2R\right) \leq\ & P\left(x<\frac{by}{a}\middle|1-\theta\lambda_{min}^{-1}>0\right)P(1-\theta\lambda_{min}^{-1}>0) \\ &+P\left(x>\frac{by}{a}\middle|1-\theta\lambda_{min}^{-1}<0\right)P(1-\theta\lambda_{min}^{-1}<0). \end{aligned} \tag{48}$$

In the following, we first focus on the calculation of the first probability $P\left(x<\frac{by}{a}\middle|1-\theta\lambda_{min}^{-1}>0\right)$ which can be expressed as

$$P\left(x<\frac{by}{a}\middle|\lambda_{min}>\theta\right)=\mathcal{E}_{\lambda_{min},\lambda_{min}>\theta}\left\{\int_0^\infty\int_0^{\frac{by}{a}}f_x(x)dxf_y(y)dy\right\}=\mathcal{E}_{\lambda_{min},\lambda_{min}>\theta}\left\{1-\frac{1}{\left(\frac{b}{a}+1\right)^{(M-1)}}\right\},$$

where $\mathcal{E}_{x\in\mathcal{D}}\{\cdot\}$ denotes the expectation operation by treating x as the variable with the constraint $\mathcal{D}$, and the last equation follows from Eq. (3.38.4) in [28]. To find the expectation of the factor in the above equation, the density function of the minimum eigenvalue λ_{min} is required. Fortunately because $\mathbf{G}$ is a square complex Gaussian matrix, the expression of its smallest eigenvalue is quite simple. As shown in [29], the pdf of the minimum eigenvalue of $\mathbf{G}^H\mathbf{G}$ is exponentially distributed with the parameter $\frac{M}{2}$. By using such a pdf, we can express the probability as

$$P\left(x<\frac{by}{a}\bigg|\lambda_{min}>\theta\right) = \int_\theta^\infty \left(1-\frac{1}{\left(\frac{\frac{\theta}{\lambda}}{1-\frac{\theta}{\lambda}}+1\right)^{(M-1)}}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda$$
$$= \int_\theta^\infty \left(1-\left(1-\frac{\theta}{\lambda}\right)^{M-1}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda. \quad (49)$$

Since $\lambda\geq\theta$, we can have $\frac{\theta}{\lambda}\leq 1$. As a result, the factor $\left(1-\frac{\theta}{\lambda}\right)^{M-1}$ can be expressed as a summation of a series as follows [28]

$$\left(1-\frac{\theta}{\lambda}\right)^{M-1} = 1-(M-1)\frac{\theta}{\lambda}+\frac{(M-1)(M-2)}{2!}\left(\frac{\theta}{\lambda}\right)^2-\cdots. \quad (50)$$

Substituting this equation to the probability $P\left(x<\frac{by}{a}\big|\lambda_{min}>\theta\right)$ can be expressed as

$$P\left(x<\frac{by}{a}\bigg|\lambda_{min}>\theta\right) = \int_\theta^\infty \left((M-1)\frac{\theta}{\lambda}-\frac{(M-1)(M-2)}{2!}\left(\frac{\theta}{\lambda}\right)^2-\cdots\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda. \quad (51)$$

In the above equation, the key integral will be $\int_\theta^\infty\left(\frac{\theta}{\lambda}\right)^n\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda$, and such an integral can be rewritten as

$$\int_\theta^\infty\left(\frac{\theta}{\lambda}\right)^n\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda \underset{\frac{M}{2}\lambda=z}{=} \left(\frac{M}{2}\right)^n\theta^n\int_{\frac{M}{2}\theta}^\infty\frac{1}{z^n}e^{-z}dz$$
$$= \left(\frac{M}{2}\right)^n\theta^n\left(\frac{M\theta}{2}\right)^{-n/2}e^{-\frac{M\theta}{4}}W_{-\frac{n}{2},\frac{1-n}{2}}\left(\frac{M\theta}{2}\right), \quad (52)$$

where $W(\cdot)$ denotes the Whittaker function and the last equation follows from Eq. (3.381.6) in [28]. By using the property of Whittaker functions $W_{-\frac{n}{2},\frac{1-n}{2}}\left(\frac{M\theta}{2}\right)=W_{-\frac{n}{2},\frac{n-1}{2}}\left(\frac{M\theta}{2}\right)$. The above integral can be expressed as

$$\int_\theta^\infty\left(\frac{\theta}{\lambda}\right)^n\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda = \left(\frac{M\theta}{2}\right)^{n/2}e^{-\frac{M\theta}{4}}W_{-\frac{n}{2},\frac{n-1}{2}}\left(\frac{M\theta}{2}\right)$$
$$= \frac{\left(\frac{M\theta}{2}\right)^n}{\Gamma(n)}\int_1^\infty e^{-\frac{M\theta}{2}x}(x-1)^{n-1}x^{-1}dx = \left(\frac{M\theta}{2}\right)^n\Gamma(-n+1,\frac{M}{2}\theta), \quad (53)$$

where the second equation follows from another integral presentation of the Whittaker function, the last equation follows from Eq. 3.383.9 in [28], $\Gamma(x)$ denotes the gamma function and $\Gamma(x,y)$ denotes the incomplete gamma function. By using the series expansion of the incomplete gamma function, the above integral can be expressed as

$$\int_{\theta}^{\infty}\left(\frac{\theta}{\lambda}\right)^{n}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda = \left(\frac{M\theta}{2}\right)^{n}\frac{(-1)^{n+1}}{(n-1)!}\left[-Ei\left(-\frac{M\theta}{2}\right)-e^{-\frac{M\theta}{2}}\sum_{m=0}^{n-2}(-1)^{m}\frac{m!}{\left(\frac{M\theta}{2}\right)^{m+1}}\right] \quad (54)$$

$$\approx \left(\frac{M\theta}{2}\right)^{n}\frac{(-1)^{n+1}}{(n-1)!}\left[-Ei\left(-\frac{M\theta}{2}\right)-(-1)^{n-1}\frac{(n-1)!}{\left(\frac{M\theta}{2}\right)^{n-1}}\right], \quad (55)$$

for $n \geq 2$, and

$$\int_{\theta}^{\infty}\left(\frac{\theta}{\lambda}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda = \left(\frac{M\theta}{2}\right)\left[-Ei\left(-\frac{M\theta}{2}\right)\right], \quad (56)$$

for the case $n = 1$, where $Ei(x)$ denotes the exponential integral function.

As a result, the addressed probability can now be expressed as

$$P\left(x < \frac{by}{a}\middle|\lambda_{min} > \theta\right) = \int_{\theta}^{\infty}\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\left(\frac{\theta}{\lambda}\right)^{n}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda \quad (57)$$

$$= \sum_{n=2}^{M-1}\binom{M-1}{n}\frac{\left(\frac{M\theta}{2}\right)^{n}}{(n-1)!}\left[-Ei\left(-\frac{M\theta}{2}\right)-(-1)^{n-1}\frac{(n-1)!}{\left(\frac{M\theta}{2}\right)^{n-1}}\right]+(M-1)\left(\frac{M\theta}{2}\right)\left[-Ei\left(-\frac{M\theta}{2}\right)\right].$$

Note that the exponential integral function can have the following approximation

$$-Ei\left(-\frac{M\theta}{2}\right) = -\mathbf{C}-\ln\left(\frac{M\theta}{2}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{M\theta}{2}\right)^{k}}{k\cdot k!} \approx -\ln\left(\frac{2^{2R}-1}{\rho}\right).$$

since $\left(\frac{2^{2R}-1}{\rho}\right)\theta \to 0$ for $\rho \to \infty$. Note that $\mathbf{C}$ denotes the Euler constant. By using such an approximation and the fact that $x\ln(x) \to 0$ for $x \to 0$, we express the probability in (57) as

$$P\left(x < \frac{by}{a}\middle|\lambda_{min} > \theta\right) = (-1)^{n-2}\sum_{n=2}^{M-1}\binom{M-1}{n}\left(\frac{M\theta}{2}\right)-(M-1)\left(\frac{M\theta}{2}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right)$$

$$\approx -(M-1)\left(\frac{M\theta}{2}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right). \quad (58)$$

On the other hand, it can be easily found that

$$P(1-\theta\lambda_{min}^{-1} < 0) = 1-e^{-\frac{M}{2}\theta} \approx \frac{M}{2}\theta. \quad (59)$$

Hence finally we can have

$$P\left(\mathcal{I}_{U_m} < 2R\right) \leq P\left(x < \frac{by}{a}\middle|\lambda_{min} > \theta\right)P(\lambda_{min} > \theta)+P\left(x < \frac{by}{a}\middle|\lambda_{min} < \theta\right)P(\lambda_{min} < \theta)$$

$$\leq -(M-1)\left(\frac{M\theta}{2}\right)\ln\left(\frac{2^{2R}-1}{\rho}\right)+\frac{M}{2}\theta,$$

and the first equation of the theorem is proved.

To obtain the diversity-multiplexing tradeoff, we first substitute $R = r\log\rho$ into the upper bound the outage probability as

$$\begin{aligned} P\left(\mathcal{I}_{U_m} < 2R\right) &\leq -\ln(2)\frac{3M(M-1)}{4}\left(\frac{2^{2r\log\rho}-1}{\rho}\right)\log\left(\frac{2^{2r\log\rho}-1}{\rho}\right) \\ &= -\ln(2)\frac{3M(M-1)}{4}\rho^{-(1-2r)}\log\rho^{-(1-2r)}. \end{aligned} \tag{60}$$

The achievable diversity-multiplexing tradeoff can be obtained by calculating the following limit

$$\lim_{\rho\to\infty}\frac{\log\left[P\left(\mathcal{I}_{U_m} < 2R\right)\right]}{\log\rho} \leq \lim_{\rho\to\infty}\frac{\log\left[-\rho^{-(1-2r)}\log\rho^{-(1-2r)}\right]}{\log\rho} \underset{\rho^{1-2r}=x}{=} (1-2r)\lim_{x\to\infty}\frac{\log\left[-\frac{1}{x}\log\frac{1}{x}\right]}{\log x}. \tag{61}$$

Note that provided $x\to\infty$, we can have the following limit

$$\lim_{x\to\infty}\frac{\ln\left(-\frac{1}{x}\ln\frac{1}{x}\right)}{\ln x} = -1, \tag{62}$$

which can be shown as follows. Using $y = \frac{1}{x}$, we can express the limit as

$$\lim_{x\to\infty}\frac{\ln\left(-\frac{1}{x}\ln\frac{1}{x}\right)}{\ln x} \underset{y=\frac{1}{x}}{=} \lim_{y\to 0}\frac{\ln\left(-y\ln y\right)}{\ln\frac{1}{y}}.$$

By applying the l'Hopital's rule, the limit in (62) can be obtained. By using such a limit, the second part of the theorem can be proved. ■

Proof for Theorem 2: Following the similar steps in the previous section, we can obtain a lower bound of the SNR as follows

$$SNR_{BS_m} \geq \rho\frac{\mathbf{h}_{mR}^H\mathbf{v}\mathbf{v}^H\mathbf{h}_{mR}}{1+\lambda_{min}^{-1}\mathbf{h}_{mR}^H\mathbf{h}_{mR}}. \tag{63}$$

As a result, the outage probability of the i-th stream can be expressed as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq P\left(\frac{\mathbf{h}_{iR}^H\mathbf{v}\mathbf{v}^H\mathbf{h}_{iR}}{1+\lambda_{min}^{-1}\mathbf{h}_{iR}^H\mathbf{h}_{iR}} < \epsilon\right), \tag{64}$$

where $\epsilon = \frac{2^{2R}-1}{\rho}$. Using similar definitions, we can upper bound the outage probability as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq P\left(x < \frac{\epsilon+by}{a}\middle| 1-\epsilon\lambda_{min}^{-1} > 0\right)P(1-\epsilon\lambda_{min}^{-1} > 0) + P(1-\epsilon\lambda_{min}^{-1} < 0), \tag{65}$$

Compared the above equation with (48), the main difference is $P\left(x < \frac{\epsilon+by}{a}\middle| 1-\epsilon\lambda_{min}^{-1} > 0\right)$ which can be expressed as

$$\begin{aligned} P\left(x < \frac{\epsilon+by}{a}\middle|\lambda_{min} > \epsilon\right) &= \mathcal{E}_{\lambda_{min},\lambda_{min}>\epsilon}\left\{\int_0^\infty\int_0^{\frac{\epsilon+by}{a}} f_x(x)dxf_y(y)dy\right\} \\ &= \mathcal{E}_{\lambda_{min},\lambda_{min}>\epsilon}\left\{1-\frac{e^{-\frac{\epsilon}{a}}}{\left(\frac{b}{a}+1\right)^{(M-1)}}\right\}. \end{aligned} \tag{66}$$

To simplify notations, we define $P_1 = P\left(x < \frac{\epsilon+by}{a}\middle|\, \lambda_{min} > \epsilon\right)$. Note that the pdf of the minimum eigenvalue is exponentially distributed, $f_{\lambda_{min}}(\lambda) = \frac{M}{2}e^{-\frac{M}{2}\lambda}$. By using such a pdf, we can express the probability as

$$P_1 = \int_{\epsilon}^{\infty} \left(1 - e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\left(1-\frac{\epsilon}{\lambda}\right)^{M-1}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda. \tag{67}$$

Again the binomial expansion is applied to the above integral and we can obtain

$$\begin{aligned} P_1 &= \int_{\epsilon}^{\infty}\left(1-e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\left(1-\sum_{n=0}^{M-1}\binom{M-1}{n}(-1)^n\left(\frac{\epsilon}{\lambda}\right)^n\right)\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda \\ &= \int_{\epsilon}^{\infty}\left(1-e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}+e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\left(\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\left(\frac{\epsilon}{\lambda}\right)^n\right)\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda. \end{aligned} \tag{68}$$

To enable the results developed in the previous section to be applicable, we upper bound the probability as follows

$$\begin{aligned} P_1 &\leq 1-\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda+\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\left(\sum_{l=1}^{\lfloor\frac{M-1}{2}\rfloor}\binom{M-1}{l}\left(\frac{\epsilon}{\lambda}\right)^{2l-1}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda \\ &\leq 1-\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda+\int_{\epsilon}^{\infty}\left(\sum_{l=1}^{\lfloor\frac{M-1}{2}\rfloor}\binom{M-1}{l}\left(\frac{\epsilon}{\lambda}\right)^{2l-1}\right)\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda. \end{aligned}$$

By applying Whittaker functions and their series presentation as in (57), the above equation can be expressed as

$$P_1 \leq 1-\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda-(M-1)\left(\frac{M\epsilon}{2}\right)\ln\epsilon. \tag{69}$$

The probability $\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda$ in the above equation can be evaluated as

$$\begin{aligned} \int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda &\underset{\lambda-\epsilon=x}{=} \int_{0}^{\infty}e^{-\frac{\epsilon x+\epsilon^2}{x}}\frac{M}{2}e^{-\frac{M}{2}x-\frac{M}{2}\epsilon}dx \\ &= \frac{M}{2}e^{-\epsilon}e^{-\frac{M}{2}\epsilon}\sqrt{\frac{4\epsilon^2}{\frac{M}{2}}}K_1\left(\sqrt{4\epsilon^2\frac{M}{2}}\right), \end{aligned} \tag{70}$$

where $K_1(\cdot)$ denotes the modified Bessel function of the second kind. Note that the Bessel function can be approximated as $K_1(x) \approx \frac{1}{x}$, for $x \to 0$. Hence at high SNR, we can have the following approximation [28]

$$\int_{\epsilon}^{\infty}e^{-\frac{\epsilon\lambda}{\lambda-\epsilon}}\frac{M}{2}e^{-\frac{M}{2}\lambda}d\lambda \approx \frac{M}{2}e^{-\epsilon}e^{-\frac{M}{2}\epsilon}\frac{1}{\frac{M}{2}} \approx 1-\left(\frac{M}{2}+1\right)\epsilon. \tag{71}$$

By substituting the above approximation into (69), we can obtain

$$P_1 \leq \left(\frac{M}{2}+1\right)\epsilon-(M-1)\left(\frac{M\epsilon}{2}\right)\ln\epsilon. \tag{72}$$

Now following the steps similar to the proof for Theorem 1, the theorem can be proved. ■

About the upper bound of $1/\theta_1$: First rewrite the expectation as

$$1/\theta_1 \leq \frac{1}{Q}\mathcal{E}\{\sum_{m=1}^{M}\lambda_{\mathbf{HH}^H,m}\lambda_{(\mathbf{GG}^H)^{-1},m}\} \leq \frac{M}{Q}\mathcal{E}\{\text{trace}\{\mathbf{HH}^H\}\}\mathcal{E}\{\lambda^{-1}_{\mathbf{GG}^H,min}\}, \tag{73}$$

where $\lambda_{\mathbf{HH}^H,m}$ is the m-th largest eigenvalue of $\mathbf{HH}^H$, $\lambda_{(\mathbf{GG}^H)^{-1},m}$ is defined in a similar way, and $\lambda_{\mathbf{GG}^H,min}$ is the smallest eigenvalue of $\mathbf{GG}^H$. The first inequality in the above equation follows the results developed in [30] and the second inequality follows from the fact that the two channel matrices are independent. The expectation of the trace of a Wishart matrix can be easily obtained as $\mathcal{E}\{\text{trace}\{\mathbf{HH}^H\}\} = NM$ as shown in [21]. On the other hand, by applying the results from [29], the expectation of the inverse of the smallest eigenvalue can be obtained as

$$\begin{aligned}\mathcal{E}\{\lambda^{-1}_{\mathbf{GG}^H,min}\} &= 2^{Q-N+1}c_{m,n}\int_0^\infty \frac{1}{x}\cdot x^{Q-N}e^{-Nx}P_{m,n}(2x)dx \\ &= 2^{Q-N+1}c_{m,n}\sum_{i=0}^{(Q-N)(N-1)} 2^i a_i \frac{(Q-N+i-1)!}{Q^{Q-N+i}},\end{aligned} \tag{74}$$

where $c_{m,n}$ is a constant, $P_{m,n}(x)$ is a polynomial of degree $(Q-N)(N-1)$, i.e. $P_{m,n}(x) = \sum_{i=0}^{(Q-N)(N-1)} a_i x^i$. So as a result, it can be shown that the variable $1/\theta$ is indeed upper bounded by a constant as following

$$1/\theta_1 \leq 2^{Q-N+1}c_{m,n}NM\sum_{i=0}^{(Q-N)(N-1)} 2^i a_i \frac{(Q-N+i-1)!}{Q^{Q-N+i}} \leq \infty. \tag{75}$$

■

Proof for Lemma 5: Following the steps in the previous section, we can simplify the expression of SNR_{U_m} as follows

$$SNR_{U_m} = \frac{\theta_2\theta_1\rho}{1+\theta_2\frac{1}{\mathbf{h}_{mR}(\mathbf{I}-\tilde{\mathbf{P}}_m)\mathbf{h}_{mR}}} = \frac{\theta_2\theta_1\rho}{1+\theta_2\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m}}, \tag{76}$$

where $\tilde{\mathbf{\Lambda}}$ is the diagonal matrix containing the eigenvalues of $(\mathbf{I}-\tilde{\mathbf{P}}_m)$, $\tilde{\mathbf{h}}_m = \mathbf{U}_m^H\mathbf{h}_{mR}$ and $\mathbf{U}_m$ consists of the eigenvectors of the matrix. As discussed before, all elements of $\tilde{\mathbf{h}}_m$ are still independent and identically Rayleigh fading because the uniform transformation does not change the density function. However, unlike the previous case, it can be easily shown that the number of non-zero eigenvalues of $(\mathbf{I}-\tilde{\mathbf{P}}_m)$ is not just one, but $(N-M+1)$. By using such this fact, the expression of the outage probability can be written as

$$\begin{aligned}P\left(SNR_{U_m} < 2^{2R}-1\right) &= P\left(\sum_{i=1}^{N-M+1}|\tilde{h}_{m,i}|^2 \leq \frac{\theta_2(2^{2R}-1)}{\theta_1\theta_2\rho-2^{2R}+1}\right) \\ &= \int_0^{\frac{2^{2R}}{\rho-2^{2R}+1}}\frac{y^{N-M}}{\Gamma(N-M+1)}e^{-y}dy = \frac{1}{(N-M)!}\sum_{k=1}^{\infty}(-1)^k\frac{\left(\frac{\theta_2(2^{2R}-1)}{\theta_1\theta_2\rho-2^{2R}+1}\right)^{N-M+1+k}}{k!(N-M+1+k)}.\end{aligned} \tag{77}$$

By applying the high SNR approximation, the first equation in the lemma can be obtained.

To obtain the outage probability at the base station, again we use the smallest eigenvalue of $\mathbf{G}\mathbf{G}^H$ and obtain a lower bound of the SNR as follows

$$SNR_i \geq \frac{\rho}{\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m} + \theta_2^{-1}\lambda_{min}^{-1}\mathbf{h}_{iR}^H\mathbf{U}\mathbf{U}^H\mathbf{h}_{iR}} = \frac{1}{\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m} + \theta_2^{-1}\lambda_{min}^{-1}\mathbf{h}_{iR}^H\mathbf{h}_{iR}}. \quad (78)$$

As a result, the outage probability of the i-th stream can be expressed as

$$P\left(\mathcal{I}_{BS_m} < 2R\right) \leq P\left(\frac{1}{\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m} + \theta_2^{-1}\lambda_{min}^{-1}\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i} < \epsilon\right), \quad (79)$$

where $\tilde{\mathbf{h}}_i$ is replaced by $\tilde{\mathbf{h}}_m$ due to the fact that $\mathbf{U}_m\mathbf{U}_m^H = \mathbf{I}_N$. where $\epsilon = \frac{2^{2R}-1}{\rho}$. Furthermore, this outage probability can be expressed as

$$\begin{aligned} P\left(\mathcal{I}BS_m < 2R\right) \leq\; & P\left(\frac{1}{\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m} + \theta_2^{-1}\lambda_{min}^{-1}\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i} < \epsilon \middle| \tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \geq c\epsilon\right) P\left(\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \geq c\epsilon\right) \\ & + P\left(\frac{1}{\frac{1}{\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m} + \theta_2^{-1}\lambda_{min}^{-1}\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i} < \epsilon \middle| \tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \leq c\epsilon\right) P\left(\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \leq c\epsilon\right), \quad (80) \end{aligned}$$

where c is a constant and $c > 1$. The above equation can be used to find a further upper bound of the outage probability as follows

$$P\left(\mathcal{I}BS_m < 2R\right) \leq P\left(\frac{1}{\frac{1}{c\epsilon} + \theta_2^{-1}\lambda_{min}^{-1}\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i} < \epsilon \middle| \tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \geq c\epsilon\right) + P\left(\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \leq c\epsilon\right). \quad (81)$$

It is important to observe that $\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i \geq \tilde{\mathbf{h}}_i^H\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_i$ which can be proved as follows

$$\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i - \tilde{\mathbf{h}}_i^H\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_i = \tilde{\mathbf{h}}_i^H\left(\mathbf{I}_N - \tilde{\mathbf{\Lambda}}\right)\tilde{\mathbf{h}}_i.$$

It can be easily seen that $\left(\mathbf{I}_N - \tilde{\mathbf{\Lambda}}\right)$ is symmetrical and has $(M-1)$ eigenvalues equal to one and $(N-M+1)$ zero eigenvalues. Hence $\left(\mathbf{I} - \mathbf{u}_m\mathbf{u}_m^H\right)$ is a positive semi-definite matrix, which means

$$\tilde{\mathbf{h}}_i^H\left(\mathbf{I}_N - \tilde{\mathbf{\Lambda}}\right)\tilde{\mathbf{h}}_i \geq 0.$$

By using this observation, we can express the upper bound of the outage probability as

$$P\left(\mathcal{I}BS_m < 2R\right) \leq P\left(\frac{1}{\frac{1}{c\epsilon} + \theta_2^{-1}\lambda_{min}^{-1}\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i} < \epsilon \middle| \tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i \geq c\epsilon\right) + P\left(\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \leq c\epsilon\right). \quad (82)$$

Now define $y = \tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_i$. The first probability can be written as

$$P\left(\frac{1}{\frac{1}{c\epsilon} + \theta_2^{-1}\lambda_{min}^{-1}y} < \epsilon \middle| y \geq c\epsilon\right) = P\left(\lambda_{min} < \frac{y}{\theta_2}\frac{\epsilon}{1-\frac{1}{c}} \middle| y \geq c\epsilon\right), \quad (83)$$

where the last equation follows the fact the constant c is larger than one.

Recall that the probability density function of y can be obtained from the chi-square distribution as follows

$$f_y(y) = \frac{y^{N-1}}{\Gamma(N)}e^{-y},$$

and the probability density function of the smallest eigenvalue of a complex Wishart matrix $\mathbf{G}\mathbf{G}^H$ can be upper bounded as [29]

$$f_{\lambda_{min}}(z) \leq \phi z^{Q-N-1} e^{-z}. \tag{84}$$

where $\phi = \frac{2^{Q-N-1}\tilde{K_{Q,N}}}{\tilde{K_{Q+1,N-1}}}$ and $\tilde{K}_{n,m}^{-1} = 2^{mn}\prod_{i=1}^{m}\Gamma(n-i+1)\Gamma(m-i+1)$. By using the above density functions, we can have

$$\begin{aligned} P\left(\frac{1}{\frac{1}{c\epsilon}+\theta_2^{-1}\lambda_{min}^{-1}y} < \epsilon \middle| y \geq c\epsilon\right) &\leq \phi \int_0^\infty \int_0^{\frac{\epsilon}{1-\frac{1}{c}}\frac{y}{\theta_2}} z^{Q-N-1}e^{-z}dz f_y(y)dy \\ &= \phi \sum_{k=0}^{\infty}(-1)^k \frac{1}{\Gamma(N)} \int_0^\infty \frac{\left(\frac{\epsilon}{1-\frac{1}{c}}\frac{y}{\theta_2}\right)^{Q-N+k}}{k!(Q-N+k)} y^{N-1}e^{-y}dy, \end{aligned} \tag{85}$$

which can be further simplified as

$$P\left(\frac{1}{\frac{1}{c\epsilon}+\theta_2^{-1}\lambda_{min}^{-1}y} < \epsilon \middle| y \geq c\epsilon\right) = \sum_{k=0}^{\infty}\frac{(-1)^k\phi}{\Gamma(N)}\frac{\left(\frac{\epsilon}{\theta_2(1-\frac{1}{c})}\right)^{Q-N+k}}{k!(Q-N+k)}\Gamma(Q+k) \approx \phi\frac{\Gamma(Q)}{\Gamma(N)}\frac{\left(\frac{\epsilon}{(1-\frac{1}{c})\theta_2}\right)^{Q-N}}{(Q-N)}. \tag{86}$$

On the other hand the remaining probability in the expression of the outage probability can be expressed as

$$P\left(\tilde{\mathbf{h}}_m\tilde{\mathbf{\Lambda}}\tilde{\mathbf{h}}_m \leq c\epsilon\right) \approx 1 - e^{-c\epsilon} \approx \frac{1}{(N-M+1)!}(c\epsilon)^{N-M+1}.$$

Combining the above equation with (86), the lemma can be proved. ■

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TABLE I

THE VALUE OF THE POWER NORMALIZATION VALUE $\mathcal{E}\left\{\frac{1}{\mathbf{h}_{1R}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{1R}}\right\}$

	M=2d	M=3	M=4	M=5	M=6	M=7	M=8	M=9	M=10
$\mathcal{E}\left\{\frac{1}{\mathbf{h}_{1R}^H(\mathbf{G}^{-1})^H\mathbf{G}^{-1}\mathbf{h}_{1R}}\right\}$	1.0058	0.4942	0.3327	0.2488	0.1991	0.1671	0.1424	0.1243	0.1113

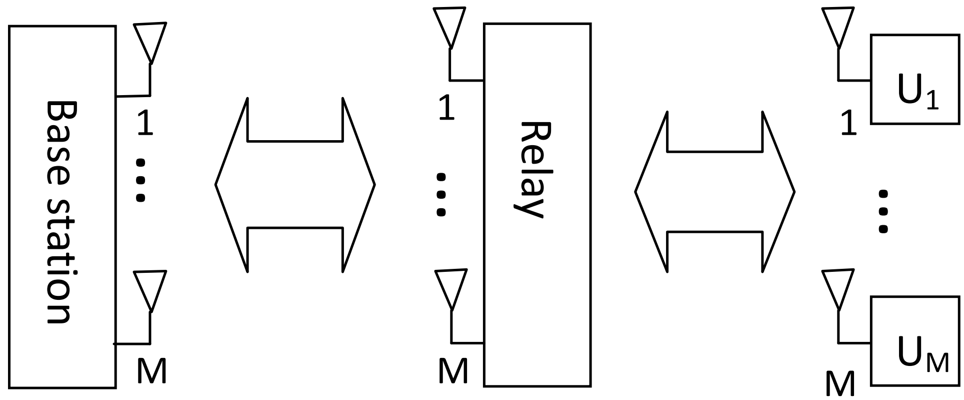

Fig. 1. A system diagram for the scenario where the base station and the relay have M antennas, and each of the M users are only equipped with a single antenna.

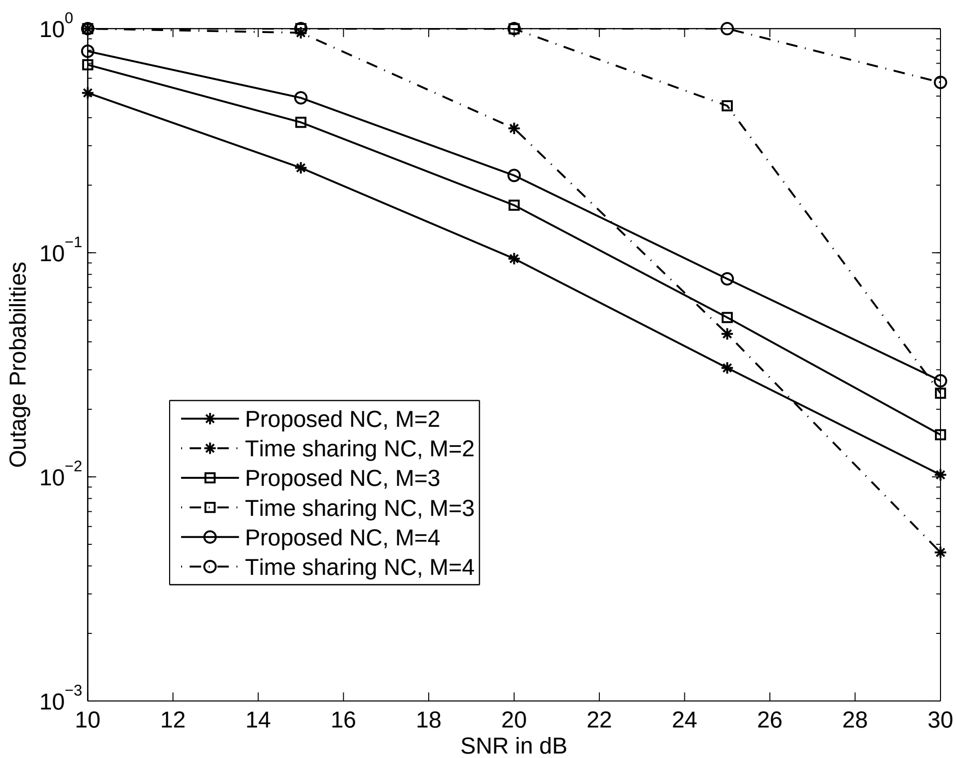

Fig. 2. Outage Probability versus the signal to noise ratio. The target data rate for all users is $R = 1$ bit per channel use (BPCU). The base station and the relay have M antennas and each of the M users has a single antenna.

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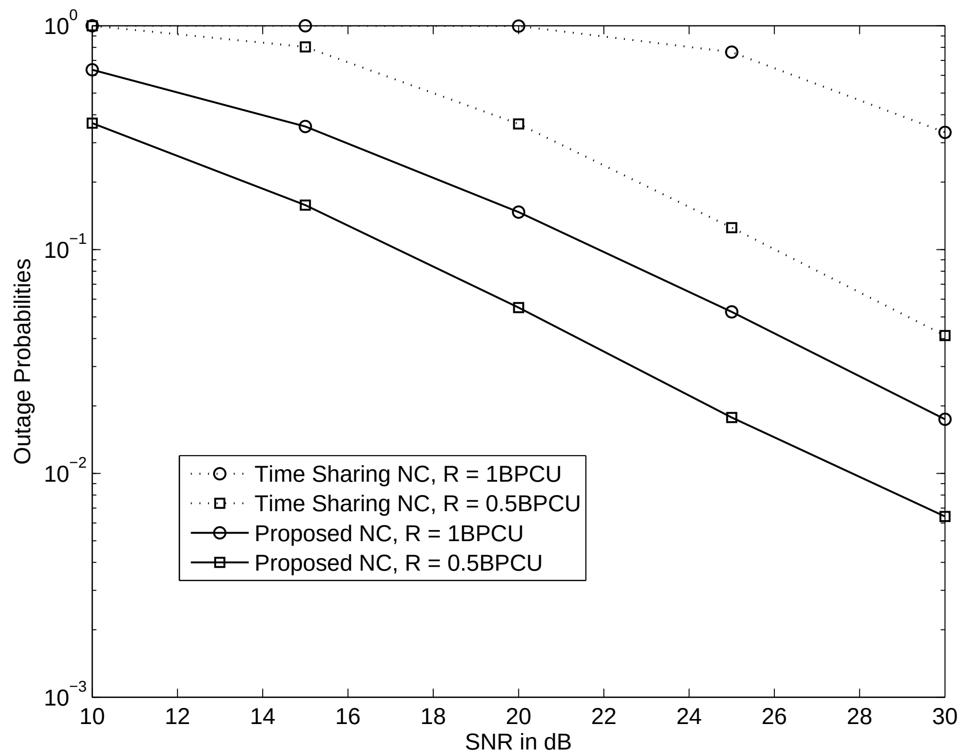

Fig. 3. Outage Probability versus the signal to noise ratio. The number of users is M. The base station and the relay have $M = 3$ antennas. Each of the M users has a single antenna.

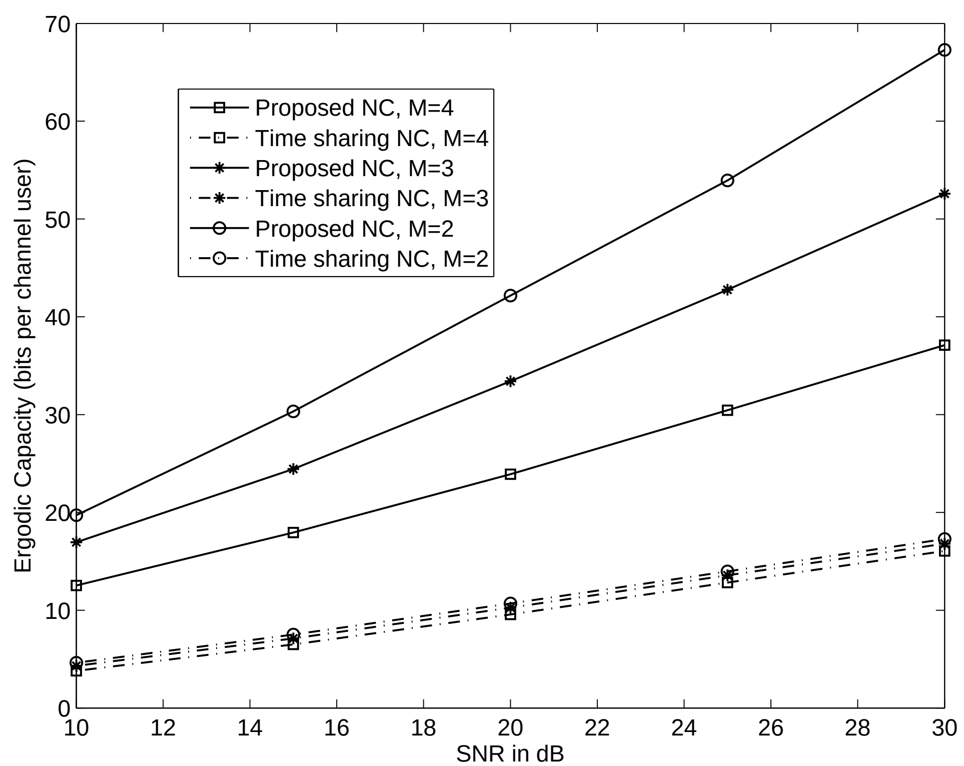

Fig. 4. Ergodic capacity versus versus the signal to noise ratio. The number of users is M. The base station and the relay have $M = 3$ antennas.

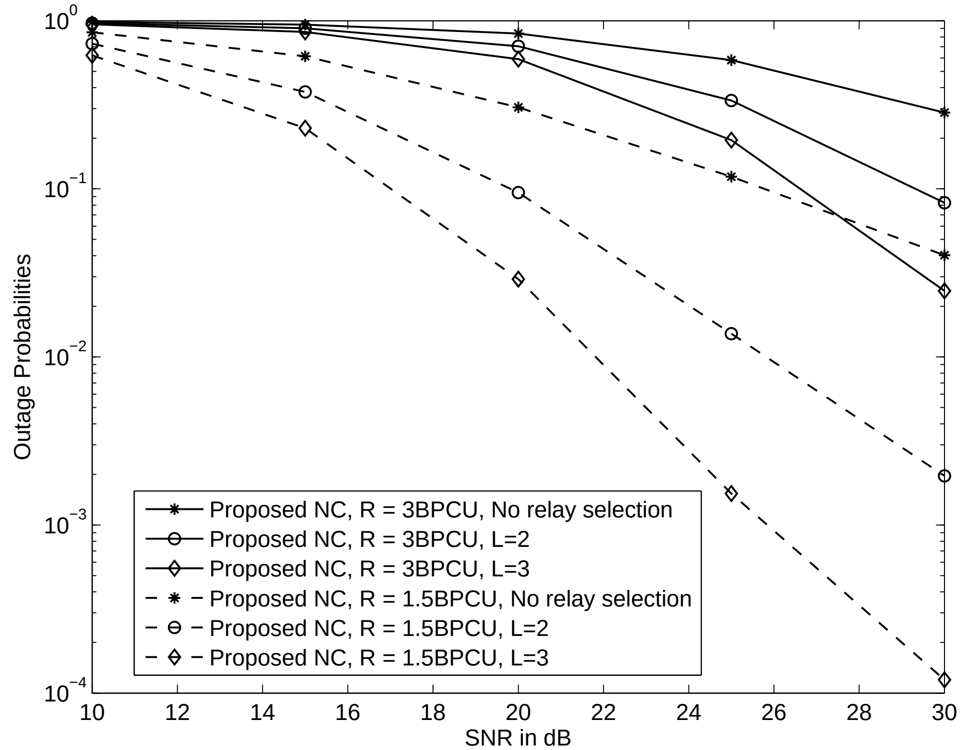

Fig. 5. Outage Probability versus the signal to noise ratio. The base station and the L relays have $M = 3$ antennas. Each of the M users are equipped with a single antenna.

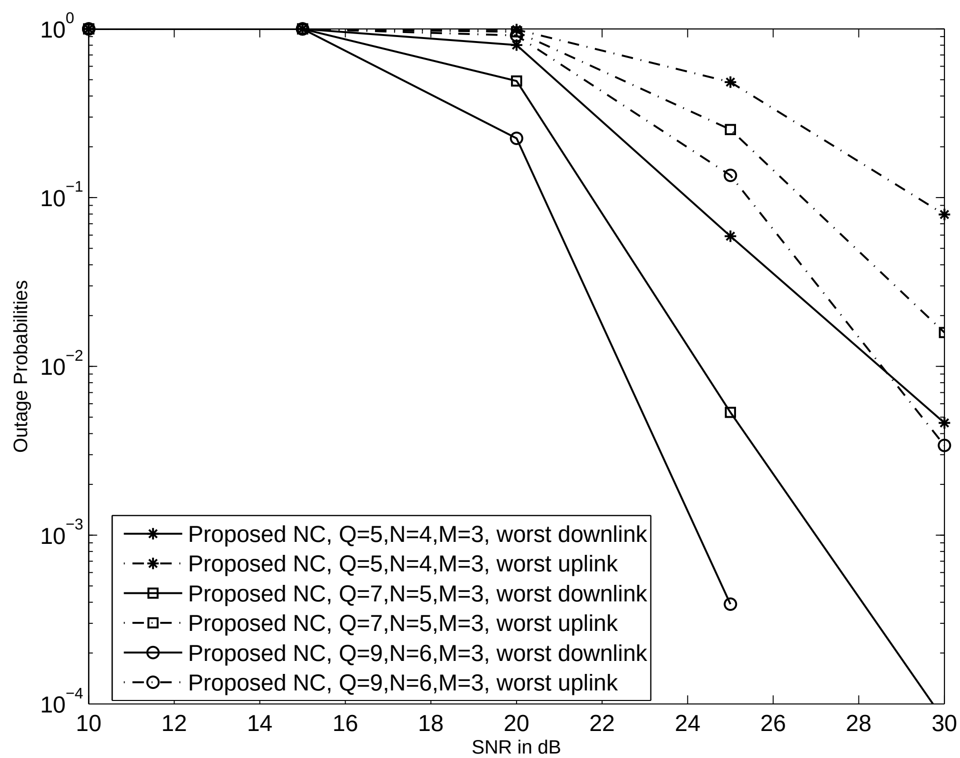

Fig. 6. Outage Probability versus the signal to noise ratio. The target data rate for all users is $R = 3$ bits per channel user (BPCU). The base station has M antennas, the relay has N antennas and there are M single antenna users.

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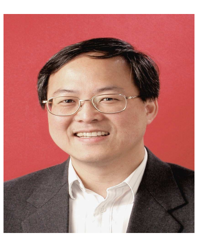

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