Impact of Network Coding on System Delay for Multi-source Multi-destination Scenarios

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***Abstract*—Existing work has shown that random coding across multi-cast sessions can reduce the system delay significantly, however, such a scheme requires the strong assumption that each source has the priori information of other sources' messages. Actually the broadcasting nature of radio propagation can provide an opportunity to realize collaboration across sessions without causing much system overhead. In this paper, we propose the application of network coding to multi-source multi-destination (MSMD) scenarios and provide formal analysis for the improvement of system delay. In particular, two types of analytical results have been developed, one based on the outage probability and the other based on the use of practical convolutional codes. Monte-Carlo simulation results have also been provided to demonstrate the delay performance of the proposed network coded protocol.**

I. Introduction

Originally developed to increase the capacity for wireline networks, network coding has recently received a lot of attentions due to its wide application to wireless communications [1]. In [2], [3], efficient network coded protocols have been developed for the two-way relaying channels, and in [4], [5] several cooperative multiple access transmission protocols based on network coding have been proposed have been proposed. For such two-way relaying and multiple access channels, it has been demonstrate that the use of network coding can not only just increase reception reliability, but also improve system throughput and delay performance.

Interference channel, also know as multi-source multi-destination scenarios (MSMD), is one of the fundamental building blocks of wireless communications. Different from other traffic patterns, MSMD is severely interference limited. [6] is one of the first tries to apply network coding to MSMD scenarios and the key idea of [6] is to encourage source nodes collaborating with each other. By applying random coding across the multiple sessions, each source transmits a mixture of all source messages, analogue to network coding. Comparing with non-cooperative schemes, random coding across multi-cast sessions is much more reliable, which is due to the reason that each source transmission can serve all destination simultaneously. However, such a scheme requires the strong assumption that each source needs the priori information of other sources' information.

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In this paper, our aim is to study the impact of network coding on the system delay in multi-source multi-destination scenarios. Different from [6], we are interested in how to achieve the collaboration among multiple source-destination pairs without causing too much system overhead. In specific, the two-hop transmission strategy is focused and the use of intermediate relays is introduced into MSMD scenarios. Instead of asking one source transmitting each time, all source nodes will broadcast their messages simultaneously. At relays, mixtures of all source messages are observed because of the broadcasting nature of radio propagation. Rather than to ask relays to separate the mixture, the idea of network coding is used and relays are allowed to forward the mixtures. In such a way, random coding across multiple sessions is realized without causing any system overhead. To further improve the performance of the proposed transmission protocol, the opportunistic use of relays is also used to exploit multi-user diversity. Various decoding methods can be utilized at destinations to solve the mixture, where the criteria of zero forcing is used due to its simplicity. Two types of analytical results are developed for the overall system delay. One is based on the outage probability, which can tightly bound the error probability of maximum likelihood for infinite length of data blocks and high SNR. In addition to such a theoretical upper bound, we also provide analytical results based on the use of practical convolutional codes. Monte-Carlo simulation results have also provided to demonstrate the performance of the proposed network coded transmission protocol with comparisons to comparable schemes.

II. Two-Hop Transmissions: Cooperative and Non-Cooperative Strategies

Consider a two-hop communication scenario with M source-destination pairs and L intermediate relays. Each source aims to send its head-of-line packet to its corresponding receiver. Assume that there is no direct link between the sources and destinations as in [7]. The cooperative coding strategy for two-hop scenarios can be described as following. At the first time slot, all sources broadcast their head-of-line packets simultaneously. Hence at this time slot, each relay receives the superposition of the M messages

$$y_{R_n} = \sum_{m=1}^{M} h_{mR_n} s_m + n_{R_n}, \quad n \in \{1, \dots, L\}. \tag{1}$$

where s_m is the message transmitted from the mth source, n_{R_n} is the additive Gaussian noise and h_{mR_n} is the coefficient for the channel between the mth source and the relay R_n. To simplify notation, we assume that each source transmits a symbol each time slot rather than a packet. In this paper, all wireless channels are assumed to be independent identical Raleigh fading. After this first transmission, all relays received a mixture of the M transmitted messages with different combination coefficients. For the next hop transmission, each relay will broadcast its received mixture to all destinations, which is analogous to the strategy of random coding across multicast sessions [6]. Thanks for the broadcasting nature of radio propagation, cooperation among multi-cast secessions does not consume extra bandwidth resource and the next hop transmission can benefit from such random coding without any loss of bandwidth efficiency.

Due to the dynamic nature of radio propagation, the channel quality of different relays varies. Ideally relays should be scheduled to transmit in a way that a relay with better connection to the sources and destinations should be used earlier, where the details for relay selection will be discussed at the end of this section. The amplify-forward strategy is used here for relay transmission. During the next n time slots, the scheduled relays will take their turns to forward the mixture to the destination, and the m-th destination receives

$$y_{D_{l,m}} = h_{R_l D_m} \hat{y_{R_l}} + n_{l+1}, \quad l = 1, \ldots, n, \tag{2}$$

where $\hat{y_{R_l}} = y_{R_l}/\mu_l$, $\mu_l = \sqrt{M + 1/\rho}$ and ρ is denoted as signal-to-noise ratio (SNR). Note that the averaged transmission power constraint is applied to each relay. So after n time slots, the observations at the destination can be expressed as

$$\mathbf{y}_{m,n} = \mathbf{D}_{m,n}\mathbf{H}_n\mathbf{s} + \mathbf{n}_{m,n} \tag{3}$$

where $\mathbf{y}_{m,n} = \begin{bmatrix} y_{D_{1,m}} & \cdots & y_{D_{n,m}} \end{bmatrix}^T$, $\mathbf{D}_{m,n} = \text{diag}\{\frac{h_{R_1 D_m}}{\sqrt{M}} \ldots \frac{h_{R_n D_m}}{\sqrt{M}}\}$, $\mathbf{s} = \begin{bmatrix} s_1 & \cdots & s_M \end{bmatrix}^T$ $\mathbf{H}_n$ is the channel matrix whose element at its ith row and jth column is the channel from the ith source to the jth relay and $\mathbf{n}_{m,n} = \begin{bmatrix} n_{1,m} + \frac{h_{R_1 D} n_{R_1}}{\sqrt{M}} & \cdots & n_{n,m} + \frac{h_{R_n D} n_{R_n}}{\sqrt{M}} \end{bmatrix}^T$ The relays will keep forwarding their received mixture until all destinations have correctly received their corresponding source messages, where the criterion for successful transmission will be discussed later. It is possible that all relays have been scheduled to transmit, but at least one destination can not decode its source messages, e.g., $n \geq L$. In such a case, the relays will be reused, which means that there are some repeated rows in the matrices, $\mathbf{D}_{m,n}$ and $\mathbf{H}_n$.

There are many choices of the criteria to determinate whether one destination can receive the source message correctly. In this paper, we will use the principle of zero forcing [8], [9], a linear receiver which is not only easy to implement, but also helpful to simplify the development of analytical results. Applying the principle of zero forcing detection, a simplified signal model can be obtained as

$$\begin{aligned} \mathbf{H}_n^\dagger \mathbf{D}_{m,n}^{-1}\mathbf{y}_{m,n} &= \mathbf{s} + (\mathbf{H}_n^H\mathbf{H}_n)^{-1}\mathbf{H}_n^H\mathbf{D}_{m,n}^{-1}\mathbf{n}_{m,n} \\ &= \mathbf{s} + \tilde{\mathbf{n}}_{m,n} \end{aligned} \tag{4}$$

The signal-to-noise ratio (SNR) for the i-th source message at the m-th destination after n time slots can be expressed as $\rho_{i,m}^n = \frac{P}{E\{\tilde{n}_{i,m,n}\}}$, where $\tilde{n}_{i,m,n}$ is the i-th element of the vector $\tilde{\mathbf{n}}_{m,n}$. The noise power $E\{\tilde{n}_{i,m,n}\}$ can be obtained from the noise covariance matrix as

$$\mathbf{C}_{m,n} = (\mathbf{H}_n^H\mathbf{H}_n)^{-1}\mathbf{H}_n^H\tilde{\mathbf{C}}_{m,n}\mathbf{H}_n(\mathbf{H}_n^H\mathbf{H}_n)^{-1} \tag{5}$$

where $\tilde{\mathbf{C}}_{m,n} = \mathbf{D}_{m,n}^{-1}\bar{\mathbf{C}}_{m,n}(\mathbf{D}_{m,n}^{-1})^H$ and $\bar{\mathbf{C}}_{m,n} = \text{diag}\{1 + |h_{R_1 D_m}|^2/M, \cdots, 1 + |h_{R_n D_m}|^2/M\}$. So we can have $\tilde{\mathbf{C}}_{m,n} = \text{diag}\{1 + M/|h_{R_1 D_m}|^2, \cdots, 1 + M/|h_{R_n D_m}|^2\}$. Based on these SNRs, $\rho_{i,m}^n$, the performance of the addressed protocol, in terms of delay and stability, can be obtained.

A. Distributed scheduling strategy of relay forwarding

Different from those scenarios with single S-D pair, the design of relay scheduling/selection for the addressed multicast sessions is more complicated. Due to the use of the random coding cross the sessions, one relay has to serve more than one S-D. Assume that each relay has the access to its local channel state information (CSI), $[h_{1R_m} \cdots h_{MR_m} h_{R_m D_1} \cdots h_{R_m D_M}]$. Among possible choice of the criteria for relay quality, the one used in this paper is to order the relays in a descending order according to its worst link, e.g.,

$$|h_{R_1,min}|^2 \geq \cdots \geq |h_{R_L,min}|^2,$$

where $|h_{R_m,min}|^2 = \min(|h_{1R_m}|^2 \cdots |h_{R_m D_M}|^2)$. Consider that there is only one S-D pair, and recall that a well known relay selection criterion for amplify forward protocols is the harmonic mean of the incoming and outgoing channels of each relay,

$$\frac{|h_{1R_m}|^2 |h_{R_m D_1}|^2}{|h_{1R_m}|^2 + |h_{R_m D_1}|^2}.$$

For such a special case, the proposed criterion is to select relays according to the minimum of their incoming and outgoing channels, $\min\left(|h_{1R_m}|^2, |h_{R_m D_1}|^2\right)$, which is the same as the harmonic mean.

III. Performance Analysis for the Two-hop Transmissions

In this paper, we are interested in the impact of the proposed transmission protocol on the system delay and stability. Define $\bar{N}$ as the averaged number of retransmissions required to deliver the M head-of-line packets to their associated destinations. So the averaged delay and the maximum stable arrival rate for the addressed two-hop scenario can be obtained as

$$\bar{T} = \bar{N}T_p, \quad \lambda = \frac{M}{\bar{N}},$$

where T_p is the time duration of one time slot. Obviously the key step to study the delay and stability is to find the averaged number of required time slots. Define $P(N = n)$ as the probability for the event that n re-transmissions ensure all messages correctly decoded by all destinations, but $n - 1$ transmissions can not. Then the averaged number of the minimum re-transmissions can be obtained as

$$\bar{N} = \sum_{n=1}^{\infty} nP(N = n). \tag{6}$$

The probability, $P(N = n)$, can be obtained in different ways dependent on the definition of the error probability. In this paper, we will use two types of error probability, outage probability and block error probability of Viterbi decoding of convolutional codes. Conditioned on high SNR and infinity length of coding, the outage probability can tightly bounded the ML probability of detection error. Although the assumption is not practical, the use of outage probability can provide us insights. So in the following, the outage probability is first to use in order to get some closed-form expression of $P(N = n)$. Later the error probability based on practical length of coding will be studied.

A. Analytical results based on outage probability

An outage event occurs when the mutual information supported by the instantaneous receive SNR is less than the target data rate. Hence based on the simplified model in (4), the probability $P(N = n)$ can be defined as

$$P(N = n + 1) = P(\rho_{min}^{n-1} \leq \phi, \rho_{min}^{n} \geq \phi) \tag{7}$$

where $\phi = 2^R - 1$, R is the target data rate, and ρ_{min}^n is the minimum SNR among the M subchannels, e.g., $\rho_{min}^n = \min(\rho_{m,m}^n)$, $\forall m \in \{1, \ldots, M\}$. Note that a symmetric network is considered here, where the target rates for all sessions are the same. The use of relay scheduling means that the elements in $\mathbf{H}_n$ and $\mathbf{D}_{m,n}$ are no longer complex Gaussian distributed. To obtain tractable analytical expressions, we first construct a new signal model as following

$$\bar{\mathbf{y}}_n = \mathbf{s} + \bar{\mathbf{n}}_n, \tag{8}$$

which has the new noise covariance matrix as

$$\begin{aligned}\bar{\mathbf{C}}_n &= (\mathbf{H}_n^H\mathbf{H}_n)^{-1}\mathbf{H}_n^H(1+M)\mathbf{I}_M\mathbf{H}_n(\mathbf{H}_n^H\mathbf{H}_n)^{-1} \quad (9)\\ &= (1+M)(\mathbf{H}_n^H\mathbf{H}_n)^{-1}.\end{aligned}$$

Conditioned on the assumption that there are an infinity number of relays, the use of the strategy of relay scheduling can ensure that no relay is scheduled twice and those used relays can have good enough outgoing channels, $\frac{1}{|h_{R_m D_m}|^2} \geq 1$. So with infinity relays, we can have the following inequality

$$\bar{\mathbf{C}}_n(i,i) \geq \mathbf{C}_{m,n}(i,i), \quad \forall i, m \in \{1, \ldots, M\}, \tag{10}$$

which means that the use of such a simplified signal model could result in more transmissions than the original model. Hence in the following, we will focus on this simplified signal model to develop tractable analytical results. Define $\tilde{N}$ as the total transmission number required to deliver source messages to all destinations by using the simplified model in (8), and we can have the following inequality

$$\bar{N} = \sum_{n=1}^{\infty} nP(N = n) \leq \sum_{n=1}^{\infty} nP(\tilde{N} = n). \tag{11}$$

To obtain tractable expressions, the probability $P(\tilde{N} = n+1)$ is first expressed as

$$P(\tilde{N} = n + 1) = P(\max\{N_1, \cdots, N_M\} = n + 1) \tag{12}$$

where N_m denotes the number of transmissions required for reliable communication between the mth source-destination pair by using the simplified model in (8). Define $\gamma_{m,n}$ as the effective channel gain at the m-th subchannel of the model in (8) after n time slots, and hence the corresponding SNR can be written as $\frac{\gamma_{m,n}\rho}{M+1}$. To further simplify the development, it is assumed that the elements in the matrix $\mathbf{H}_n$ are i.i.d complex Gaussian distributed, which has no effect to the inequality in (10). Provided that $\mathbf{H}_n$ is a Gaussian random matrix, the density function of the effective channel gain at m-th subchannel after n transmissions $\gamma_{m,n}$ can be expressed as [8], [9]

$$f_{\gamma_{m,n}}(\gamma) = \frac{e^{-\gamma}}{(n-M)!}(\gamma)^{n-M}$$

which is Chi-square distributed with $2(n - M + 1)$ degree of freedom. And the probability $P(N_m = n)$ can be expressed as

$$P(N_m = n + 1) = P(\gamma_{m,n-1} \leq \epsilon, \gamma_{m,n} \geq \epsilon), \tag{13}$$

where $\epsilon = \frac{(2^R-1)(M+1)}{\rho}$. While the density of $\gamma_{m,n-1}$ and $\gamma_{m,n}$ can be easily found, obviously the two variables are not independent to each other and it is not clear how to get their joint density function which is needed to obtain the probability $P(N_m = n)$. Define $\Delta_{m,n} = \gamma_{m,n} - \gamma_{m,n-1}$. An intuition is that $\Delta_{m,n}$ should be exponentially distributed and independent to $\gamma_{m,n-1}$ *if* $\gamma_{m,n-1}$ is the sum of $(n - M)$ i.i.d complex Gaussian variables and $\gamma_{m,n}$ is the addition of $\gamma_{m,n-1}$ with another i.i.d Gaussian variable. Although the relationship between the two variable is not as explicit as expected, the intuition for the density of $\Delta_{m,n}$ is still valid, as shown in the following lemma.

Lemma *1:* Consider a $N \times M$ complex Gaussian matrix $\mathbf{H}_N$. Define

$$\gamma_{m,N} = \frac{1}{\left[(\mathbf{H}_N^H\mathbf{H}_N)^{-1}\right]_{m,m}},$$

where $[\mathbf{A}]_{m,m}$ denotes the ith element on the diagonal of $\mathbf{A}$. It can be proved that the difference between $\gamma_{m,N}$ and $\gamma_{m,N+1}$, denoted as $\Delta_{m,N}$, is independent to $\gamma_{m,N}$, and its cumulative density function is

$$f_{\Delta_{m,N}}(x) = 1 - e^{-x}.$$

Proof: Please refer [10]. ■

By using Lemma 1, the probability $P(N_m = n + 1)$ can be found as

$$P(N_m = n+1) = P(\gamma_{m,n-1} \leq \epsilon, \gamma_{m,n} \geq \epsilon) = \frac{e^{-\epsilon}}{(n-M)!}\epsilon^{n-M}.$$

Consider that the numbers of required time slots for M source-destination pairs, $N_1, \cdots, N_M$, can be sorted in descending order as

$$N_{(1)} \leq N_{(2)} \cdots \leq N_{(M)}$$

By using order statistics, we can find the probability of $N_{(M)}$, the largest number of transmissions

$$P(\tilde{N} = n+1) = \left(\sum_{k=M}^{n} P(N_i = k)\right)^M - \left(\sum_{k=M}^{n-1} P(N_i = k)\right)^M. \tag{14}$$

And the expectation of the largest number of re-transmissions can be upper bounded as

$$\bar{N}_{(M)} \leq \sum_{n=1}^{\infty} nP(\tilde{N}=n) = (M+1)\left(P(N_i=M)\right)^M + \sum_{n=M+2}^{\infty} n\left(\left(\sum_{k=M}^{n-1} P(N_i=k)\right)^M - \left(\sum_{k=M}^{n-2} P(N_i=k)\right)^M\right) \tag{15}$$

Recall that the use of order statistics in (14) requires that all variables $N_1, \cdots, N_M$ are independent. It is important to note that the multiple sub-channels for linear receivers are not strictly independent, however, the approximation of independence is used here to make the analytical results tractable as in [11].

To obtain insights of the performance achieved by the proposed protocol, we use the exponential expansion and the assumption of medium SNR to obtain some approximations. The probability $P(N_i = n)$ can be simplified as

$$P(N_i = n+1) \approx \frac{1-\epsilon}{(n-M)!}\epsilon^{n-M}.$$

And based on this approximation, the expected number of required time slots can be upper bounded

$$\begin{aligned}\bar{N} \leq \bar{N}_{(M)} &\approx (M+1)(1-M\epsilon) + (M+2)M\epsilon \\ &= M(1+\epsilon)+1,\end{aligned} \tag{16}$$

conditioned on the assumption that there is infinity number of relays.

B. Performance analysis based on convolutional coding

Assuming that data sequence from each source has been convolutional encoded by a convolutional code. Still the relays forward the messages to destinations by employing amplify-forward strategy until all destinations have decoded correctly. Let $\mathcal{O}_0^n$, $\mathcal{O}_u^n$ and $\mathcal{O}_d^n$ denote, respectively, the events "decoded sequence contains no errors", "decoded sequence contains undetected errors" and "decoded contains detected errors" when there are n transmissions. Clearly, $P(\mathcal{O}_0^n)+P(\mathcal{O}_u^n)+P(\mathcal{O}_d^n) = 1$. Further assuming $\mathcal{O}_u^n$ is actually negligible, hence $\mathcal{O}_d^n$ can be approximated by

$$P(\mathcal{O}_d^n) = 1 - P(\mathcal{O}_0^n). \tag{17}$$

Notice that the joint probability $P(\mathcal{O}_d^1, \mathcal{O}_d^2, \cdots, \mathcal{O}_d^n)$ can be upper bounded as

$$P(\mathcal{O}_d^1, \mathcal{O}_d^2, \cdots, \mathcal{O}_d^n) \leq P(\mathcal{O}_d^n). \tag{18}$$

Now the error probability that the m-th user's message can be decoded N transmissions can be expressed as

$$P(N=n+1) = P(\mathcal{O}_d^1, \cdots, \mathcal{O}_d^n, \bar{\mathcal{O}}_d^{n+1}) \tag{19}$$

By considering (17) and (18), we have $P(N = n+1)$ bounded by

$$P(N=n+1) = P(\mathcal{O}_d^n) - P(\mathcal{O}_d^{n+1}) \tag{20}$$

For a frame length K convolutional coded data, $P(\mathcal{O}_d^n)$ can be bounded by

$$P(\mathcal{O}_d^n) \geq 1 - \left(1 - P\left(E^n\right)\right)^K \tag{21}$$

where $P(E^n)$ is the probability of a decoding error event of Viterbi decoding after the nth transmission. According to the Viterbi decoding convolutional codes bounds, the mth sub-sessions error probability $P(E^n)$ can be upper bounded as

$$P\left(E^n\right) < \sum_{d=d_{free}}^{\infty} \beta_d Q(\sqrt{2\rho_{m,n}R_c d}) \tag{22}$$

where β_d, d_{free} and R_c denote, respectively, distance spectra, free distance and the rate of employed convolutional code. the Q function is defined as $Q(\alpha) = \frac{1}{\sqrt{2\pi}}\int_\alpha^\infty e^{-\frac{x^2}{2}}dx \leq \frac{1}{2}e^{-\frac{\alpha^2}{2}}$. Substituting (21)(22) into (20) and using exponential expansion, we may rewrite $P(N = n + 1)$ approximately as

$$\begin{aligned}P(N=n+1) \leq \ & \textstyle\sum_{d=d_{free}}^{\infty} \frac{K\beta_d e^{-\rho_{m,n-1}R_c d}}{2} \\ & -\textstyle\sum_{d=d_{free}}^{\infty} \frac{K\beta_d e^{-\rho_{m,n}R_c d}}{2}\end{aligned} \tag{23}$$

Since $\rho_{m,n}$ are Chi-square distributed with $2(n-M+1)$ degree of freedom, $P(N = n+1)$ can be formulated by its expectation after interchanging the integrating and sum operation

$$\begin{aligned}P(N=n+1) \leq & \sum_{d=d_{free}}^{\infty}\int_0^\infty \left(\frac{K\beta_d\gamma^{(n-M-1)}e^{-(\rho R_c d+1)\gamma}}{2(n-M-1)!}\right)\mathrm{d}\gamma \\ & -\sum_{d=d_{free}}^{\infty}\int_0^\infty \left(\frac{K\beta_d\gamma^{(n-M)}e^{-(\rho R_c d+1)\gamma}}{2(n-M)!}\right)\mathrm{d}\gamma\end{aligned}$$

The above results can be integrated and further simplified as

$$P(N=n+1) \leq \sum_{d=d_{free}}^{\infty}\frac{K\beta_d}{2}\left(\frac{\rho R_c d}{(\rho R_c d+1)^{n-M+1}}\right) \tag{24}$$

by combining (15) and let $\epsilon_{c0} = \sum_{d=d_{free}}^{\infty}\frac{K\beta_d}{2}\left(\frac{1}{\rho R_c d+1}\right)$, $\epsilon_{c1} = \sum_{k=M}^{n-2}\sum_{d=d_{free}}^{\infty}\frac{K\beta_d}{2}\left(\frac{\rho R_c d}{(\rho R_c d+1)^{k-M+1}}\right)$ and $\epsilon_{c2} = \sum_{d=d_{free}}^{\infty}\frac{K\beta_d}{2}\left(\frac{\rho R_c d}{(\rho R_c d+1)^{n-M}}\right)$, the expectation of largest number of re-transmission under convolutional coded case can be written as

$$\begin{aligned}\bar{N}_{(M)} = & \quad (M+1)(1-\epsilon_{c0})^M \\ & +\textstyle\sum_{n=M+2}^{\infty} n\left((\epsilon_{c1}+\epsilon_{c2})^M - \epsilon_{c1}^M\right)\end{aligned} \tag{25}$$

At high SNR region, (25) can be approximated as

$$\bar{N}_{(M)} \approx (M+1)(1-M\epsilon_{c0}) + (M+2)M\epsilon_{c1}^{M-1}\epsilon_{c2} \tag{26}$$

Notice when SNR goes to large, ϵ_{c0},ϵ_{c1} and ϵ_{c2} would be extremely small, which indicates the average number of transmission of proposed protocol should converge to $M + 1$ eventually.

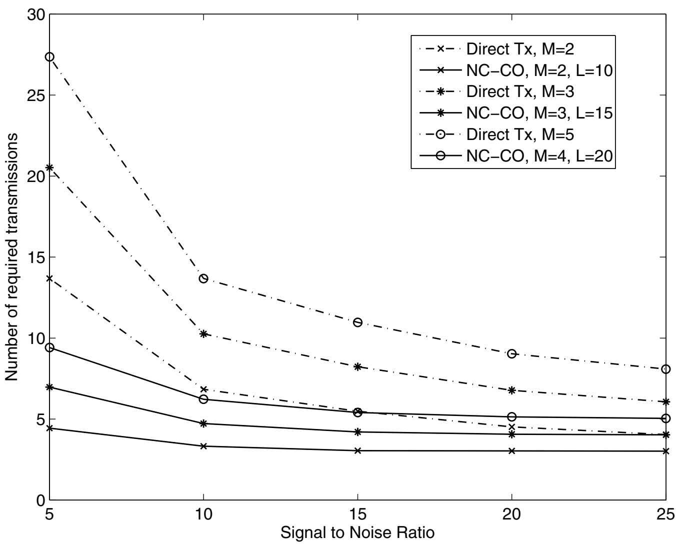

Fig. 1. Averaged number of required retransmissions vs SNR. The targeted data rate is $R = 1$ bit/s/Hz.

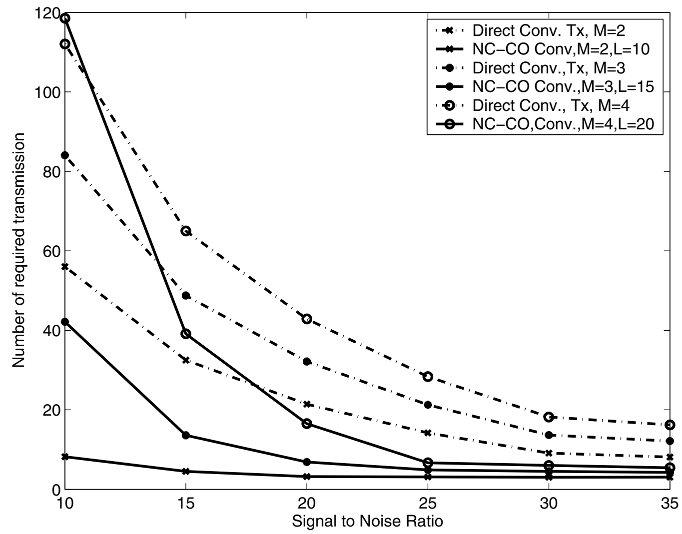

Fig. 2. Averaged number of required retransmissions vs SNR under $(5, 7)_{oct}$ systematic convolutional coed case. The targeted data rate is $R = 1$ bit/s/Hz.

IV. Numerical Results

In this section, the performance of the proposed two-hop transmission protocol is evaluated with the comparison to the two comparable schemes, the two-hop non cooperative direct transmission scheme and the two-hop best-relay scheme. First the expected number of required transmissions based on the *outage probability* is studied for the three transmission schemes. The targeted data rate is set as $R = 1$bits/s/Hz. In Fig. 1, the performance of the proposed cross-coding scheme is compared with the direct transmission. As can be seen from Fig. 1, the proposed scheme can achieve significantly performance gain over the non cooperative scheme particularly in low SNR range.

In the *convolutional coded* case, a $(5, 7)_{oct}$ systematic recursive convolutional code is considered with 100 information bits in each coded frame. Fig. 2 shows the performance comparison between convolutional coded cooperative transmission and direct transmission. Slightly different from the ideal random codes case, the practical coded transmission system may suffer the more transmissions at low SNR (e.g. less than 10dB) when M goes to large duo to the poor cooperative gain in low SNR region. But at medium to high SNR region, our protocol outperforms the direct transmission case significantly in terms of the number of transmission. Especially, the number of transmission would converge to $M+1$ while the increasing of SNR. Comparing Fig. 2 with the previous figure, we can observe that the number of transmissions based on convolutional coding is much larger than that based on outage probability, particularly at low SNR. And provided the use of more sophisticated error control codes, such as turbo or LDPC codes, the performance gap between the cooperative and non cooperative schemes can be reduced.

V. Conclusion

In this paper, we have proposed the application of network coding to multi-source multi-destination (MSMD) scenarios and provided formal analysis for the improvement of system delay. In particular, two types of analytical results have been developed, one based on the outage probability and the other based on the use of practical convolutional codes. These analytical results have shown that the use of network coding is helpful to the robustness of multi-session transmissions. Monte-Carlo simulation results have also been provided to demonstrate the delay performance of the proposed network coded protocol.

References

[1] R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Information Theory*, vol. 46, pp. 1204–1217, Jul. 2000.

[2] S. Zhang, S. Liew, and P. Lam, "Physical layer network coding," in *Proc. 12th Annual International Conference on Mobile Computing and Networking (ACM MobiCom 2006)*, Sept. 2006.

[3] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "On the study of network coding with diversity," *IEEE Trans. on Wireless Communications*, vol. 8, pp. 1247C–1259, Mar. 2009.

[4] Y. Chen, S. Kishore, and J. Li, "Wireless diversity through network coding," in *Proceeding of IEEE Wireless Communications and Networking Conference (WCNC)*, Mar. 2006, pp. 1681 – 1686.

[5] Z. Ding, T. Ratnarajah, and K. K. Leung, "On the study of network coded af transmission protocol for wireless multiple access channels," *IEEE Trans. on Wireless Communications*, vol. 8, pp. 118C–123, Jan. 2009.

[6] R. Cogill, B. Shrader, and A. Ephremides, "Stability analysis of random linear coding across multicast sessions," in *Proc. IEEE International Symposium on Information Theory (ISIT-08)*, Toronto, Canda, Jul. 2008.

[7] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels : performance limits and space-time signal design," *IEEE Journal on Select. Areas in Comm.*, vol. 22, pp. 1099–1109, Aug. 2004.

[8] D. A. Gore, R. W. Heath, and A. J. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Communications Letters*, vol. 6, pp. 491–493, Nov. 2002.

[9] M. Rupp, C. Mecklenbrauker, and G. Gritsch, "High diversity with simple space time block codes and linear receivers," *Proc. GLOBECOM*, vol. 2, pp. 302–306, Dec. 2003.

[10] Z. Ding, M. Zheng, and K. Leung, "On the application of network coding to multi-source multi-destination scenarios," *IEEE Trans. on Communications*, (submitted).

[11] A. Hedayat and A. Nosratinia, "Outage and diversity of linear receivers in flat fading MIMO channel," *IEEE Trans. on Signal Processing*, vol. 55, pp. 5868–5873, Dec. 2007.