Optimal Transmission Probabilities in VANETs with Inhomogeneous Node Distribution

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Abstract – In a Vehicular Ad-hoc Network (VANET), the amount of interference from neighboring nodes to a communication link is governed by the vehicle density dynamics in vicinity and transmission probability of terminals. It is obvious that vehicles are distributed non-homogeneously along a road segment due to traffic controls and speed limits at different portions of the road, the common assumption of homogeneous node distribution in the network in most of the previous work in mobile ad-hoc networks thus appears to be inappropriate in VANETs. To capture the density dynamics in generic urban routes, we utilize a fluid model to characterize the general vehicular traffic flow, and a stochastic model to capture the randomness of individual vehicles, from which we can acquire respectively the densities of the mean number of vehicles along the road and the probability distribution. With the knowledge of the vehicular density dynamics from the stochastic traffic model, we determine the throughput and progress performances of a routing strategy, and confirm the accuracy of the analytical results through simulations. The analytical model proposed in this paper serves as a fundamental building block for performance analysis of other transmission protocols and network configurations, we also demonstrate that the optimal transmission probability for optimized network performance can be found from our results, which provides insights into system engineering and protocol designs in VANETs.

I. INTRODUCTION

In a Vehicular Ad-hoc Network (VANET), vehicles communicate with each other and possibly with road-side infrastructure nodes. The amount of interference to a communication link depends on the number of concurrent transmissions in vicinity, which is governed by the vehicular density dynamics and the transmission probability of terminals.

There are a number of studies on analyzing capacity or throughput and forward progress in Mobile Ad-hoc Networks (MANETs). For instance, references [1-3] explore how network capacity scale with the number of nodes in the network. [4] investigates the optimal transmission radii for maximized expected forward progress in multi-hop packet radio networks, while [5] analyzes the performance of several transmission strategies in with transmission radius control. For VANET research, [6,7] investigate the capacity of VANETs and its scalability in highway and urban grid structures.

However, all of these studies mainly assume that node density is homogeneous throughout the whole network, and lack of a general approach for handling terminals that spatially distribute in a heterogeneous manner, which is obviously inappropriate in VANETs. In fact, vehicle movements, particularly in urban environments, are restricted by the road topologies, buildings, etc., and affected by traffic density, which is determined by road capacity, traffic control and driver behavior. Thus, we will expect the density of cars at road junctions (where traffic signals are located) behaves quite differently from that at the middle of a road segment, such spatial distribution is governed by the space and time dynamics of moving vehicles.

To capture such dynamics and thus the heterogeneity of the density distribution of vehicles, we proposed in [8,9] a stochastic traffic model for modeling vehicular traffic in generic urban road systems. The stochastic traffic model uses a fluid model to characterize the space and time dynamics of vehicle movements, which is driven by a velocity profile as a function of space and time. In real practice, empirical velocity measurements from GPS devices can serve as an input to the model. The mean density profile, again as a function of space and time, is readily computable from the conservation equations in the fluid model. The randomness of individual vehicle is captured by a stochastic model. The actual number of vehicles in a given road section at a certain time instance has Poisson distribution according to previous results in [10,11] given that the arrival of vehicles follow a non-homogeneous Poisson process.

We consider two channel access protocols, slotted ALOHA and CSMA, and a routing strategy that aims to minimize the number of hops from the source to the destination in this paper. Through the vehicular density dynamics computed from the stochastic traffic model, we determine the distribution of a node's location on the urban route, and derive the local throughput and average progress of the routing strategy in different transmission protocols. From the analytical results, we can identify the optimal transmission probability for maximized throughput and progress as a function of the location space, which is significant for system engineering and network planning in VANETs.

II. STOCHASTIC TRAFFIC MODEL

In this section, we define the system model for the analysis in this paper, and provide background information of the stochastic traffic model [8,9] that captures the vehicular density dynamics in generic urban routes.

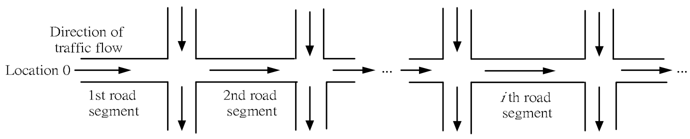


Figure 1. The road configuration considered in this paper.

We consider traffic in a one-way, single-lane, semi-infinite urban road (or route) as shown in Figure 1. Although the road is fed with traffic from adjacent streets, the one-way road under consideration is the one running from the left to the right in the figure. More complicated road topology can be represented by superposing multiple versions of urban routes. Let our location space to be the interval $[0, \infty)$, the boundary point 0 is the spatial origin, and it marks the starting point of the road. The arrival process $\{A(t) \mid -\infty < t < \infty\}$ counts the number of arrivals to the first segment of the route up to time t, which we assume is finite with probability 1, and is characterized by a non-negative and integrable external arrival rate function $\alpha(t)$.

Furthermore, the route consists of a number of road segments indexed by $i = 1, 2, 3, \ldots$, vehicles can leave and join the route at the junctions of road segments.

A. Deterministic Fluid Model

The fluid model is a kind of continuum traffic flow models, which reduces laws of traffic to a partial differential equation (PDE) that may be studied as elegantly and simply as other physical phenomena that are also governed by PDE's.

The major difference between our fluid model and other continuum models is that we model vehicle motions with a velocity profile, vehicles at location x and time t move forward the route according to a velocity field $v(x, t)$. Stopping or slowing down of vehicles at road junctions or traffic signals can be reflected and modeled by the velocity profile. However, continuum model alone is unable to capture traffic instability and the randomness of individual vehicle, therefore, we couple the fluid model with the stochastic model as a remedy.

We will first describe the fluid dynamic conservation equations and corresponding notations that hold for the general systems. Let $N(x, t)$ be the number of vehicles in location (0, x] at time t, and $n(x, t)$ be the density of vehicles in location (0, x] at time t. Let $Q(x, t)$ be the number of vehicles moving past position x before time t, and $q(x, t)$ be the flow rate. Thus,

$$n(x,t) = \frac{\partial N(x,t)}{\partial x} \text{ and } q(x,t) = \frac{\partial Q(x,t)}{\partial t}. \quad (1)$$

Let $C^+(x, t)$ and $C^-(x, t)$ be the number of vehicles arriving to and departing from the route in location (0, x] during time interval (-∞, t], respectively. Then the associated rate densities are respectively

$$c^+(x,t) = \frac{\partial^2 C^+(x,t)}{\partial x \partial t} \text{ and } c^-(x,t) = \frac{\partial^2 C^-(x,t)}{\partial x \partial t}. \quad (2)$$

Assuming all traffic moves only from left to right down the positive real line, then the four variables N, Q, C^+, C^- satisfy the following conservation relation:

$$C^+(x,t) = N(x,t) + Q(x,t) + C^-(x,t) \quad (3)$$

By applying the operator $\partial^2/(\partial x \partial t)$ to (3), we have the partial differential equation

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = c^+(x,t) - c^-(x,t). \quad (4)$$

According to traffic flow theory [12], we have

$$q(x,t) = n(x,t)v(x,t). \quad (5)$$

By substituting (5) into (4), we have

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial [n(x,t)v(x,t)]}{\partial x} = c^+(x,t) - c^-(x,t). \quad (6)$$

The resulting partial differential equation (6) is a one-dimensional version of the generalized conservation law for fluid motion. This equation governs the mean behavior of any stochastic traffic model.

We assume that vehicles arrive at the first route segment according to an external arrival rate function $\alpha(t)$. Let us use $\xi_i(t)$ to denote the external arrival rate of vehicles at the i-th junction at time t. Then we have

$$c^+(x,t) = \alpha(t)\delta(x) + \sum_i \xi_i(t)\delta(x - x_i). \quad (7)$$

As for vehicles leaving the route, we use $\rho_i(t)$ to denote the fraction of vehicles departing when they pass by the i-th junction at time t. If these departing vehicles leave at the same velocity as they move forward along the route, then

$$c^-(x,t) = \beta(x,t)n(x,t),$$
$$\text{where } \beta(x,t) = v(x,t)\sum_i \rho_i(t)\delta(x - x_i). \quad (8)$$

B. Stochastic Model

In contrast to the deterministic fluid model, the stochastic model captures the stochastic fluctuations of the quantities of interest. When the two models are coupled with each other to form the stochastic traffic model, the solutions from the PDE's describe the expected number of vehicles, and the actual number of vehicles is captured by the additional distribution information from the stochastic model.

From now on, the densities $n(x, t)$ and $q(x, t)$ are defined as the partial derivatives of expected values, that is,

$$n(x,t) = \frac{\partial E[N(x,t)]}{\partial x} \text{ and } q(x,t) = \frac{\partial E[Q(x,t)]}{\partial t}.$$

Similarly, the rate densities $c^+(x, t)$ and $c^-(x, t)$ are the second partial derivatives of expected values, that is,

$$c^+(x,t) = \frac{\partial^2 E[C^+(x,t)]}{\partial x \partial t} \text{ and } c^-(x,t) = \frac{\partial^2 E[C^-(x,t)]}{\partial x \partial t}.$$

The general stochastic model can be of any distributions depending on the arrival process of vehicles, and the equations in the fluid model continue to hold regardless the distribution of the stochastic model. In this paper, we specifically consider the Poisson arrival location model (PALM) [10,11]. Again, the fluid dynamic model is not dependent on the Poisson assumption; they hold as long as the arrival process A is an arbitrary point process with time-dependent arrival-rate function α.

With PALM, the arrival process $\{A(t) \mid -\infty < t < \infty\}$ for vehicles to arrive at the first road segment of the route is a *non-homogeneous Poisson process* with non-negative and integrable external arrival rate function $\alpha(t)$. That is, the number of arrivals in the interval $(t_1, t_2]$ is Poisson with mean $\int_{t_1}^{t_2} \alpha(s)ds$.

According to [10,11], we can construct $N(x, t)$, the random number of vehicles within the range (0, x] at time t, via stochastic integration starting with the Poisson process A, where $A(t)$ counts the number of vehicles arriving to the road segment up to time t.

$$N(x,t) = \int_{\sigma(x,t)}^{t} 1_{\{L_s(t) \in (0,x]\}} dA(s) = \sum_{n=A(\sigma(x,t))}^{A(t)} 1_{\{L_{\hat{A}_n}(t) \in (0,x]\}} \quad (9)$$

Thus, for all real t, $\{N(x, t) \mid x \geq 0\}$ is a Poisson process with

$$E[N(x,t)] = \int_{\sigma(x,t)}^{t} \alpha(s)ds. \quad (10)$$

where $\hat{A}_n$ is the nth jump time of A, counting backward from time t. 1_B is an indicator function such that it returns 1 if B is true and 0 otherwise. $L_s(t)$ is the location process, which specifies the position of the vehicle on the road segment at time t that arrived at time s. Let $\sigma(x, t)$ denote the route entrance time for a vehicle to be in position x at time t. For vehicles that arrive to the route before $\sigma(x, t)$, it will be past position x by time t. On the other hand, for vehicles arrive after $\sigma(x, t)$, it will be still in position x at time t.

Therefore, as long as we model the traffic flow through a deterministic velocity field as a function of space and time such that vehicles do not interact with each other, the Poisson distributional conclusion in [10,11] remains valid.

III. NETWORK MODELS

A. Channel Access Protocols

The protocols considered in this paper are slotted ALOHA and non-persistent carrier-sense-multiple-access (CSMA) [13].

For slotted ALOHA, time is divided into slots of duration equal to the transmission time of a packet. In every slot, each node tries to transmit according to a Bernoulli process with parameter p, where $0 < p \leq 1$. That is, it is in transmit mode with probability p and not in transmit mode with probability $1 - p$. It is assumed that all nodes always have packets waiting to be sent, and a separate channel is available for acknowledgement traffic. We further assume that the system is independent from slot to slot such that whenever there is a packet waiting to be sent, it is equally likely that this packet will be destined to any other node, no matter whether it is a new packet or retransmitted packet.

For slotted non-persistent CSMA, the constant packet transmission time is chosen as the unit of time, and the length of a mini-slot, denoted by τ, represents the propagation delay. Hence, the length of a successful transmission period is $1+\tau$ or $T + 1$ mini-slots, where $T = 1/\tau$, as shown in Figure 3b. We assume that all nodes within a Carrier-Sensing Range (*CSRange*) of the transmitter recognize the transmission in one mini-slot, and they hear the transmission one mini-slot more after the completion of transmission. Again, all nodes always have packets waiting to be sent. In every mini-slot, each node listens to the channel with probability p (except during transmission), and does not with probability $1 - p$. If the channel is idle, it begins transmission in the same slot with probability 1. If the channel is sensed busy, it suppresses the transmission, and stops sensing the channel until the end of the current transmission.

B. Routing Strategy

In many applications of VANETs, cars exchange traffic information in order to predict travel time and prevent congestions and accidents. We consider the scenario that vehicles aim to inform the following cars of the preceding road condition for travel time prediction, and to warn following cars to slow down in case of any accidents, thus, the direction of packet transmission is opposite to that of vehicular traffic flow.

Each node is assumed to use the same transmission radius R and know the position of those vehicles within the transmission range. Given a packet and its final destination, a node transmits to the most distanced neighbor within R in the backward direction (where we define the direction of vehicular traffic flow as the forward direction). If no nodes are within the backward transmission range R at all, it does not transmit in that slot.

Define *progress* as the distance between the transmitting and receiving nodes projected onto a line drawn from the transmitter toward the final destination (or simply the distance between the transmitter and receiver for one-dimensional networks), we denote this routing strategy as *Most Progress within R* (*MPR*).

C. Interference Model

We assume the following power-transfer relationship: $P(a, b) = cP_a/|a - b|^\gamma$, where $P(a, b)$ is the power received by node b from node a, P_a is the transmit power of node a, $|a - b|$ is the distance between nodes a and b (for brevity, we also use a and b to denote the nodes' positions), $\gamma > 2$ is the path-loss exponent, and c is a constant.

Consider the Protocol Model in [1]. For a link (a, b) to transmit successfully, we require

1) Node a is in transmitting mode and node b is not;
2) $|a - b| \le R$; and
3) $P_a|i - b|^\gamma > \beta P_i|a - b|^\gamma$ for every other node i simultaneously transmitting, where β is the SIR requirement.

In MPR, since all nodes use the same transmission power/radius, the transmission of node i will interfere with the signal reception at node b from node a if

$$|i - b| \le \beta^{1/\gamma}R. \tag{11}$$

Let $R_I = \beta^{1/\gamma}R$ be the interference range. We define that two nodes are *neighbors* if they are within a distance R_I of each other. Hence, a transmission from nodes a to b will be successful only if there are no neighboring nodes of b transmit concurrently.

IV. THROUGHPUT AND PROGRESS ANALYSIS

For analytical simplicity, we consider a specific time instance t_0 and cease to focus on time dynamics in this section. For this reason, the variable t is simply dropped from our previously defined notations. In practice, probabilities over a period of time can be obtained by taking the time-average of multiple time instances.

Consider a one-dimensional road segment of length L. From the fluid model, we can acquire the mean vehicular density profile $n(x)$, such that the expected number of vehicles within a region $(x_1, x_2]$ in the road segment is

$$E[N(x_1, x_2)] = \int_{x_1}^{x_2} n(x)dx. \tag{12}$$

Moreover, we can derive the pdf

$$f_L(x) = n(x)/E[N(L)] \tag{13}$$

where $f_L(x)\Delta x$ represents the probability that a random node in the road segment is located in the small region $(x, x + \Delta x]$. For notation simplicity, let us denote $E[N(x_1, x_2)]$ as $\bar{N}_{x_1}^{x_2}$, and $\int_{x_1}^{x_2} f_L(x)dx$ as $m_{x_1}^{x_2}$, which is the probability that a random node in the network is located in the region $(x_1, x_2]$.

Let C_a be the event that given the position of a transmitter a, it can be associated with a receiving node b in the backward direction. i.e., there is at least one node located in the region $(a, a - R]$. According to the Poisson property of the stochastic traffic model, we have

$$P(C_a) = 1 - \exp(-\bar{N}_{a-R}^{a}). \tag{14}$$

Let the random variable $r_a = |a - b|$ represent the distance between nodes a and b. For MPR, the cumulative distribution function of r_a assuming that C_a holds is

$$F_{r_a}(r) = P(r_a \le r) = \frac{P(\text{no nodes in } (a-R, a-r])P(\text{at least one in } (a-r, a])}{P(C_a)}$$
$$= \frac{e^{-\bar{N}_{a-R}^{a-r}}(1 - e^{-\bar{N}_{a-r}^{a}})}{1 - e^{-\bar{N}_{a-R}^{a}}} = \frac{e^{-\bar{N}_{a-R}^{a-r}} - e^{-\bar{N}_{a-R}^{a}}}{1 - e^{-\bar{N}_{a-R}^{a}}} \tag{15}$$

A. Slotted ALOHA

For slotted ALOHA, to have the transmission from nodes a to b be successful, node b's neighboring nodes must not transmit in the same slot. Let node i be one of b's neighbors and random variable s_i be the distance between nodes b and i. Given that there is a transmission from nodes a to b, there exists an *excluded region* such that no nodes exist. This region is a function of the position of b with respect to a, and the routing strategy used. For MPR, we know that node i cannot be located in $(a - R, a - r_a]$, otherwise, node a will transmit to node i instead of node b. Therefore, node i could be in any of the following two regions as illustrated in Figure 2. Region 1: $(a - r_a, a - r_a + R_I]$; and Region 2: $(a - r_a - R_I, a - R]$.

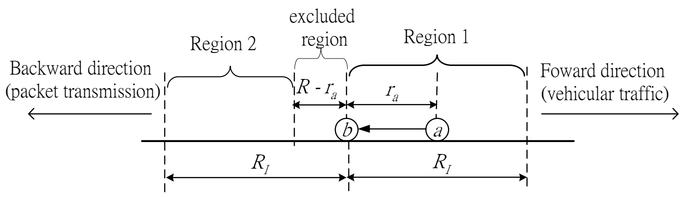


Figure 2. The regions that a neighboring node of node b could be located in under the MPR strategy.

Let $G(g)$ denote the event that node i is located in Region g, where $g = 1, 2$, and I_N the event that node i will interfere with node b, given that node i is in transmit mode in the same slot. Thus, given that $r_a = r$ and the position of node a, we have

$$P(I_N) = \sum_{g=1}^{2} P(I_N|G(g)) \cdot P(G(g)) \tag{16}$$

where $$P(G(g)) = \int_{G(g)} f_L(x)dx / (m_{a-r-R_I}^{a-r+R_I} - m_{a-R}^{a-r}) \tag{17}$$

$$P(I_N \mid G(g)) = \int_s P(C_i \mid s_i = s) f_{s_i}^{G(g)}(s)ds \tag{18}$$

where C_i is the event that node i can be associated with a receiving node in the backward direction, $f_{si}^{G(g)}(s)$ denotes the pdf of s_i (or the distance $|b - i|$) given that node i is in Region g.

Let q be the probability that neither transmitter a nor receiver b is within node i's transmission range in the backward direction. When neither nodes a nor b is within the backward transmission range of node i, node i will be able to transmit with probability $1 - \exp(-\bar{N}_{i-R}^{i})$ when it is in transmit mode. On the other hand, when either nodes a or b is within node i's backward transmission range, node i can always find a receiver. Thus, given the position of node i, we have

$$P(C_i) = (1-q) + q(1-e^{-\bar{N}_{i-R}^{i}}) = 1 - qe^{-\bar{N}_{i-R}^{i}} \quad (19)$$

Given that $r_a = r$, $s_i = s$, and the position of node a, if node i is located in Region 1, $q = m_{a+R}^{a-r+R_I} / m_{a-r}^{a-r+R_I}$ and the position of node i is $a - r + s$; while if it is in Region 2, q simply equals to 1 since nodes a and b are not located in the backward direction of node i, and node i's position is $a - r - s$. Thus, (16) becomes

$$P(I_N) = \frac{1}{m_{a-r-R_I}^{a-r+R_I} - m_{a-R}^{a-r}} \left[\begin{array}{l} \int_0^{R_I} (1-qe^{-\bar{N}_{a-r+s-R}^{a-r+s}}) f_L(a-r+s)ds \\ +\int_{R-r}^{R_I} (1-e^{-\bar{N}_{a-r-s-R}^{a-r-s}}) f_L(a-r-s)ds \end{array} \right] \quad (20)$$

Let E_i be the event that neighbor i does not interfere with node b. E_i will occur when node i is not in transmit mode, or it is in transmit mode but it cannot find any receiving nodes.

$$P(E_i) = (1-p) + p(1-P(I_N)) = 1 - pP(I_N) \quad (21)$$

Let $a \rightarrow b$ be the event that the transmission from nodes a to b is successful given that nodes a transmits to b, and N_k be the event that node b has k neighbors, not including node a.

$$P(a \to b \mid N_k) = P(b \text{ does not transmit AND the } k \text{ neighbors do not interfere with } b)$$

$$= (1-p)\prod_{n=1}^{k} P(E_i) = (1-p)(1-pP(I_N))^k. \quad (22)$$

With regard to the Poisson property of the stochastic traffic model, we have the number of vehicles within a road region be a Poisson process, since the k neighbors of node b cannot be located in the excluded region $(a - R,\ a - r_a]$ given the transmission from nodes a to b, we have

$$P(N_k \mid r_a = r) = \frac{\left(\bar{N}_{a-r-R_I}^{a-r+R_I} - \bar{N}_{a-R}^{a-r}\right)^k}{k!} e^{-\left(\bar{N}_{a-r-R_I}^{a-r+R_I} - \bar{N}_{a-R}^{a-r}\right)}. \quad (23)$$

Thus, $\quad P(a \to b) = \sum_{k=0}^{\infty} P(a \to b \mid N_k) \cdot P(N_k)$

$$= (1-p)\sum_{k=0}^{\infty} (1-pP(I_N))^k \cdot \int_0^R P(N_k \mid r_a = r) dF_{r_a}(r)$$

where $F_{ra}(r)$ is given in (15). Substitute (23) into it, we have

$$P(a \to b) = (1-p)\int_0^R \exp\left[-pP(I_N)\left(\bar{N}_{a-r-R_I}^{a-r+R_I} - \bar{N}_{a-R}^{a-r}\right)\right] dF_{r_a}(r). \quad (24)$$

To evaluate the throughput and progress of the network, we compute ρ, the local throughput (or single-hop throughput), which is the average number of successful transmission per slot from a node, and π, the average progress in unit distance per slot from a node, which is a measure of the packet propagation rate. The local throughput of the transmitting node located at a is

$$\rho_a = P(C_a) \cdot P(a \text{ transmits}) \cdot P(a \to b) \quad (25)$$

Substitute (14) and (24) into it, we have

$$\rho_a = (1-e^{-\bar{N}_{a-R}^{a}}) p(1-p)\int_0^R e^{-pP(I_N)\left(\bar{N}_{a-r-R_I}^{a-r+R_I} - \bar{N}_{a-R}^{a-r}\right)} dF_{r_a}(r). \quad (26)$$

The average progress for a node located at a, π_a can be obtained similarly by inserting the term r into the integration above,

$$\pi_a = (1-e^{-\bar{N}_{a-R}^{a}}) p(1-p)\int_0^R e^{-pP(I_N)\left(\bar{N}_{a-r-R_I}^{a-r+R_I} - \bar{N}_{a-R}^{a-r}\right)} r dF_{r_a}(r). \quad (27)$$

Therefore, the average local throughput and progress for the whole network in the road segment $(0, L]$ is given by the following weighted sums.

$$\rho = \int_0^L \rho_x f_L(x)dx \text{ and } \pi = \int_0^L \pi_x f_L(x)dx. \quad (28)$$

B. Slotted non-persistent CSMA

When analyzing the CSMA protocol, we introduce the assumption that the actual transmission occurs as a result of channel sensing are independent Bernoulli trials. That is, for every mini-slot (except during transmission), each node transmits a packet with probability p' (and does not with probability $1 - p'$). A similar assumption was used in [4,13], and the validity of the results will be claimed by comparing the throughput values against simulation in the next section. The parameter p' is the transmission rate per mini-slot. With reference to the appendix in [4], the sensing probability for a transmitter located at x, $p(x)$, can be expressed in terms of p', τ (the propagation delay), and $N_{CS}(x)$ (the expected number nodes within the carrier-sensing range of x) as

$$p(x) = \frac{p'(1+\tau-e^{-p'N_{CS}(x)})}{\tau e^{-p'N_{CS}(x)}} \quad (29)$$

where $N_{CS}(x) = \bar{N}_{x-CSRange}^{x+CSRange}$ can be found from $n(x)$.

A transmission is successful when no neighbors of the receiver transmit during the transmission period $1 + \tau$ or $T + 1$ mini-slots. For MPR, potential interfering (or neighboring) nodes are located in Regions 1, 2, or 3 as illustrated in Figure 3a. Since neighboring nodes of b in Regions 1 and 2 are within the *CSRange* of transmitter a (where $CSRange \geq R$), they can recognize the transmission in one mini-slot, and collision can be avoided if they do not transmit in the same slot as node a. While for neighbors located in Region 3, they are hidden to node a, it is required that they are silent throughout the vulnerable period of length $2 + \tau$ or $2T + 1$ mini-slots as shown in Figure 3b. The first T mini-slots are included for preventing collisions with any ongoing transmissions, while the following $T + 1$ mini-slots are included for not being interfered with any newly started transmissions. Therefore, given that $r_a = r$, we have

$$P(a \to b \mid r_a = r) = P(b \text{ does not transmit in the same slot}) \cdot$$
$$P(\text{no transmission from Reg 1 and 2 during a mini-slot} \mid r_a = r) \cdot$$
$$P(\text{no transmission from Reg 3 during } 2T+1 \text{ mini-slots} \mid r_a = r)$$
$$= (1-p') \cdot e^{-p'P(I_N|G(1))\bar{N}_{a-r}^{a-r+R_I}} \cdot e^{-p'P(I_N|G(2))\bar{N}_{a-CSRange}^{a-R}} \cdot e^{-(2T+1)p'P(I_N|G(3))\bar{N}_{a-r-R_I}^{a-CSRange}} \quad (30)$$

where $P(I_N \mid G(g))$ is given by (18).
Hence, the local throughput and expected progress for the transmitter node located at a are given by

$$\rho_a = (1-e^{-\bar{N}_{a-R}^{a}})\frac{p'}{\tau}\int_0^R P(a \to b \mid r_a = r) dF_{r_a}(r) \quad (31)$$

$$\pi_a = (1-e^{-\bar{N}_{a-R}^{a}})\frac{p'}{\tau}\int_0^R P(a \to b \mid r_a = r) r dF_{r_a}(r). \quad (32)$$

The average local throughput and progress for the whole network can be found accordingly by (28). In the next section, we aim to optimize the expected progress π by finding the optimal transmission probability p' (or p for slotted ALOHA), and the corresponding optimal sensing probability.

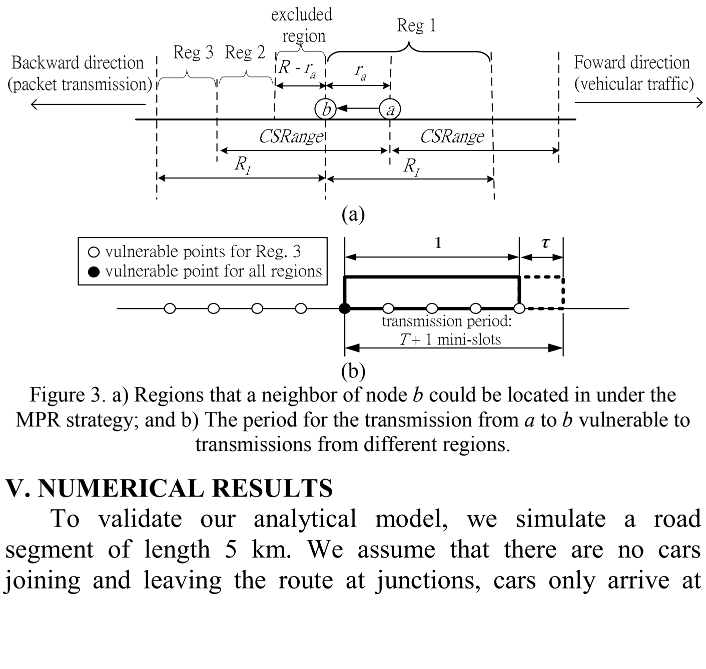
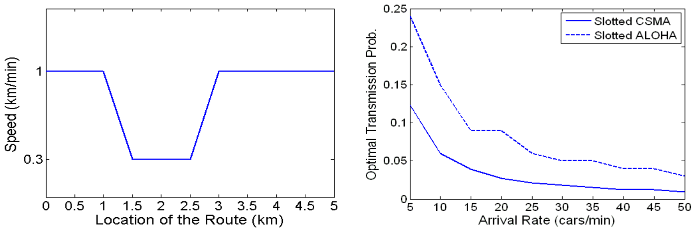


Figure 3. a) Regions that a neighbor of node b could be located in under the MPR strategy; and b) The period for the transmission from a to b vulnerable to transmissions from different regions.

V. NUMERICAL RESULTS

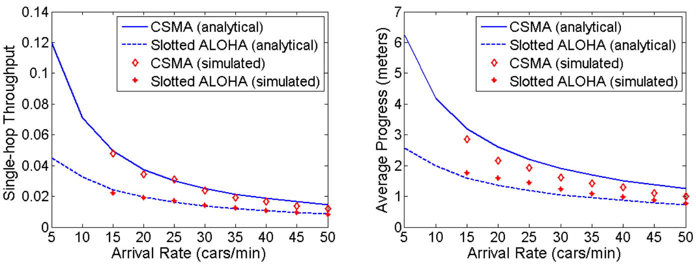
To validate our analytical model, we simulate a road segment of length 5 km. We assume that there are no cars joining and leaving the route at junctions, cars only arrive at location 0 at a constant rate (denoted by α). Every vehicle moves along the road with respect to a velocity profile, $v(x)$ as shown in Figure 4a, which describes a slowdown from positions 1 to 3 km. Intuitively, we know that the slowdown region will result in higher node density than the other parts of the road, where the mean density profile $n(x)$ can be computed by the fluid model. In the simulations, we have $R = 0.1$ km, $\beta = 10$, $\gamma = 4$. For CSMA, $\tau = 0.25$ and *CSRange* = 0.178.



(a) (b)

Figure 4. a) Velocity profile used for examining the non-homogeneous density case; and b) transmission probability that maximize the average progress

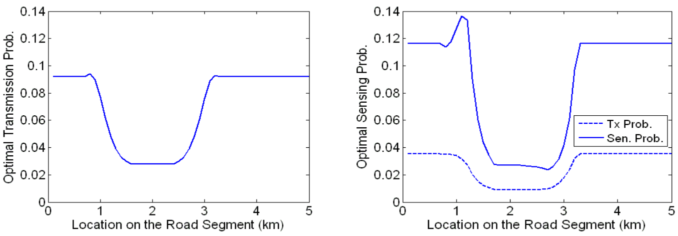
From the analytical results, the optimal transmission probability p for slotted ALOHA or p' for CSMA is sought to maximize the average progress π in the network, which are plotted in Figure 4b against the arrival rate of vehicles to the road segment. We can see from the figure that CSMA has a smaller optimal transmission probability than slotted ALOHA, it is because nodes suppress their transmissions to avoid collisions when the channel is sensed busy. The optimal p and p' found are served as inputs to the simulations. For CSMA, the corresponding optimal sensing probability $p(x)$ is found based on the optimal p' and density profile $n(x)$ according to (29).



(a) (b)

Figure 5. a) Single-hop throughput; and b) average progress against vehicle arrival rate for slotted ALOHA and CSMA protocols.

The analytical and simulated results of the maximized average progress π, and the corresponding single-hop throughput ρ for slotted ALOHA and CSMA are plotted in Figure 5b and Figure 5a respectively as a function of the arrival rate of cars to the road (For proper comparison, the average progress and single-hop throughput of slotted ALOHA is divided by $1 + \tau$ to include the propagation time in a slot). Each simulated data is obtained by averaging the results of 500 simulation runs, we can see from the figures that the analytical results match closely with the simulated results, which confirms our analysis.



(a) (b)

Figure 6. a) Optimal transmission probability for slotted ALOHA; and b) Optimal sensing probability for CSMA as a function of the Location Space.

Instead of maximizing the average local throughput π, and progress ρ for the whole network, we can optimize π_x and ρ_x for a specific location point x in the road segment. The optimal transmission probabilities $p(x)$ for slotted ALOHA, $p'(x)$ for CSMA (and the corresponding optimal sensing probability) are shown in Figure 6a and Figure 6b respectively. From the figures, we can see that region with higher node density (the slowdown region from 1 to 3 km) results in lower optimal transmission and sensing probabilities, which is intuitively correct. Our results here serve as a fundamental building block for the design of protocols in VANETs with adaptive transmission/sensing rate according to the vehicular density in the future.

VI CONCLUSION

Nowadays, most of the vehicles are installed with GPS devices that can collect velocity information. Such information can serve as an initial input to the stochastic traffic model for computing the vehicular density dynamics.

In this paper, we have analyzed the throughput and progress of VANETs in a generic urban route with inhomogeneous node distribution based on the density information provided by the stochastic traffic model. We have confirmed the accuracy of our analysis through simulations, and demonstrated that the optimal transmission probability (or sensing probability) for optimized performance can be identified through the analytical model, which is significant for system engineering and network planning in VANETs.

In general, the analysis in this paper is applicable to more elaborated urban traffic models. For example, routes with arrival and departure of vehicles at road junctions, multiple lanes, bi-directional traffic. More complicated urban road network (e.g., two-dimensional road topology) can be represented by superposing multiple versions of urban routes. Moreover, we can consider more practical velocity profile, or even velocity profile that is a function of vehicular density to approximate interactions between vehicles. In addition, the throughput and progress performance of other routing strategies and channel access protocols can be evaluated similarly with the approaches presented in this paper.

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