Density-Based Optimal Transmission for Throughput Enhancement in Vehicular Ad-Hoc Networks

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Abstract—Vehicular ad-hoc networks (VANETs) have received a lot of research and industrial attention, including the approval of the IEEE 802.11p standard. However, resource allocation in the standard still makes use of the traditional mechanisms (e.g., carrier sensing) without exploiting the unique characteristics of VANETs. This provides the motivation for this work. As a first step toward the goal and by considering vehicle density, this paper investigates how transmission probability can be determined to optimise throughput of VANETs. A challenging design issue of VANETs is to deal with node (vehicle) mobility, which causes various vehicular densities within the same network and consequently influences the connectivity and capacity of the network. This work shows that it is indeed possible to follow the dynamics of a network and consequently adapt the transmission probability at the MAC layer to reduce the interference and maximise the single-hop throughput between adjacent nodes. By exploiting the characteristics of VANETs, we introduce approximations in order to derive closed-form expressions of the network throughput and other performance metrics in terms of transmission probability, which would otherwise be impossible. Our extensive simulations validate the approximations and the proposed analytical model thus can serve as a promising tool to improve VANETs performance. For example, the optimal transmission probability can be used to develop efficient MAC protocols using vehicle density estimation in VANETs for our future work.

Keywords—Vehicular Ad-Hoc Networks, Throughput, transmission probability, MAC layer, 802.11p.

I. INTRODUCTION

Vehicular ad-hoc networks (VANETs) are highly mobile wireless networks formed of vehicles that communicate with each other through a multi-hop ad-hoc connection. The integration of communication technology with transportation systems creates a self-organising and rapidly deployable network that ultimately does not require a permanent infrastructure [1], [2]. Furthermore, hybrid networks can also be supported, since vehicles can communicate with road side access point.

VANETs have been employed in many scenarios and their applications can be divided into two different classes: civilian and military domains. In civilian applications they are mostly used with safety related issues, such as collision warning, where the object is to prevent imminent car accidents through coordination between vehicles when visual range is limited by conditions such as rain or fog ultimately limiting the situation awareness [1], [2], [3]. In addition, vehicle-to-vehicle (V2V) communications can be exploited for applications such as intelligent cruise control, traffic information system and internet access. Vehicular ad hoc networks are also extremely important in military operations such as target acquisition, rescue missions and tracking operations that often include the combined use of Unmanned Aerial Vehicles (UAVs) in addition to the traditional ad-hoc network [4], [5].

The relevance of this kind of networks has been confirmed by the development of a specific IEEE standard, purposely adapted to vehicular characteristics and operating scenarios. In particular, the IEEE 802.11p is a wireless area network (WLAN) standard for dedicated short-range communication (DSRC) among vehicles [3], [6], [7]. It defines protocols for the physical and MAC layers and has a 75 MHz bandwidth allocated at 5.9 GHz.

Many issues arise due to the high mobility of nodes (vehicles) within VANETs. The rapid change in the network topology is difficult to handle because it significantly affects the performance of the network as well as the frequent fragmentation into multiple clusters that usually takes place in vehicular traffic scenarios. An additional limitation of the VANETs is related to the low latency that is often required by many applications, especially for the implementation of safety systems. Ergo, it becomes imperative to maintain the network connectivity in order to achieve reliable communication across all of the nodes. Connectivity and capacity are very important issues that have been widely investigated in [8], [9], [10], [11], [12] and both connectivity and capacity are closely influenced by vehicular density. VANETs are expected to properly enable communications among vehicles under various conditions of vehicular density. Unfortunately, both dense or sparse vehicle density scenarios are difficult to handle. It is so because when vehicle density is too high, the communication needs among vehicles may not be supported by the limited network capacity. On the other hand, for low vehicle density, excessive distance between adjacent vehicles can cause connectivity problems for communications. As a result, efficient resource allocation (e.g., right to transmit) should strike a balance for the optimal network performance by directly considering vehicle density. Having said this, the medium-access-control (MAC) protocol in the IEEE 802.11p still makes use of indirect mechanisms through carrier sense. Specifically, the standard uses Enhanced distributed channel access (EDCA) that employs carrier sense multiple access with collision avoidance (CSMA/CA). The latter is characterised by a back off window and a fixed transmis-
transmission probability $p_t$. The need for distributed communications and dynamic topology requires improvement of the classic protocols adapted to the dynamics of highly mobile networks. In [3] for instance, a variable back off window is proposed, while in [10] the problem of choosing the transmission power to reduce the network interference is investigated. Optimisation of VANETs performance is also performed and analysed in [13], [14], [15]. In [13], the authors derive the best transmission range to minimise the energy usage over uniformly distributed networks. Authors of [14] investigate the optimal transmission radius to enhance the packet progress, assuming a Poisson distribution of the terminals to derived an improved model. The work considers two MAC protocols: slotted ALOHA and CSMA. It is not worthy to note that the model does not take into account VANET scenario with highly mobile nodes. In [15] the matter of position-related optimal transmission probability for packet progress enhancement is investigated. Unfortunately, due to the high mobility of nodes, vehicular density can change rapidly creating heterogeneous scenarios. Hence, it is not always possible to have a priori knowledge of the density distribution based on vehicles locations as it is assumed in [15].

As a first step to devise efficient MAC protocols for VANETs, we focus in this paper on optimising the transmission probability to enhance the single-hop throughput for connection between adjacent nodes, in order to keep the network connected (i.e., maintaining minimally connected [11]). A closed form solution is indeed achievable under some assumptions that help to handle such a complex interference scenario. As a proof of the model validity, extensive simulations are compared with it, showing that the proposed model can be a powerful tool to improve the performance of a vehicular ad-hoc network, enabling the nodes to follow the highly changing network conditions and adapt accordingly. To gain the design insights, a simple VANET is considered here, which consists of a single-lane road with one traveling direction where vehicles arrive according to a Poisson process at the entrance point of the road and move at a constant speed on the road. Vehicle movement can be characterised by a fluid mobility model. It is assumed that all vehicles use the slotted Aloha protocol with a uniform transmission probability to control their channel access. Using the information from the mobility model, the expression for the network throughput is derived analytically. Consequently, the optimal transmission probability can be evaluated and obtained based on the vehicle density. This result reveals that it is possible to improve VANET performance by adapting the transmission probability depending on the vehicular density. In fact, the optimal transmission probability can be determined and thus achieving the best network throughput in a distributed manner by vehicles (nodes\(^1\)) through sensing and estimating vehicle density in the vicinity area. This work represents our first step toward the design of an efficient MAC protocol specifically tailored for VANETs, in contrast to the CSMA protocol in the 802.11p, to adjust the backoff window according to vehicle density. The rest of the paper is organised as follows. Section II presents the network model, describing the mobility, interference and connectivity and throughput models. Section III shows numerical results and comparisons with our model. Finally conclusions are given in section VI.

II. SYSTEM MODEL

A. Mobility Model

We characterise the traffic source and its underlying assumptions. An infinite single lane, one direction road is considered in this work, as shown in Figure 1. The one-dimensional scenario can be helpful to give a good insight into more complex scenarios. As in [8], [10], [14], [15], a Poisson arrival process with an integrable rate function $\lambda(t)$, is considered to be a good traffic generation model in order to describe the approaching vehicles distribution at the entrance of the road section in a free flow state scenario. Vehicles can join the network only through the main entrance (located at the far left hand side) as in Figure 1, and they are not supposed to enter or leave the network along the road. After entering the network, vehicles proceed from left to right as shown in Figure 1 and maintain a constant velocity $v$ during the time interval $(0, t]$.

Finally, no interactions between vehicles are taken into account, as a result the locations of every node depend only on vehicle arrival times.

By assumption of Poisson arrival process, the expected value of the number of vehicles entering the road in the time interval $(a, b]$ is:

$$E[V(t)] = \int_a^b \lambda(t)dt. \quad for \ t \geq 0 \quad (1)$$

To describe vehicle movement on the road, a fluid model is used where vehicles are treated as continuum fluid with variables $\zeta$ and $q$ denoting the vehicle density and flow rate, respectively.

Considering that vehicles move at a constant velocity $v$ on the road and that there are no interactions among vehicles, the flux is defined as

$$q = v \zeta. \quad (2)$$

Consequently, due to the Poisson arrivals assumption, we obtain that the distance between adjacent vehicles is exponentially distributed with a probability density function (pdf)

$$f(x) = \zeta e^{-\zeta x}. \quad (3)$$

Given the exponentially distributed distance between any two adjacent nodes, the distance $(x)$ between any two non-adjacent

\(^1\)In this paper we refer to both terms vehicle and node as synonyms.
vehicles will follow an Erlang distribution (i.e., the sum of exponentially distributed random variable). This distribution has the following pdf

\[
f(x) = \frac{e^x}{(k-1)!}x^{k-1}e^{-\zeta r},
\]

where \(k\) represents the number of vehicles located between the two non-adjacent vehicles under consideration and \(\zeta\) is the vehicular density in the network.

**B. Interference Model**

The analysis of the co-channel interference requires the consideration of the following assumptions. To consider the characteristics of certain applications, let us focus on data transmission from a vehicle to another vehicle that is traveling right next to the former one. Specifically, data communication can flow both directions and we consider that the transmission is established between adjacent nodes, as to maintain the connectivity of the network and reduce the interference simultaneously. For example, node \(N_i\) is transmitting towards the receiving node \(N_j\), while an arbitrary node is denoted by \(N_k\), as shown in Figure 2.

The transmission power \(P_t\) is set to be identical on every node. Signal attenuation is assumed solely due to distance with a power exponent \(\alpha > 2\) in the operating environment. The system is assumed to be interference limited, thus thermal noise is neglected in the analysis.

Slotted Aloha protocol is used to allow channel access for analytic simplicity, rather than the more sophisticated CSMA required in vehicular standard 802.11p. In fact, this work is only a first step with the sole purpose of proving that it is indeed possible, under certain circumstances, to find a closed form solution to optimise network parameters, i.e. the transmission probability, based on system density dynamics; hence at this stage of the research Slotted Aloha protocol is adopted for analytic simplicity. Finally, half-duplex communication is considered such that each node can either transmit or receive signal, but not both, at any given time.

The received power \(P_r\) at a node depends on the transmission power \(P_t\) of the transmitted packet and the path loss \(\gamma\) from the transmitting node to the receiving one as follows

\[
P_r = P_t \left(\frac{1}{d(N_i, N_j)}\right)^\alpha \text{ with } \alpha > 2,
\]

where \(d(N_i, N_j)\) represents the distance in metres between a transmitter and a receiver. The signal-to-interference ratio (SIR) at the receiving node, for \(M\) interfering nodes and under the interference-limited assumption is

\[
SINR = \frac{P_r}{\sum_{k=1}^{M} I_k P_r(k)},
\]

where the denominator is the total interference received at the receiving node and \(I_k\) is an indicator of 1 or 0, corresponding to whether node \(k\) is transmitting or not, respectively. The indicator \(I_k\) reflects whether node \(k\) is allowed to transmit according to the slotted Aloha protocol under consideration. The probability mass function (pmf) for \(I_k\) is given by

\[
f(\omega) = \begin{cases} p_t & \text{for } \omega = 1 \\ 1 - p_t & \text{for } \omega = 0. \end{cases}
\]

Inserting (5) into (6) we obtain the general expression for SIR at \(N_j\) when \(N_i\) is transmitting

\[
SIR_{i,j} = \frac{P_t \left(\frac{1}{d(N_i, N_j)}\right)^\alpha}{\sum_{k=1}^{M} I_k P_t \left(\frac{1}{d(N_k, N_j)}\right)^\alpha}.
\]

**C. Connectivity and Throughput Model**

The following assumptions are used in order to evaluate the connectivity of the system. Although data packets can be transmitted and forwarded from one vehicle to another via multi-hops, we focus on the connectivity between any two adjacent (neighbouring) vehicles. That is, we consider whether a packet can be received by the vehicle immediately in front of or behind a given transmitting vehicle.

The communication range \(R_c\) is defined as the distance from a given transmitting vehicle within which any other vehicle can receive the signal with a power level exceeding a threshold (referred to as the receiver sensitivity). For our analysis, the communication range is assumed to be identical for all nodes in the network, [12], [13], [14], [15].

For any two adjacent nodes \(N_i\) and \(N_j\), Figure 2 shows two separate segments of the road, namely \(S_L\) and \(S_R\), where possible interfering nodes are located that can interfere with transmission between \(N_i\) and \(N_j\).

**Connectivity Requirements:** Two adjacent nodes are considered to be connected if two conditions are fulfilled, such as the vehicles are located within each other’s communication range \(R_c\), which is referred as the event \(E\), and the communication link between them has a SIR exceeding a prefixed threshold \(\beta\). Therefore, the connectivity conditions are given by

\[
d(N_i, N_j) \leq R_c,
\]

and

\[
SIR_{i,j} \geq \beta.
\]

Given the exponential distribution for distance between two adjacent vehicles in (3), the probability of event \(E\), that (9) is valid, is given by

\[
P[E] = 1 - e^{-\xi R_c}.
\]

By the definition of SIR in (8), the threshold in (10) can be expressed as,

\[
P_{r(i,j)} \geq \beta \sum_{k=1}^{M} I_k P_{r(k,j)}.
\]

Where \(P_{r(i,j)}\) denotes the received power at node \(N_j\) from \(N_i\). Unfortunately, to determine whether the condition in (12) is satisfied requires complicated calculation because of the random variables involved, including the transmission indicators \(I_k\) and the distribution of distance between the vehicles. As a
result, it is not possible to obtain closed-form expression for the performance metrics of interest. To overcome this difficulty, we propose to replace the connectivity requirement in (12) by a set of single-interferer conditions. That is, the SIR at the receiving node \( N_j \), associated with a transmission from node \( N_i \), satisfies the following expression for every interfering vehicle \( \hat{N}_k \)

\[
P_{r(i,j)} \geq I_k \beta P_{r(k,j)} \quad \forall k. \quad (13)
\]

Clearly, replacing the requirement in (12) by a set of conditions in (13) represents an approximation, which is intuitively reasonable for VANETs. This is so because vehicles tend to spread along the roads and vastly different path losses between the interfering nodes and the receiving node often result into strong and weak interferers. Therefore, considering the effect of dominant interferers in (13) can closely approximate the effect of all interferers combined, as in (12). Extensive simulation in a later section validates this approximation. By using the path loss formula in (5), (13) can be expressed as,

\[
d(N_k, N_j) \geq I_k \beta^{1/\alpha} d(N_i, N_j) \quad \forall k. \quad (14)
\]

The above inequality represents that the distance \( d(N_k, N_j) \) between node \( N_k \) and node \( N_j \) exceeds the distance \( d(N_i, N_j) \) by a factor of \( \beta^{1/\alpha} \). This condition needs to be verified for every possible interfering \( k \)th node in the network in order to guarantee a successful reception in terms of SIR in (13).

Let us now define an event \( F_k \) where the condition in (13) or equivalently (14) is satisfied for a given interfering node \( N_k \). Clearly, the event \( F_k \) occurs when either node \( N_k \) is not transmitting (i.e., \( I_k = 0 \)) or if it does, \( d(N_k, N_j) \geq \beta^{1/\alpha} d(N_i, N_j) \). Therefore, we have

\[
P[F_k] = P\{I_k = 0 \lor (I_k = 1 \land d(N_k, N_j) \geq \beta^{1/\alpha} d(N_i, N_j))\} \quad (15)
\]

Evaluating the probability in (15) again requires complicated calculation because distances between two vehicles are random variables. Consequently, it is impossible to obtain a closed-form expression for the probability. To overcome the difficulty, we observe that the random distance between node \( N_i \) and \( N_j \), \( d(N_i, N_j) \), is characterized by (3). In order to meet the requirement of receiver sensitivity as defined for the communication range, \( d(N_i, N_j) \) has its maximum value of \( R_c \), as shown in (9). Replacing \( d(N_i, N_j) \) by \( R_c \) in (15) provides the approximate probability as

\[
P[F_k] = P\{(I_k = 1 \land d(N_k, N_j) \geq R_c \beta^{1/\alpha}) \lor I_k = 0\} \quad (16)
\]

For convenience, we set \( R_f = R_c \beta^{1/\alpha} \). If node \( N_k \) is located beyond \( R_f \) from the receiving node \( N_j \), the interference condition in (13) is satisfied for node \( \hat{N}_k \). Therefore, \( R_f \) is referred as the interference range below. As a first step towards the evaluation of such probability in (16), we need information regarding the density dynamics, in term of \( \zeta \), that can be evaluated through the fluid model shown in (2).

As the next step, let us determine the probability of an event \( A \) that node \( N_k \) is located outside the interference range \( R_f \) of node \( N_j \). As shown in Figure 2, any interfering node \( N_k \) can be located in road segments, \( S_L \) and \( S_R \), to the left and right of node \( N_j \). Given the Poisson vehicle arrivals, the distance between any two adjacent nodes (vehicles) is exponentially distributed as given in (3). Therefore, the distance between two non-adjacent nodes has an Erlang distribution in (4) where the value of \( k-1 \) represents the number of nodes between the non-adjacent nodes. Combining this fact with the indexing scheme \( k \) for interfering nodes as shown in Figure 2, the probability that node \( N_k \) lies beyond \( R_f \) from node \( N_j \) is

\[
P[A] = \sum_{n=0}^{k-1} \left(\frac{(\zeta R_f)^n}{n!}\right) e^{-\zeta R_f} \quad \forall k. \quad (17)
\]

As the access protocol assumption in (7), the probability that node \( N_k \) does not transmit is

\[
P[I_k = 0] = 1 - p_t \quad (18)
\]

Substituting (17) and (18) into (16) yields

\[
P[F_k] = 1 - p_t [1 - \left(\sum_{n=0}^{k-1} \left(\frac{(\zeta R_f)^n}{n!}\right) e^{-\zeta R_f}\right)], \quad (19)
\]

It is worth noting that the probability in (19) is that for satisfying the single-interferer SIR condition in (13) or equivalently (14) for node \( N_k \). The probability of satisfying (14) for all interfering nodes \( N_k \) located in the road segment \( S_L \) is thus given by

\[
P[L_k] = \prod_{k=1}^{S} P[F_k] \quad (20)
\]

because all nodes transmit or not independently. Similarly, the corresponding probability for all interfering nodes \( N_k \) in the road segment \( S_R \) is

\[
P[R_q] = \prod_{k=2}^{Q} P[F_k]. \quad (21)
\]

Using (20) and (21), the probability of meeting all of the interference conditions in (13) or (14) for transmission from node \( N_i \) to \( N_j \) (defined as the event \( G_{ij} \)), despite of all possible interfering nodes \( N_k \), is given by

\[
P[G_{ij}] = \prod_{k=1}^{M \rightarrow \infty} \left[ (1 - p_t) + p_t \sum_{n=0}^{k-1} \left(\frac{(\zeta R_f)^n}{n!}\right) e^{-\zeta R_f}\right]^{2} \frac{1}{(1 - p_t) + p_t e^{-\zeta R_f}}. \quad (22)
\]
Optimal Throughput: Data throughput from node $N_i$ to its adjacent node $N_j$ is defined as successful reception subject to satisfying conditions in (9) and (14). This definition also include that node $N_i$ is transmitting while its adjacent node $N_j$ is not transmitting (i.e., receiving). Combining all these factors, the throughput from node $N_i$ to node $N_j$ is given by

$$T_h = P \{ E \land G_{ij} \land I_i = 1 \land I_j = 0 \}.$$  

Substituting (11), (18) and (22) into the above yields the single-hop throughput from node $N_i$ to its neighboring node $N_j$ as

$$T_h = p_t(1 - e^{-\zeta R_i})(1 - p_t),$$

$$\prod_{k=1}^{\infty} \left[ (1 - p_t) + (p_t \sum_{n=0}^{k-1} \frac{(\zeta R_i)^n}{n!} e^{-\zeta R_i}) \right]^2$$

It is important to note from the above equation that the only control variable is the transmission probability, $p_t$, and all other variables are constants for a given network and communication equipment. Naturally, it is useful to maximize the throughput with respect to $p_t$ by the first derivative of (24). In terms of protocol operations, the optimal $p_t$ can be determined from (24) as a function of the vehicle density, $\zeta$, which can be estimated by vehicles.

III. Numerical Results

Simulations are used to validate the proposed analytical model. In this work, it is assumed a one-lane and single-direction road with 5 Km in length, and data packets can travel in both directions, as shown in Figure 2. Furthermore, it is assumed that vehicles arrive and join the network only at the entrance (located at the far left hand side) of the road with a constant arrival rate $\lambda(t)$. All vehicles move forward on the road with a constant velocity $v$. The fluid model provides the vehicle density information in the network, which is then used as an input parameter in the analytical model. The transmission range is assumed to be $R_t = 100 m$, $\beta = 4$ and the path loss exponent $\alpha = 4$.

Figure 3 shows the probability $P(G_{ij})$ of meeting all of the interference conditions in (13) or (14) for a single-hop transmission. The simulation results, design to meet the SIR condition in (12) and obtained by carrying out 100,000 Monte Carlo simulation, are compared with the analytical model in (22). Four cases are considered with different vehicle arrival rates: 5, 10, 20 and 40 vehicle/min, respectively. It is important to note that the curves in Figure 3 show a close match between simulation and analytic results, validating the approximations used in the model proposed in this work. It can be observed that the $P(G_{ij})$ decreases when the probability of packet transmission $p_t$ increases. This is so because a higher transmission probability causes more interference. Moreover, the performance in terms of SIR in (13) or (14), tends to be better in situations with a smaller vehicular density, as seen in [11], as there are less vehicles able to interfere.

From (24) the optimal transmission probability $p_t$ to maximise the network throughput on a single-hop connection can be evaluated. The result is displayed in Figure 4 as a function of the increasing arrival rate of vehicles at the entrance of the network. The optimal transmission probability $p_t$ decreases as the vehicle arrival rate increases. This is due to a higher node density and thus more interfering vehicles. Consequently, the high chance of collisions can be decreased by reducing the transmission probability at every node.

Finally, Figure 5 shows the maximised throughput obtained from the previously chosen values of transmission probability. It can be observed that the throughput reaches a maximum at a certain value of transmission probability. Throughput does not depend only on the transmission probability, but is also influenced by the vehicular density and the average distance between vehicles that affect whether vehicles can properly communicate. Fortunately, all these parameters have been included in the throughput expression (24). The plot also shows that the analytical and simulated results are very close.
revealing the possibility of adapting the optimal transmission probability \( p_t \) from the proposed analytic model to enhance the network performance, according to the estimates of vehicle arrival rate or equivalently the vehicle density.

IV. CONCLUSION

As a first step toward the design of efficient MAC protocol tailored for VANETs, we have investigated whether it is possible to adapt the transmission probability for the MAC layer depending on vehicle density in order to reduce the interference and maximise the single-hop throughput between adjacent nodes (vehicles). To gain initial insights, a simple roadway scenario with one lane and one single travel direction has been considered. By considering SIR expressed as a set of pairwise conditions, connectivity among vehicles in the network can be determined and an expression for network throughput for one-hop communications has been obtained. This closed form expression is then exploited to determine the optimal transmission probability and hence maximise network throughput. The accuracy of the analytic approximation approach has also been verified with extensive simulations. This work can be extended in a couple of ways. First, the optimal transmission probability can be integrated into the CSMA protocol. Moreover, the results shown in this work can be incorporated in the design of a new MAC protocol that makes use of the optimal transmission probability for throughput enhancement. Either way, the performance improvements over the existing 802.11p protocol can be quantified to reveal the merits of our new protocol designs for VANETs. Future extensions can also include the analysis of more challenging vehicular scenarios.

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