Clique-Based Utility Maximization in Wireless Mesh Networks

Erwu Liu, *Member, IEEE*, Qinqing Zhang, *Senior Member, IEEE*, and Kin K. Leung, *Fellow, IEEE*

***Abstract*—This study considers utility-based resource allocation in backbone wireless mesh networks (WMNs). Unlike single-hop cellular networks, a WMN has multi-hop transmissions with multiple contending links, and thus requires more careful design for resource allocation. To this end, we provide a clique-based method with efficient spatial reuse, which is then incorporated into proportionally fair scheduling (PFS) for fair resource management in WMNs. We call it a clique-based proportionally fair scheduling (CBPFS) algorithm.**

The linear and/or logarithmic rate models used to analyze PFS in single-hop cellular networks cannot be used to analyze CBPFS in backbone WMNs. Using stochastic approximation and recent results on rate modeling for Rayleigh fading channels, we conduct mathematical analysis and obtain a closed-form model to quantify CBPFS performance, without the need of the highly time-consuming ordinary differential equation (ODE) analysis. We use the derived analytical framework to estimate the link throughput of CBPFS and compare it with simulations.

It is the first time a closed-form analytical model is developed to quantify the throughput of links in a multi-hop network where links are proportionally fair scheduled.

***Index Terms*—Clique-based scheduling, wireless mesh networks, spatial reuse, proportional fairness, Rayleigh fading.**

I. Introduction

THIS paper considers network resource management and optimization problem in wireless mesh networks (WMNs). A WMN is a communication network consisting of radio nodes organized in a mesh topology. Different from the single-hop cellular network, nodes in WMNs can communicate with each other directly or through one or more intermediate nodes. A WMN is a hierarchical wireless network with a static multi-hop backbone wireless network overlaid on a mobile wireless Ad-hoc network. In this paper, we limit the discussion to the backbone WMN where the topology is relatively static and nodes are with limited mobility [1]. We address the throughput and fairness issues in such networks [2].

Resource allocation mechanism relies on a suitable performance metric. Broadly, there are two types of performance metrics used in resource allocation: total-rate-based performance metric and utility-based performance metric. Total-rate-based performance metric is known to cause severe unfairness among the nodes in wireless networks [3]. Most recent work on resource allocation has shifted to a utility-based framework whose objective is to maximize the aggregate utility in the network. In this paper, we study the utility-based approach to resource allocation in backbone WMNs.

In a utility-based framework, each link l is associated a utility function $U(R_l)$, over the link throughput R_l. The function U is typically assumed concave and non-decreasing for all l. By defining different $U(\cdot)$, different fairness criteria of interest, such as proportional fairness (PF) or max-min fairness (MMF), can be achieved [4], [5]. Radunovic and Boudec [6] have proved that the MMF allocation has fundamental efficiency problem and results in all links receiving the rate of the worst link. For efficiency and fairness, we consider the PF allocation in backbone WMNs.

When designing schedulers for WMNs, one need to consider the impact of spatial reuse. While a single-hop cellular network has only one link activated at a time, a WMN could have multiple concurrent links at at time. This obviously complicates the analysis, especially when we implement in WMNs the proportionally fair allocation which is difficult to analyze and only limited analytical results are available in single-hop cellular networks.

To adapt PF in WMNs while remaining the benefit of spatial reuse in such networks, we introduce the concept of clique (refer to Section II for details): the backbone WMN is divided into multiple non-overlapping cliques; each clique represents a maximal number of concurrent, non-contending links; these cliques are scheduled in a proportionally fair manner. Note that one will not be able to extend existing results on PF to backbone WMNs when using cliques, as the linear and/or logarithmic rate models, commonly used in analyzing PF in single-hop cellular networks [7]–[10], are no longer valid in clique-based WMNs. Specifically, the linear rate model is valid only for very small signal-to-interference-plus-noise ratio (SINR) and could be fairly inaccurate in typical fading scenarios [11]. As SINR is typically large in a backbone WMN, especially when directional antennas are used, the linear rate model cannot be used in this case. On the other hand, use of more accurate logarithmic rate model complicates the PF analysis and would require various simplifications (the most common ones are assuming some kind of *i.i.d.* relationship among links and/or using modified PF metric) in the PF problem for analytical tractability. For example, [10] analyzes the proportionally fair scheduling (PFS) algorithm

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E. Liu was with the Department of Electrical and Electronic Engineering, Imperial College, London, UK. He is also with the School of Electronics and Information, Tongji University, Shanghai, China (e-mail: erwu.liu@ieee.org).

Q. Zhang is with the Applied Physics Laboratory and the Department of Computer Science, Johns Hopkins University, USA (e-mail: qinqing.zhang@jhuapl.edu).

K. K. Leung is with the Department of Electrical and Electronic Engineering, Imperial College, London, UK (e-mail: kkleung@ieee.org).

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based on logarithmic rate model, using a modified PF metric different from the one seen in current PFS implementation [12]. Even if we accept the modified PF metric, we still cannot apply the result of [10] to a WMN divided into cliques, because the rate of a clique (*i.e.,* the sum of rates of all links in the clique) is not characterized by logarithmic rate model. Regarding the problems of the linear and/or logarithmic rate models, one might want to use the ODE analysis [8] to study CBPFS. While the ODE analysis applies to any rate model, the main problem of the ODE analysis is that it is highly time-consuming: it requires solving N ODE equations if there are N links in the single-hop cellular network; when there are $N > 5$ links, the ODE analysis easily goes time-prohibited, especially because the ODEs involved are nonlinear and interplay with each other in an intricate manner. This calls for further study of PF in backbone WMNs.

Telatar [13], Smith and McKay *et al.* [14], [15] have suggested that the link capacity over Rayleigh fading channels can be modeled by a Gaussian distribution with surprisingly high accuracy. With this model, we [16], [17] conduct mathematical analysis and provide closed-form expressions for the PFS throughput without the limitations mentioned above. In previous study, we focused on PFS in single-hop cellular networks. In this work, we consider extensions to backbone WMNs.

Our ultimate objective is to develop a theoretical framework to facilitate the research on throughput-optimal and fair resource allocation for WMNs with multiple contending links and multi-hop transmissions. Specifically, we want to study PF in backbone WMNs with spatial reuse. Towards this end, we propose a systematic optimization method and then derive a mathematical model without the limitations above to quick estimate the link throughput in WMNs where links are scheduled under the PF criterion. In particular, our contributions are summarized as follows.

- We propose a clique-based proportionally fair scheduling (CBPFS) algorithm which maximizes the aggregate utility of link cliques. By using cliques, CBPFS achieves efficient spatial reuse as each clique represents a maximal number of concurrent links. CBPFS is a PFS extension to WMNs and it becomes PFS when used in single-hop cellular networks where each link clique contains one and only one link.
- We provide closed-form expressions for evaluating the performance of CBPFS, without turning to the time-consuming ODE analysis (though we do use some results of [8] in the proof). To put our work on a solid base, we use results of stochastic approximation [18], and abstract the underlying fading processes with stochastic estimates in the analysis.

The rest of the paper is structured as follows. In Section II, we introduce basic terminology and concepts of clique-based scheduling in WMNs for efficient spatial reuse; we then consider PF for WMNs, formulate the problem and present CBPFS algorithm for it, together with a closed-form model for the CBPFS throughput. In Section III, we first present simulation to validate our theoretical findings, we then evaluate the performance of CBPFS by comparing it to PFS with spatial reuse (*i.e.,* MIPFS in the paper), and PFS without spatial reuse, in terms of average throughput and allocated slots. We conclude the paper in Section IV. All related proofs are put in the Appendices for ease of exposition.

II. System Model, Algorithm, and Analytic Results

A. Notation and Conventions

Consider a WMN that is represented by a connected graph, $\mathbf{G} = (\mathbf{N}, \mathbf{L})$, which has a node set $\mathbf{N}$ (with cardinality $|\mathbf{N}|$), a link set $\mathbf{L}$ (with cardinality $|\mathbf{L}|$), and $|\mathbf{F}|$ source-destination node pairs $\{s_1, d_1\}, \cdots, \{s_{|\mathbf{F}|}, d_{|\mathbf{F}|}\}$. For any link $l \in \mathbf{L}$, let C_l be the capacity, $E[C_l]$ and σ_{C_l} be the average and standard deviation of C_l, and R_l and $E[R_l]$ be the throughput and average throughput of link l, respectively. We consider time-division-multiple-access (TDMA) networks where time is divided into small scheduling intervals called slots and the network resource is shared amongst links via disjoint time slots. The end of slot t is called time t. In next time slot $t+1$, the instantaneous data rate of link j will be $C_j[t+1]$. Its throughput up to time t is denoted by $R_j[t]$. Like most prior studies, we adopt an independent Rayleigh flat fading model in the analysis: each link experiences independent Rayleigh fading, and the channel coefficient keeps constant during a slot but varies from slot to slot and from link to link.

For our discussion, we assume that each node in a WMN is equipped with one single antenna and operates in half-duplex transmit/receive mode. We limit our discussion to the backbone WMN and for interference mitigation, we assume directional antenna using a similar antenna model introduced by Tang *et.al* [19]. In the antenna model, we divide the 360-degree whole angle centered at a transmitter (or receiver) into M equal sized angles. These M angles divide the 360-degree whole angle into M cones, numbered $1, 2, \ldots, M$ in clockwise order. Each node has a transmission range for communications. A cone and a transmission range define a sector. Figure 1 illustrates the case where there are $M = 12$ sectors and transmission (or reception) is using cone 1. If a wireless node wants to transmit, it will transmit using the cone pointing to the receiver, which will in turn receive using the cone pointing to the transmitter. Because of non-ideal radiation pattern, even outside the transmitting/receiving cone, a directional antenna will still transmit/receive signals, with a much attenuated gain due to the front-to-back ratio of antenna.

We now consider a general link contention model formulation specified by a set of pairs of links that contend with each other, *i.e.,* we say that two links contend if their simultaneous transmissions need to access the same radio resource(s) or introduce unacceptable interference. With the above assumption, there are four kinds of link contention in a slot: multiple links transmitting to the same node, multiple links transmitting from the same node, the transmitting and receiving links of the same node, and link contention where different links heavily interference with each other. The first three cases correspond to the single-antenna and half-duplex limitation, while the last one is determined by the interference model.

Unlike [19] where a transmitter only produces interferences at nodes (other than the receiver) within the transmitting sector, we formulate interference using the Physical Interference

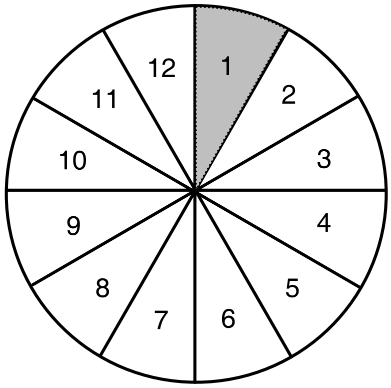

Fig. 1. Transmission (or reception) using directional antenna.

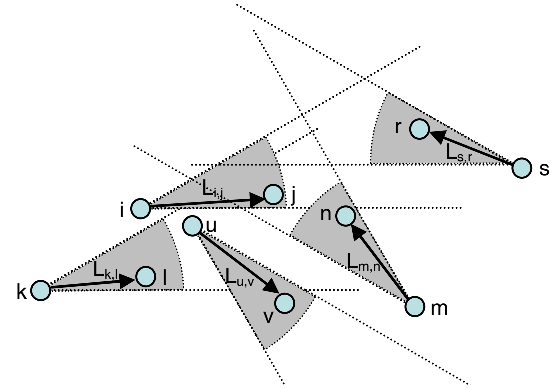

Fig. 2. Link interference.

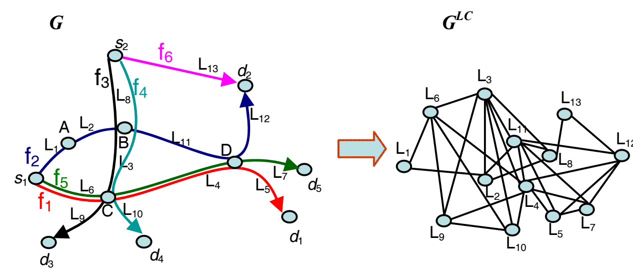

Fig. 3. Generation of link contention graph $\mathbf{G}^{LC}$.

Model introduced by Gupta [20] where a transmitter produces interferences at all nodes (other than the receiver) within the whole network. In Fig. 2 for example, nodes k, m, u, and n produce interferences at node j and we say that links $L_{k,l}$, $L_{m,n}$, and $L_{u,v}$ interfere with $L_{i,j}$. Let say transmitter i communicates with receiver j and link $L_{i,j}$ has a set of interferers denoted by $\mathbf{K}$. For a typical interferer $k \in \mathbf{K}$ located at distance $d(k,j)$ from receiver j, the interference power at j is given by

$$I(k,j) = \frac{P_t(k) \cdot G_t(k) \cdot G_r(j)}{PL(k,j)}. \quad (1)$$

where $P_t(k)$ is node k's transmit power, $G_t(k)$ is node k's transmit antenna gain, $G_r(j)$ is node j's receive antenna gain[1], and $PL(k,j)$ is the path loss from k to j determined by

$$PL(k,j) = PL_0 \cdot \left(\frac{d(k,j)}{d_0}\right)^{\alpha}. \quad (2)$$

where α is path loss exponent, and PL_0 is the reference path loss at reference distance d_0.

We should point out that, because of the front-to-back ratio of directional antenna, the interference at node j from node s will be very small as node j is outside node s's transmitting cone and s is outside j's receiving cone.

Let N_0 be the noise, the average SINR at link $L_{i,j}$ is then given by

$$\overline{SINR}_{i,j} = \frac{P_t(i) \cdot G_t(i) \cdot G_r(j) \,/PL(i,j)}{\left(N_0 + \sum_{\forall k \in \mathbf{K}} I(k,j)\right)}. \quad (3)$$

Given a minimum SINR threshold β for decoding, $L_{i,j}$ will contend with other links if $\overline{SINR}_{i,j} < \beta$. By this, one will know whether a contention occurs due to interference.

B. Clique for Spatial Reuse in WMNs

Follows we provide some concepts in graph theory before analyzing the PF allocation for WMNs. To maximize spatial reuse in WMNs, one can use the idea of clique. Given a graph, a *clique* (sometimes called a *maximal clique*) is defined as a *complete subgraph* that is not contained in any other complete subgraph. Among all maximal cliques, the largest one is the *maximum clique* $\mathbf{V}$ with cardinality $|\mathbf{V}|$ (called the *clique number* of the graph). Applying this to resource allocation in WMNs, if we construct a *link contention graph* $\mathbf{G}^{LC}$ where each pair of vertices of an edge corresponds to two contending links, then a maximum clique of the *complement graph* of $\mathbf{G}^{LC}$ represents a maximum number of concurrent links.

Refer to Fig. 3, for the network topology represented by a graph $\mathbf{G}$ shown in the left-side plot, we generate the *link contention graph* $\mathbf{G}^{LC}$ that captures the contention among links in such a way that each link is a vertex in this graph and two links that contend are adjacent. The right-side plot in Fig. 3 is the resulting link contention graph.

Using the link contention graph $\mathbf{G}^{LC}$, we construct the complement or inverse graph $\mathbf{G}^{I}$ of $\mathbf{G}^{LC}$ in such a way that two vertices in $\mathbf{G}^{I}$ are adjacent if and only if they are not adjacent in $\mathbf{G}^{LC}$. From the complement graph $\mathbf{G}^{I}$, we generate the *clique allocation graph* $\mathbf{G}^{CA}$ which is a maximum clique of $\mathbf{G}^{I}$. A maximum clique $\mathbf{V}$ is simply the set of vertices in the clique allocation graph, and represents a maximum number of concurrent links in the WMN.

C. Problem Formulation and the CBPFS Algorithm

We formulate the problem in this subsection, and then present the clique-based proportionally fair scheduling (CBPFS) algorithm for it.

Note that given a graph, there could be multiple maximum cliques each of the same size. By scheduling one from these maximum cliques at each slot, we can maximize the spatial reuse of WMNs. This method has two major problems: 1.) Fairness issue. Because different maximum cliques could be

[1]When interferer k is outside receiver j's receiving cone, it should be $G_r(j)\,/FTB(j)$ due to the front-to-back ratio FTB of receive antenna. Similarly, transmit antenna gain should be $G_t(k)\,/FTB(k)$ if receiver j is outside interferer k's transmitting cone. A good directional antenna typically has $FTB \geq 20$ dB.

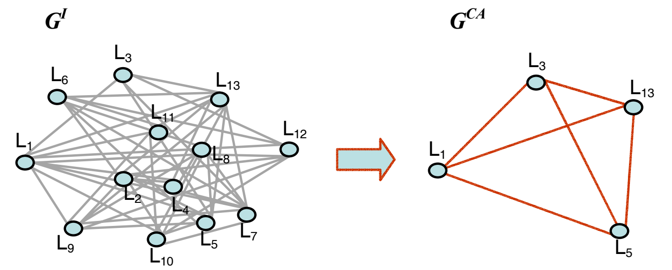

Fig. 4. Generation of clique allocation graph $\mathbf{G}^{CA}$ ($\mathbf{G}^{I}$: the complement graph of $\mathbf{G}^{LC}$).

overlapping, the overlapping links will be scheduled more frequently than those non-overlapping ones. The fairness will be even worse if these cliques do not cover all links in the network, resulting in some links (*i.e.,* those not covered by any of these cliques) not served at all; 2.) Complexity issue. The method requires enumerating all maximum cliques. Since enumerating all maximal cliques is an *NP-hard* problem, enumerating all maximum cliques is likewise an NP-hard problem, as finding a maximum clique is at least as hard as finding a maximal clique. To resolve the fairness problem and decrease the complexity, we provide the following greedy algorithm:

- Step 1). Initially, generate the 1^{st} link contention graph $\mathbf{G}^{LC}$ as in Fig. 3, then generate the 1^{st} clique allocation graph $\mathbf{G}^{CA}$ as in Fig. 4 and obtain the maximum clique $\mathbf{V}_1$;
- Step 2). generate the K^{th} link contention graph $\mathbf{G}^{LC}$ by removing $\mathbf{V}_{K-1}$ from the $(K-1)^{th}$ link content graph, then generate the K^{th} clique allocation graph $\mathbf{G}^{CA}$ and obtain the maximum clique $\mathbf{V}_K$;
- Step 3). repeat the above procedure for $K = 2, 3, \cdots$, until all links are in cliques.

Though finding the maximum clique is typically NP-hard, Tomita [21] has proved that, given a graph of n vertices and whose maximum degree is $\triangle$, if $\triangle \leq 2.493 \cdot d \cdot \lg n$ $(d \geq 1)$, then finding a maximum clique has $O(n^{2+d})$ complexity. In the above greedy algorithm, if there are $n = |\mathbf{L}|$ links in the network, K is at most n, *i.e.,* there are $O(n)$ link contention graphs. Obviously, this greedy algorithm has polynomial-time complexity of $O(n \cdot n^{2+d})$ if we limit the maximum degree of the clique allocation graph to be $\triangle$, and thus has a much less complexity than finding all maximum cliques. In the following, we simply use *clique* to denote the term *maximum clique* for ease of exposition.

With the above algorithm, we divide the WMN into K cliques $\{\mathbf{V}_i, i = 1, 2, \cdots, K\}$. By scheduling one of these K cliques at each slot, efficient spatial reuse is achieved as each clique $\mathbf{V}_i$ represents a maximum number of non-contending links in the i^{th} clique allocation graph. Once clique $\mathbf{V}_i$ is scheduled, all links in the clique can transmit simultaneously. Now we would like to add proportional fairness in the scheduling. In other words, we want to maximize the aggregate logarithmic utility of all K cliques. Formally, we have the following problem,

$$\max \sum_{i=1}^{K} \ln \left(\gamma_i[t]\right). \tag{4}$$

s.t.

$$\gamma_i[t] = \sum_{\forall l \in \mathbf{V}_i} R_l[t]. \tag{5}$$

where $\mathbf{V}_i$ is determined by the greedy algorithm; γ_i is the throughput of clique $\mathbf{V}_i$; $\forall l \in \mathbf{V}_i$, the link throughput R_l is updated by

$$R_l[t+1] = \left(1 - \frac{1}{k}\right) R_l[t] + I_i[t+1] \times \frac{C_l[t+1]}{k}. \tag{6}$$

where $C_l[t+1]$ is the estimated capacity of link l at next slot, constant $k \geq 1$ is the smoothing factor (typically $k > 50$ for an acceptable measure of throughput), and $I_i[t+1]$ is the indicator function of the event that clique $\mathbf{V}_i$ is scheduled in slot $t+1$,

$$I_i[t+1] = \begin{cases} 1 & \text{if } \mathbf{V}_i \text{ is scheduled at next slot} \\ 0 & \text{else} \end{cases}. \tag{7}$$

Define $\chi_i[t+1] \triangleq \sum_{\forall l \in \mathbf{V}_i} C_l[t+1]$ to be the estimated capacity of clique $\mathbf{V}_i$ at next slot, by (5) and (6) we have

$$\gamma_i[t+1] = \left(1 - \frac{1}{k}\right) \gamma_i[t] + I_i[t+1] \times \frac{\chi_i[t+1]}{k}. \tag{8}$$

The problem described by (4) and (8) is similar to the following optimization problem seen in an N-link single-hop cellular network,

$$\max \sum_{l=1}^{N} \ln \left(R_l[t]\right). \tag{9}$$

s.t.

$$R_l[t+1] = \left(1 - \frac{1}{k}\right) R_l[t] + H_l[t+1] \times \frac{C_l[t+1]}{k}. \tag{10}$$

where $H_l[t+1]$ is the indicator function of the event that link l is scheduled in slot $t+1$.

Kelly [4] has proved that the PFS algorithm provides the solution to the above optimization problem, *i.e.,* links are scheduled according to

$$i = \arg\max_{\forall l} \frac{C_l[t+1]}{R_l[t]} \tag{11}$$

Using this result in our case, we have the solution to the optimization problem described by (4) and (8),

$$\mathbf{V}_i = \arg\max_{\forall \mathbf{V}_j} \frac{\chi_j[t+1]}{\gamma_j[t+1]} = \arg\max_{\forall \mathbf{V}_j} \frac{\sum_{\forall l \in \mathbf{V}_j} C_l[t+1]}{\sum_{\forall l \in \mathbf{V}_j} R_l[t+1]}. \tag{12}$$

We call the algorithm described by (12) a *clique-based proportionally fair scheduling* (CBPFS) algorithm. By viewing each clique in a WMN as a link in a cellular network, the CBPFS algorithm becomes a PFS algorithm. Note that existing analytic results on PFS can not be used in the CBPFS case. In prior work on PFS, the linear or logarithmic rate model are typically assumed. The use of the linear rate model in single-hop cellular networks is mainly for analytical tractability and is valid only for very small SINR [11], and obviously not a good choice for scenarios of backbone WMNs where SINR is typically large, especially when directional antennas are used. Though analytic expression for PFS throughput has been obtained in [10] for more accurate logarithmic rate model, a PF metric different from the one used in the PFS algorithm for current 3G networks [12] is assumed in [10]. Moreover, in CBPFS we are dealing with cliques containing multiple links, thus the logarithmic rate model used to characterize link capacity is no longer valid for the capacity of a clique. All these call for new approach to analyze CBPFS.

D. Theoretical Results

Before presenting our analytic results on CBPFS, we first provide some lemmas on PFS. Some PFS results can be seen in our previous study [17], [22] but are systematically presented here with more rigorous proof.

In [4], Kelly provided the following formal definition of PF and we stick to this definition in the analysis.

Lemma 1 *A vector of throughputs $x = (x_s, s \in S)$ is proportionally fair if it is feasible and if for any other feasible vector x^*, the aggregate of proportional changes is zero or negative:*

$$\sum_{s\in S}(x_s^* - x_s)/x_s \le 0. \tag{13}$$

Smith and McKay *et al.* [14], [15] have revealed that link capacity C in a Rayleigh fading channel can be modeled by a normal distribution with extreme high accuracy, with average and variance determined by

$$E[C] = \int_0^\infty \log_2\left(1 + \overline{SINR} \times \lambda\right) \times e^{-\lambda} d\lambda. \tag{14}$$

$$\sigma_C^2 = \int_0^\infty \left(\log_2\left(1+\overline{SINR}\times\lambda\right)\right)^2 \times e^{-\lambda} d\lambda - (E[C])^2. \tag{15}$$

where $\overline{SINR}$ denotes the average SINR.

Using (14) and (15), we have the following lemma.

Lemma 2 *For link capacity C in a Rayleigh fading channel, σ_C* w.r.t. *$E[C]$ is monotonically increasing, concave.*

Proof: Refer to Appendix A. ■

With Lemmas 1 and 2, we have the following inequality.

Lemma 3 *In a Rayleigh fading network, given $E[C_i] \ge E[C_j]$ for two links i, j, we have $\sigma_{C_i}/\sigma_{C_j} \le E[R_i]/E[R_j] \le E[C_i]/E[C_j]$, under the PF criterion* (13).

where $E[R_i]$, $E[R_j]$ are the mean throughputs of links i and j, respectively.

Proof: Refer to Appendix B. ■

The following two lemmas are from the ODE analysis of PFS [8].

Lemma 4 *In a PFS-enabled, N-link single-hop cellular network, link throughput $R_i[t]$ $(1 \le i \le N)$ converges weakly to the set of limit points of the solution of the ODE*

$$\dot{\theta}_i = \bar{h}_i(\theta) - \theta_i, 1 \le i \le N. \tag{16}$$

where $\bar{h}_i(\theta)$ is Link i's average data rate conditional on the event $C_i/\theta_i > C_j/\theta_j, \forall j \ne i$

$$\bar{h}_i(\theta) = E[C_i | C_i/\theta_i > C_j/\theta_j, \forall j \ne i, 1 \le j \le N]. \tag{17}$$

Lemma 5 *The limit point $(\bar{\theta}_i)$ of* (16) *is unique, irrespective of the initial condition, and equals average throughput $E[R_i]$. So the process $R_i[t]$ converge to $E[R_i]$ as $t \to \infty$.*

The following inequality is not related to PFS and is the last lemma used in our analysis.

Lemma 6 *Let $Y_k(x)$ be a non-negative, monotonically non-decreasing function of x $(k = 1, 2, \ldots, N)$. If it satisfies that 1) $x \ge 0$, and 2) $c_i/c_j \le b_i/b_j \le a_i/a_j$ $(\forall a_i \ge a_j)$, with positive $a_i, a_j, b_i, b_j, c_i, c_j$ $(\forall i, j = 1, 2, \ldots, N)$, then*

$$\prod_{\forall i\ne j, i=1}^{N} Y_i\left(\frac{b_i}{b_j}x\right) \le \prod_{\forall i\ne j, a_i \ge a_j} Y_i\left(\frac{a_i}{a_j}x\right) \prod_{\forall i \ne j, a_i < a_j} Y_i\left(\frac{c_i}{c_j}x\right). \tag{18}$$

$$\prod_{\forall i\ne j, i=1}^{N} Y_i\left(\frac{b_i}{b_j}x\right) \ge \prod_{\forall i\ne j, a_i \ge a_j} Y_i\left(\frac{c_i}{c_j}x\right) \prod_{\forall i \ne j, a_i < a_j} Y_i\left(\frac{a_i}{a_j}x\right). \tag{19}$$

Proof: Refer to Appendix C. ■

Lemmas 1-6 are used to obtain Theorem 1 on PFS and then extended to get Theorem 2 on CBPFS. To summarize, we use the convergence property of PFS throughput and results from stochastic approximation, together with the Gaussian rate model for fading channels [14], [15], to derive analytic expressions for PFS and CBPFS throughputs in Rayleigh fading environments.

Theorem 1 *In an N-link single-hop cellular network with independent Rayleigh fading, the average PFS throughput of link l is*

$$E[R_l] = \frac{E[C_l]}{N} \times \left(1 - [\phi(-m_l)]^N\right) + \sigma_{C_l} \times \int_{-m_l}^{\infty} y\rho(y) \times \phi(y)^{N-1} dy. \tag{20}$$

where $m_l = E[C_l]/\sigma_{C_l}$, $\rho(\cdot)$ and $\phi(\cdot)$ are the pdf and cdf of zero mean, unit variance standard normal distribution, respectively.

Proof: Refer to Appendix D. ■

By modeling the capacity of a Rayleigh fading link with a normal distribution [14], [15], Theorem 1 provides an analytic expression for the PFS throughput over Rayleigh fading channels. Since the capacity of a clique is defined to be the overall capacity of all links in the clique, the clique capacity can be modeled by a normal distribution as well. Applying Theorem 1 to cliques and after some algebraic manipulation, we have

Theorem 2 *In a backbone WMN with independent Rayleigh fading, the average CBPFS throughput of link l is*

$$E[R_l] = \frac{E[C_l]}{K} \times \left(1 - [\phi(-M_i)]^K\right) + \frac{E[C_l] \times \sqrt{\sum_{\forall l \in \mathbf{V}_i} \sigma_{C_l}^2}}{\sum_{\forall l \in \mathbf{V}_i} E[C_l]} \times \int_{-M_i}^{\infty} y\rho(y) \times \phi(y)^{K-1} dy. \tag{21}$$

where $M_i = \sum_{\forall l \in \mathbf{V}_i} E[C_l] \Big/ \sqrt{\sum_{\forall l \in \mathbf{V}_i} \sigma^2_{C_l}}$, *K is the number of cliques and* $\mathbf{V}_i$ *is the* i*-th clique determined by the greedy algorithm presented in Subsection II-C.*

Proof: Refer to Appendix E. ■

Remark 1 *In an extreme case, all links content each other and each clique* $\mathbf{V}_i$ *will contain one and only one link. In this case the CBPFS algorithm described by* (12) *becomes the PFS algorithm described by* (11)*. In fact, the WMN in such case is a single-hop cellular network and* (21) *reduces to* (20)*. In other words, the CBPFS algorithm and corresponding results apply to both WMNs and single-hop cellular networks.*

Theorem 2 is our main result for the CBPFS performance in WMNs in Rayleigh fading scenarios. Built upon stochastic approximation theory and recent results on rate modeling for Rayleigh fading processes, Theorem 2 provides an analytic formula for the CBPFS throughput.

In next section, we evaluate the performance of CBPFS and justify our analytic model by simulations. Note that in CBPFS, we use clique-based method for spatial reuse. In simulation, we use two baseline models for comparison. The first baseline model is the traditional PFS algorithm without spatial reuse, while the second one implements PFS with the following method for spatial reuse.

- Step 1). Initially, let the remaining link set **N** contain all links; Set current concurrent-link set $\mathbf{C} = \varnothing$;
- Step 2). Move from **R** to **C** the link L with the next minimum interference. If this moving produces contention in **C**, return the link L from **C** to **R**. Repeat Step 2) until no more link could be added into **C**;
- Step 3). Record **C** and empty **C**, repeat Step 2) until **N** is empty.

When all nodes have the same transmit power, Step 2) is indeed the heuristic policy introduced by ElBatt[23] which suggests deferring the link with the minimum SINR as an attempt to lower the level of interference from simultaneous transmissions. Like CBPFS, this baseline model schedules cliques using the PF criterion, and is thus referred to as a *minimum interference proportionally fair scheduling* (MIPFS) algorithm.

III. Simulations

Refer to Fig. 5, 11 nodes are placed in an area of 600×800 m^2 and there are 13 links $L_1 \sim L_{13}$, 6 flows $f_1 \sim f_6$. Each node uses 30-degree directional antenna for half-duplex transmission/reception. For the sake of brevity, only transmit antennas are shown in Fig. 5.

Our experiment uses the same setup in the CDMA 1xEV-DO system [12]: 1.67 ms slot duration and stationary Rayleigh fading with constant and white external noise. All links experience independent fading. We use *Mathematica* from Wolfram [24] to build up the system-level simulation platform. In the simulation, the data rate of link l is characterized by $C_l = W \times \log_2[1 + \overline{SINR}_l \times |h_l|^2]$, where W is the bandwidth, and the channel gain h_l for link l is a normalized complex Gaussian random variable to model Rayleigh fading. The simulator uses the Physical Interference Model [20] detailed

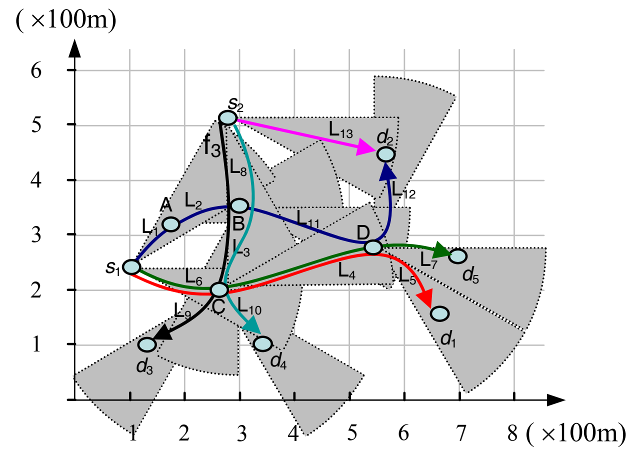

Fig. 5. Network topology.

TABLE I
Simulation Parameters

Frequency	5.8 GHz	Tx range	320 m
Bandwidth	10 MHz	Tx antenna gain	22 dB
Tx power	20 dBm	Rx antenna gain	22 dB
SINR threshold	1.0 dB	Tx ftb ratio	20 dB
Path loss exponent	3.0	Rx ftb ratio	20 dB
Reference distance	1.0 m	Noise floor	−174 dBm/Hz
Reference path loss	48 dB	Noise figure	15 dB

in Subsection II-A to calculate interference and average SINR, and it implements three schedulers, CBPFS, MIPFS, and PFS for comparison. We assume the same transmission power and transmission range for all nodes. System parameters are presented in Table I. The smoothing factor is set as $k = 500$ and the simulation runs for 4000 slots.

For the network depicted in Fig. 5, the instantaneous CBPFS throughput (curved lines) from simulation and the average CBPFS throughput (straight lines) from analysis are illustrated in Fig. 6. For ease of presentation, only links L_1, L_3, and L_4 are plotted for slots $1000 \sim 4000$. The average CBPFS throughputs from simulation and analysis are presented in Table II, which suggests that our analytical results match with the simulation ones with a relative error of less than 2%.

Table II additionally compares CBPFS, MIPFS and PFS in terms of average throughput. Not surprisingly, both CBPFS and MIPFS significantly outperform PFS due to spatial reuse. Moreover, we observed that CBPFS achieves higher throughput than MIPFS. This is because the minimum-inference policy in MIPFS does not always guarantee a maximum number of concurrent links, while CBPFS assures this by using the concept of clique from graph theory and thus provides higher throughput.

Figure 7 depicts the number of slots allocated to each link under different algorithms. With PFS each link in our experiment is allocated about $\frac{4000}{13} \approx 308$ slots. This is consistent with existing observation that PFS provides each link the same share of time slots in the long run [25]. As shown in Fig. 7, this equal-timeshare property holds for CBPFS and MIPFS as well, with allocated slots of about 800 and 600 for each link, respectively. We can see that CBPFS achieves higher spatial reuse than MIPFS. Indeed, for the given topology shown in Fig. 5, MIPFS produces 6 cliques while CBPFS produces

TABLE II
AVERAGE THROUGHPUT (MBPS): CBPFS VS. MSPFS, PFS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
CBPFS, anal	27.3	27.6	24.8	23.1	22.6	24.5	23.3
CBPFS, sim	27.2	27.5	24.8	22.7	22.9	24.8	23.5
MSPFS, sim	23.8	22.7	24.2	19.3	18.2	22.1	22.2
PFS, sim	13.1	12.3	11.7	9.5	11.4	11.6	11.6
	L_8	L_9	L_{10}	L_{11}	L_{12}	L_{13}	
CBPFS, anal	24.2	26.0	29.3	22.1	25.4	18.9	
CBPFS, sim	24.8	25.9	28.7	22.7	25.5	19.2	
MSPFS, sim	20.0	21.4	19.9	20.2	17.6	16.9	
PFS, sim	11.5	11.4	12.5	9.9	11.3	9.5	

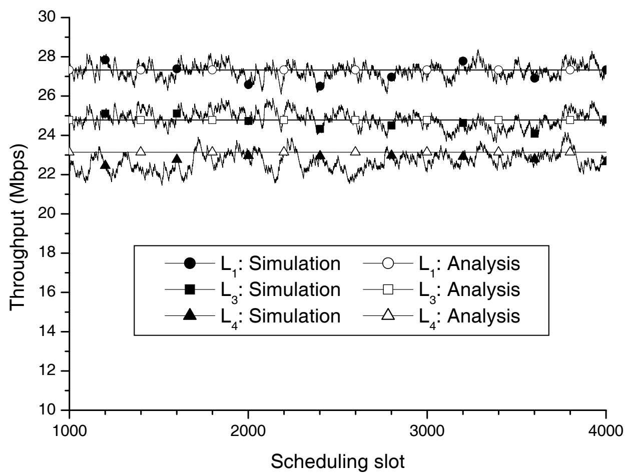

Fig. 6. CBPFS: Instantaneous throughput vs. average throughput.

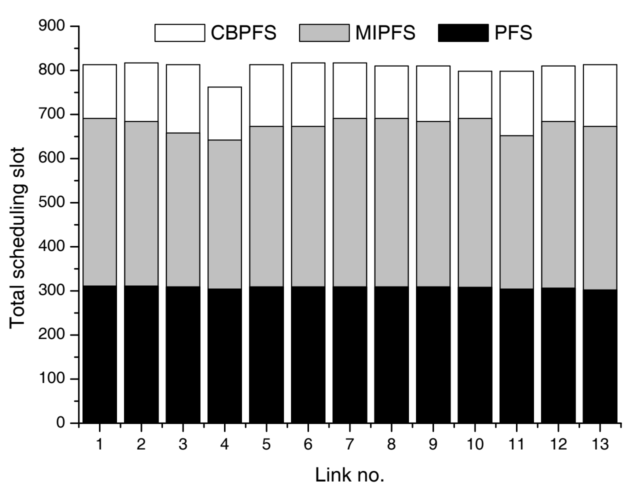

Fig. 7. Slots allocated to each link: CBPFS vs. MSPFS, PFS.

5 cliques, $\mathbf{V_1} = \{L_1, L_3, L_5, L_{13}\}$, $\mathbf{V_2} = \{L_2, L_6, L_7\}$, $\mathbf{V_3} = \{L_8, L_9, L_{12}\}$, $\mathbf{V_4} = \{L_{10}, L_{11}\}$, and $\mathbf{V_5} = \{L_4\}$, respectively. The decrease in the number of cliques translates into higher spatial reuse for CBPFS.

IV. CONCLUSIONS

This paper considers utility-based resource allocation for WMNs. Different from single-hop cellular networks, WMNs have multi-hop transmissions with multiple contending links, and thus ask for more careful design for resource allocation.

To this end, we extend the concept of proportional fairness to backbone WMNs and propose the CBPFS scheme, where links are grouped into link cliques which are proportionally fair scheduled to achieve maximized utility. CBPFS is a PFS extension and can be used in both single-hop cellular networks and backbone WMNs. In addition, simulations verify that CBPFS achieves higher spatial reuse than MIPFS, and thus performs better than MIPFS in terms of throughput and allocated slots for all links.

As the linear rate model or logarithmic rate model used in existing research on PF is valid only in single-hop cellular networks, but not in backbone WMNs that use cliques for spatial reuse, we cannot extend existing results on PF to analyze CBPFS. Instead, we use a novel approach different from existing ones and present a new tool to analyze CBPFS. Built upon stochastic approximation theory and advances in rate modeling for Rayleigh fading channels, we analyze CBPFS and provide closed-form expression for it, without turning to the highly time-consuming ODE analysis. It turns out that our analyses match quite well with simulations.

By incorporating both proportional fairness and spatial reuse, CBPFS provides a promising solution to efficient resource allocation for backbone WMNs. Furthermore, we present for CBPFS an analytic framework, which would provides guideline on system design, simulation-based modeling and performance evaluation of the CBPFS algorithm.

Though this work is promising, there are still lots of challenges we did not address in this paper. For example, throughout this paper, we focused on link scheduling in resource allocation and did not consider other aspects such as power control or routing. In addition, we only considered backbone WMNs where the topology is relative static, and we assume that channel feedback is error-free and the scheduler has perfect knowledge of the capacity of each link at each slot. In future work, we will address these issues in various network topology such as client WMNs where nodes could be mobile, and would like to explore CBPFS in the context of cross-layer design.

APPENDIX A
PROOF OF LEMMA 2

With (14) and (15), one can obtain the expressions for $\frac{d\sigma_C}{dE[C]}$ and $\frac{d}{dE[C]}\left(\frac{d\sigma_C}{dE[C]}\right)$ after tedious manipulation. The details of mathematical reasoning are not necessary for the development of the proof. From (14) and (15), we finally have $E'[C] = \frac{dE[C]}{dSINR} > 0$, $\sigma'_C = \frac{d\sigma_C}{dSINR} > 0$, and $\frac{d}{dSINR}\left(\frac{d\sigma_C}{dSINR}/\frac{dE[C]}{dSINR}\right) < 0$. By the chain rule for derivatives, we have $\frac{d\sigma_C}{dE[C]} = \frac{d\sigma_C}{dSINR}/\frac{dE[C]}{dSINR}$ and $\frac{d}{dE[C]}\left(\frac{d\sigma_C}{dE[C]}\right) = \frac{d}{dSINR}\left(\frac{d\sigma_C}{dSINR}/\frac{dE[C]}{dSINR}\right)/\frac{dE[C]}{dSINR}$. We then have that $\frac{d\sigma_C}{dE[C]} > 0$ and $\frac{d}{dE[C]}\left(\frac{d\sigma_C}{dE[C]}\right) < 0$. With the properties of the first and second derivative tests, we conclude that σ_C *w.r.t.* $E[C]$ is increasing, concave. This completes the proof.

APPENDIX B
PROOF OF LEMMA 3

For a well-designed scheduling algorithm in fading environments, one can readily verify that larger average data rate $E[C]$ (*i.e.,* better average channel quality) produces larger average throughput $E[R^*]$. On the other hand, Holtzman [11] has shown that links with more fading variability get more average throughput, *i.e.,* larger σ_C produces larger $E[R^*]$. In wireless networks, one can justify that both channel fluctuation (*i.e.,* σ_C) and average channel quality (*i.e.,* $E[C]$) contribute to average throughput $E[R^*]$. Without lost of generality, a very small increase in average throughput can be written as $\Delta E[R^*] = f^* \cdot E[C] \cdot \Delta\overline{SINR} + g^* \cdot \sigma_C \cdot \Delta\overline{SINR}$ where $f^* > 0$, $g^* > 0$ represent the weights of $E[C]$ and σ_C, respectively.

In an N-user cellular network, let say links i, j in PFS are provided average throughputs of $E[R_i]$ and $E[R_j]$. We assume $E[R_i] \geq E[R_j]$.

If a scheduling algorithm other than PFS is able to increase $E[R_i]$ to $E[R_i] + \Delta E[R_i^*]$ while keeping Link k's ($k = 1, 2, \cdots, N, k \neq i, j$) average throughput unchanged, by Lemma 1 we know that this scheduling algorithm will decrease $E[R_j]$ to $E[R_j] - \Delta E[R_j^*]$, where $\Delta E[R_j^*]$ must satisfy

$$\frac{\Delta E[R_j^*]}{E[R_j]} \geq \frac{\Delta E[R_i^*]}{E[R_i]}. \tag{22}$$

By Lemma 2, we have $\frac{\sigma_{C_i}}{E[C_i]} \leq \frac{\sigma_{C_j}}{E[C_j]}$. Since $\frac{\Delta E[R_i^*]}{\Delta E[R_j^*]} = \frac{f^* E[C_i] + g^* \sigma_{C_i}}{f^* E[C_j] + g^* \sigma_{C_j}}$, we then have

$$\frac{\Delta E[R_i^*]}{\Delta E[R_j^*]} \geq \frac{\sigma_{C_i}}{\sigma_{C_j}}. \tag{23}$$

Combining (22) and (23), we obtain

$$\frac{E[R_i]}{E[R_j]} \geq \frac{\sigma_{C_i}}{\sigma_{C_j}}. \tag{24}$$

Similarly, if this scheduling algorithm decreases $E[R_i]$ to $E[R_i] - \Delta E[R_i^*]$ while keeping Link k's ($k = 1, 2, \cdots, N, k \neq i, j$) average throughput unchanged, by Lemma 1 we know that this scheduling algorithm will increase $E[R_j]$ to $E[R_j] + \Delta E[R_j^*]$, where $\Delta E[R_j^*]$ must satisfy

$$\frac{\Delta E[R_j^*]}{E[R_j]} \leq \frac{\Delta E[R_i^*]}{E[R_i]}. \tag{25}$$

By Lemma 2, we have $\frac{\sigma_{C_i}}{E[C_i]} \leq \frac{\sigma_{C_j}}{E[C_j]}$. Similarly, we obtain

$$\frac{\Delta E[R_i^*]}{\Delta E[R_j^*]} \leq \frac{E[C_i]}{E[C_j]}. \tag{26}$$

Combining (25) and (26) yields

$$\frac{E[R_i]}{E[R_j]} \leq \frac{E[C_i]}{E[C_j]}. \tag{27}$$

.

Putting together (24) and (27) completes the proof.

Appendix C
Proof of Lemma 6

Proof: Let $B = \prod_{\forall i \neq j, i=1}^{N} Y_i (b_i x / b_j)$. Since $c_i / c_j \leq b_i / b_j \leq a_i / a_j$ ($\forall a_i > a_j$), for non-negative, monotonically increasing $Y_k(\cdot)$ ($\forall k = 1, 2, \ldots, N$), we have

$$\begin{aligned} B^2 &\leq \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{a_i}{a_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{b_i}{b_j}x\right) \\ &\quad \times \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{b_i}{b_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{c_i}{c_j}x\right) \\ &= B \times \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{a_i}{a_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{c_i}{c_j}x\right). \end{aligned} \tag{28}$$

$$\begin{aligned} B^2 &\geq \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{c_i}{c_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{b_i}{b_j}x\right) \\ &\quad \times \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{b_i}{b_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{a_i}{a_j}x\right) \\ &= B \times \prod_{\forall i \neq j, a_i \geq a_j} Y_i\left(\frac{c_i}{c_j}x\right) \prod_{\forall i \neq j, a_i < a_j} Y_i\left(\frac{a_i}{a_j}x\right). \end{aligned} \tag{29}$$

This completes the proof. ■

Appendix D
Proof of Theorem 1

For first-order wide-sense stationary R_l, applying *Bayes'* theorem in (10), we have

$$\begin{aligned} &E\left[R_l[t]\right] = E\left[C_l[t+1] | H_l[t+1] = 1\right] Pr\left(H_l[t+1] = 1\right) \\ &= Pr\left(H_l[t+1] = 1\right) \int_0^\infty x f_{C_l}\left(x | H_l[t+1] = 1\right) dx \\ &= \int_0^\infty x f_{C_l}(x) Pr\left(H_l[t+1] = 1 | C_l[t+1] = x\right) dx. \end{aligned} \tag{30}$$

where $Pr(H_l[t+1] = 1)$ is the probability that link l will be scheduled in slot $t+1$, $Pr(H_l[t+1] = 1 | C_l[t+1] = x)$ is the conditional probability with respect to the event $C_l[t+1] = x$, and $f_{C_l}(\cdot)$ is the probability density function (pdf) of C_l. According to (11), the conditional probability $Pr(H_l[t+1] = 1 | C_l[t+1] = x)$ is given by

$$\begin{aligned} &Pr\left(H_l[t+1] = 1 | C_l[t+1] = x\right) \\ &= Pr\left(\frac{x}{R_l[t]} > \frac{C_i[t+1]}{R_i[t]}, \forall i \neq l, 1 \leq i \leq N\right). \end{aligned} \tag{31}$$

By Lemmas 4 and 5, R_l weakly converges to a unique asymptotically stable value $E[R_l]$. So we have

$$\begin{aligned} &\lim_{t \to \infty} Pr\left(H_l[t+1] = 1 | C_l[t+1] = x\right) \\ &= \lim_{t \to \infty} Pr\left(\frac{x}{R_l[t]} > \frac{C_i[t+1]}{R_i[t]}, \forall i \neq l, 1 \leq i \leq N\right) \\ &= Pr\left(\frac{x}{E[R_l]} > \frac{C_i}{E[R_i]}, \forall i \neq l, 1 \leq i \leq N\right). \end{aligned} \tag{32}$$

Since C_i and C_l ($\forall i \neq l$) are independently distributed random variables, (32) can be rewritten as

$$\begin{aligned} &\lim_{t \to \infty} Pr\left(H_l[t+1] = 1 | C_l[t+1] = x\right) \\ &= \prod_{\forall i \neq l, i=1}^{N} F_{C_i}\left(E[R_i] \times x / E[R_l]\right). \end{aligned} \tag{33}$$

where $F_{C_i}(\cdot)$ is the cumulative distribution function (cdf) of C_i.

Combining (30) and (33), we have

$$E[R_l] = \lim_{t\to\infty} E[R_l[t]] = \int_0^\infty x f_{C_l}(x) \prod_{\forall i\neq l, i=1}^{N} F_{C_i}\left(E[R_i]\times x/E[R_l]\right) dx. \quad (34)$$

By Lemma 3, we have $\frac{\sigma_{C_i}}{\sigma_{C_l}} \leq \frac{E[R_i]}{E[R_l]} \leq \frac{E[C_i]}{E[C_l]}$ if $E[C_l] \leq E[C_i]$, and $\frac{\sigma_{C_i}}{\sigma_{C_l}} \geq \frac{E[R_i]}{E[R_l]} \geq \frac{E[C_i]}{E[C_l]}$ if $E[C_l] \geq E[C_i]$. Since $F_{C_i}(x)$ is non-negative, non-decreasing with respect to x, directly applying Lemma 6 we have

$$\prod_{\forall i\neq l, i=1}^{N} F_{C_i}\left(\frac{E[R_i]}{E[R_l]}x\right) \leq \prod_{\forall i\neq l, E[C_i]\geq E[C_l]} F_{C_i}\left(\frac{E[C_i]}{E[C_l]}x\right) \times \prod_{\forall i\neq l, E[C_i]<E[C_l]} F_{C_i}\left(\frac{\sigma_{C_i}}{\sigma_{C_l}}x\right). \quad (35)$$

$$\prod_{\forall i\neq l, i=1}^{N} F_{C_i}\left(\frac{E[R_i]}{E[R_l]}x\right) \geq \prod_{\forall i\neq l, E[C_i]<E[C_l]} F_{C_i}\left(\frac{E[C_i]}{E[C_l]}x\right) \times \prod_{\forall i\neq l, E[C_i]\geq E[C_l]} F_{C_i}\left(\frac{\sigma_{C_i}}{\sigma_{C_l}}x\right). \quad (36)$$

Define $m_l = E[C_l]/\sigma_{C_l}$. Substituting (35) and (36) in (34) and after straightforward algebraic manipulation, we have

$$E[R_l] \leq \sigma_{C_l}\int_{-m_l}^{\infty} \left(y\sigma_{C_l} + E[C_l]\right) f_{C_l}\left(y\sigma_{C_l} + E[C_l]\right) \times \prod_{\forall i\neq l, E[C_i]\geq E[C_l]}^{N} F_{C_i}\left(y\frac{E[C_i]}{E[C_l]}\sigma_{C_l} + E[C_i]\right) \times \prod_{\forall i\neq l, E[C_i]<E[C_l]}^{N} F_{C_i}\left(y\sigma_{C_i} + \frac{\sigma_{C_i}}{\sigma_{C_l}}E[C_l]\right) dy. \quad (37)$$

$$E[R_l] \geq \sigma_{C_l}\int_{-m_l}^{\infty} \left(y\sigma_{C_l} + E[C_l]\right) f_{C_l}\left(y\sigma_{C_l} + E[C_l]\right) \times \prod_{\forall i\neq l, E[C_i]<E[C_l]}^{N} F_{C_i}\left(y\frac{E[C_i]}{E[C_l]}\sigma_{C_l} + E[C_i]\right) \times \prod_{\forall i\neq l, E[C_i]\geq E[C_l]}^{N} F_{C_i}\left(y\sigma_{C_i} + \frac{\sigma_{C_i}}{\sigma_{C_l}}E[C_l]\right) dy. \quad (38)$$

With Lemma 3, we have $\frac{\sigma_{C_i}}{\sigma_{C_l}} \leq \frac{E[C_i]}{E[C_l]}$ if $E[C_l] \leq E[C_i]$, and $\frac{\sigma_{C_i}}{\sigma_{C_l}} \geq \frac{E[C_i]}{E[C_l]}$ if $E[C_l] \geq E[C_i]$. Since $F_{C_i}(x)$ *w.r.t.* x is monotonically non-decreasing, one can prove that the following expression lies between the bounds given by (37) and (38),

$$\sigma_{C_l}\int_{-m_l}^{\infty} \left(y\sigma_{C_l} + E[C_l]\right) f_{C_l}\left(y\sigma_{C_l} + E[C_l]\right) dy \times \prod_{\forall i\neq l, i=1}^{N} F_{C_i}\left(y\sigma_{C_i} + E[C_i]\right). \quad (39)$$

According to [14], [15], data rate R over Rayleigh fading channels can be modeled by a normal distribution with high accuracy. We then have

$$f_{C_i}(x) = \frac{1}{\sigma_{C_i}} \times \rho\left(\frac{x - E[C_i]}{\sigma_{C_i}}\right). \quad (40)$$

$$F_{C_i}(x) = \phi\left(\frac{x - E[C_i]}{\sigma_{C_i}}\right). \quad (41)$$

Substituting (40) and (41) into (39) yields

$$E[R_l] = \frac{E[C_l]}{N} \times \left(1 - [\phi(-m_l)]^N\right) + \sigma_{C_l} \times \int_{-m_l}^{\infty} y\rho(y) \times \phi(y)^{N-1} dy. \quad (42)$$

This completes the proof.

Appendix E
Proof of Theorem 2

According to the greedy algorithm presented in Subsection **II-C**, link set $\mathbf{L}$ is divided into K cliques and each link $l \in \mathbf{L}$ belongs to one and only one clique, which is mathematically expressed as

$$\begin{cases} \mathbf{V}_i \bigcap \mathbf{V}_i = \varnothing, \forall i \neq j \\ \bigcup_{i=1}^{K} \mathbf{V}_i = \mathbf{L} \end{cases}. \quad (43)$$

The proof of Theorem 1 for PFS relies on the assumption that the capacity of a link is independently modeled by a normally distributed random variable, with average and variance determined by (14) and (15). Because each clique $\mathbf{V}_i$ contains a number of independent links, with the property of normal distribution, the capacity χ_i of $\mathbf{V}_i$ is likewise a normally distributed random variable, with average and variance given by

$$\begin{cases} E[\chi_i] = \sum_{\forall l\in \mathbf{V}_i} E[C_l] \\ \sigma_{\chi_i}^2 = \sum_{\forall l\in \mathbf{V}_i} \sigma_{C_l}^2 \end{cases}. \quad (44)$$

According to (43) and (44), cliques $\mathbf{V}_i$ and $\mathbf{V}_j$ have independently, normally distributed capacity, $\forall i \neq j$. Applying Theorem 1 to these K cliques, we obtain the capacity of $\mathbf{V}_i$

$$E[\gamma_i] = \frac{E[\chi_i]}{K} \times \left(1 - [\phi(-M_i)]^K\right) + \sigma_{\chi_i} \times \int_{-M_i}^{\infty} y\rho(y) \times \phi(y)^{K-1} dy. \quad (45)$$

where $M_i = \sum_{\forall l\in \mathbf{V}_i} E[C_l] \Big/ \sqrt{\sum_{\forall l\in \mathbf{V}_i} \sigma_{C_l}^2}$.

According to CBPFS, once a clique is scheduled, all links in it will be scheduled simultaneously. From this we obviously have $E[R_l] = E[\gamma_i] \times \frac{E[C_l]}{E[\chi_i]}$. Substituting into (45) yields (21). This completes the proof.

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Erwu Liu received his M.S. and Ph.D degrees from the Department of Opto-Electronic Engineering, Huazhong University of Science & Technology, CHINA, in 1998 and 2001, respectively. He started his career at Alcatel-Lucent as a project manager in 2001, and then a senior research scientist at the Research & Innovation (R&I) Center there. In 2007, he left Alcatel-Lucent and joined Imperial College London as a researcher, working on wireless networks. Currently, his research interests include: stochastic geometry, cooperative and cognitive networks, scheduling and opportunistic resource allocation, network utility maximization, and cross-Layer optimization, *etc*. Dr. Liu is a member of the Alcatel-Lucent Technical Academy (ALTA), the ACM and the IEEE.

Qinqing Zhang received her Ph.D. degrees in electrical engineering from University of Pennsylvania, Philadelphia, PA, USA. She joined the Milton Eisenhower Research Center at the Johns Hopkins University Applied Physics Laboratory (JHU/APL) in May 2007. Prior to that, she was with Bell Labs, Alcatel-Lucent Technologies in New Jersey from 1998 to 2007.

Dr. Zhang is the recipient of numerous awards and scholarships, including the Bell Labs President's Gold Award in 2002. She is a senior member of IEEE. She serves on the editorial board of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. She was the co-chair of workshop on Cooperative Communications and Networking - Theory, Practice and Applications for ICC2008, and the track organizer and chair in Milcom2007, Milcom2008, Milcom2009, and Milcom2010. She has been serving in technical program committees of various IEEE conferences, including IEEE Infocom, Globecom, ICC, WCNC, VTC, MWC, ICCNC, etc.

Her current research interests are mobile ad-hoc networks, cooperative communications and networks, and underwater acoustic communication systems and networks.

Kin K. Leung received his B.S. degree from the Chinese University of Hong Kong in 1980, and his M.S. and Ph.D. degrees in computer science from University of California, Los Angeles, in 1982 and 1985, respectively. He joined AT&T Bell Labs in 1986 and worked at its successor companies, AT&T Labs and Bell Labs of Lucent Technologies, until 2004. Since then, he has been the Tanaka Chair Professor in Internet Technology at Imperial College in London. He serves as the Head of Communications and Signal Processing Group and as the Deputy Director for the University Defense Research Center in Signal Processing in the Electrical Engineering Department at Imperial College. His research interests include networking, protocols, optimization and modeling issues for wireless broadband, sensor and ad-hoc networks.

He received the Distinguished Member of Technical Staff Award from AT&T Bell Labs in 1994, and was a co-recipient of the 1997 Lanchester Prize Honorable Mention Award. He was elected as an IEEE Fellow in 2001. He receives the Royal Society Wolfson Research Merits Award from 2004 to 2009. He has actively served on many conference committees. He is a member of the IEEE Fellow Evaluation Committee for Communications Society (2009 to 2011). He was a guest editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, IEEE WIRELESS COMMUNICATIONS and the MONET journal, and as an editor for the JSAC: Wireless Series and IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. Currently, he is an editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, the *International Journal on Sensor Networks* and *ACM Computing Survey*.