Resource Allocation for Frequency-Selective Fading, Multi-Carrier Systems with Fairness Constraints

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***Abstract*—We consider the problem of fair resource allocation for multi-carrier systems. Opportunistic scheduling exploits the time-varying, location-dependent channel conditions to achieve multi-user diversity. Previous work in this area has focused on the single-user scheduling in single-carrier systems over a narrowband flat-fading channel, where only one node is scheduled at a time. In wideband multi-carrier systems, multiple nodes can be scheduled concurrently over multiple narrowband channels. In this paper, we analyze proportional fair scheduling (PFS) in multi-carrier systems over a wideband frequency-selective channel. In particular, we first derive analytical expressions for the throughput of opportunistic scheduling under proportional fairness constraints in a frequency-selective channel, for both single-user and multi-user systems. Furthermore, we provide closed-form expression to quantify the throughput benefit of the multi-user PFS over the single-user PFS in frequency-selective systems. This research is an extension of our previous theoretical work on opportunistic scheduling over flat-fading channel in narrowband single-carrier systems.**

I. Introduction

Scheduling policies significantly affect system performances such as throughput, delay, fairness and loss rate in wireless networks [1]. Different from wired ones, scheduling in wireless networks needs to consider the unique characteristics of time-varying and location-dependent channel status due to multi-path fading. From an information-theoretic viewpoint, Knopp and Humblet [2] showed that the system capacity can be maximized by exploiting inherent multi-user diversity in the wireless channel, i.e., the system throughput can be increased by giving a higher transmission probability to a node that is experiencing a better channel condition. However, as pointed out in [3], this greedy strategy could result in a highly unfair resource allocation, thus potentially violating the quality of service (QoS) requirements of some nodes. Hence, one has to carefully consider the trade-off between system efficiency and fairness when scheduling nodes in wireless networks. In the considerable efforts to deal with the trade-off between optimal system throughput and fairness, the proportional fair scheduling (PFS) algorithm is proposed. Since its presence [4], there has been substantial interest in the PFS algorithm in wireless networks (see [5], [6] and the references therein). By exploiting multi-user diversity and game-theoretic equilibrium, the PFS algorithm is widely conceived as an attractive solution in fading wireless environment and currently implemented in 3G wireless networks for high data rate delay-tolerant services [7]. Though the PFS algorithm has got so much attention, one can only see very limited analysis results on PFS [8]–[11] whose derivations typically assume a simple *linear rate model* or *logarithm rate model* and/or use a simplified form of the original PFS preference metric. The *linear rate model* is only valid for networks where the signal-to-noise ratio (*SNR*) is very small [8], while the *logarithm rate model* can only be used for single-input-single-output (SISO) communications [11]. The use of *linear rate model* or *logarithm rate model* is a reasonable modeling convention. However, when examining throughput performance, it does not seem satisfactory to assume such simplified rate models.

In order to provide a general system framework for the study of PFS without the above limitations, we in [12] presented theoretical results for PFS in general fading environments. The analytic results of [12] apply to both SISO and single-user multiple-input-multiple-output (MIMO) over a flat-fading channel. The merit of [12] is that it for the first time reveals a very interesting property of PFS: the average throughput of a user solely depends on its own channel statistics when its instantaneous capacity is Gaussian.

Though [12] developed a theoretical framework for the research on PFS, like most related work, it only considered the single-user PFS problem in single-carrier flat-fading scenarios. In single-user PFS, only a single node can access the channel at a given time, i.e., time division multiple access (TDMA). Various techniques are increasingly being deployed to schedule multiple nodes simultaneously on a number of separate subcarriers. Orthogonal frequency-division multiplexing (OFDM) is a popular multi-access scheme widely used in WiMAX, DVB, and ultra wideband (UWB) systems. It

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is also a promising modulation scheme of choice proposed for future cellular networks. In OFDM, the total bandwidth is divided into multiple narrowband orthogonal subcarriers, to combat frequency-selective fading and achieve higher spectral utilization. OFDMA, a multi-user version of OFDM, allows multiple nodes to transmit simultaneously on the parallel orthogonal sub-channels [13].

In this paper, we analyze PFS for multi-user OFDMA systems. We build on our previous work by going from the single-carrier to the multi-carrier case, and from the single-user to the multi-user scheduling. We derive the analytic expressions for the throughput of opportunistic scheduling under proportional fairness constraints, which are similar in form to those of [12], but adapted to the setting of multi-user scheduling, multi-carrier systems.

It is generally believed that, by applying opportunistic scheduling from one-dimensional time or frequency scale (*i.e.,* TDMA or FDMA) to two-dimensional time-frequency scale (*i.e.,* OFDMA), multi-user diversity improvement can be further achieved. Our analysis in this research quantifies this for PFS over multi-carrier systems.

The rest of the paper is structured as follows. In Section II, we first describe the assumptions made in our study of the problem and the resulting problem formulation, then we present a multi-user PFS algorithm. After that, analytical results which extend our prior work on PFS are presented in Section III. Finally, in Section IV, we present simulation results to validate the theoretical findings under Rayleigh frequency-selective fading OFDMA environments, followed by the conclusion in Section V.

II. Related Work and System Model

A. Related Work

Since its presence in Kelly's seminal paper [4], significant efforts has been put into the study of the PFS algorithm. Holtzman [8] conducted the asymptotic analysis of the PFS algorithm, with a result that all other things being equal, the user class with more fading variability gets more throughput. Kushner *et al.* [9] investigated the convergence property of the algorithm. They stated that the limiting behavior of the throughput converges to the solution of an ordinary differential equation, and found the limit throughput is proportional to the average capacity for Rayleigh fading by assuming the instantaneous capacity is proportional to the received *SNR* which is *i.i.d* for all nodes. Also, [10] presented some results on PFS for the scenario where the relative rate fluctuations are statistically identical, stating that each node would receive same amount of the time slots.

To simplify the problem, most existing analytic results are assuming some kind of *i.i.d* relationship among nodes [8]–[11]. Moreover, *linear rate model* and *logarithm rate model* are the two rate models commonly used for analyzing the performance of PFS [8]–[11]. The *linear rate model* is usually a reasonable approximation for small *SNR* and is not accurate when multiple modulations or codings are used [8]. On the other hand, the *logarithm rate model* can only be used for SISO links and is a very rough approximation when used for MIMO links [11].

To remove the above limitations, [12] developed a general theoretical framework to study PFS in single-channel wireless networks. From [12], we have the following important results:

Lemma 1 (PFS Throughput) *For an N-node cellular network implementing the PFS algorithm, each node's average throughput is solely determined by its own channel statistics when the instantaneous capacity is Gaussian. In particular, the average throughput of node i is given by*

$$E[\mu_i] \approx \frac{E[R_i]}{N} \times \Big(1 - \big[F_{(0,1)}\left(-M_i\right)\big]^N\Big) + \int_{-M_i}^{\infty} y\sigma_i f_{(0,1)}(y) \times \big[F_{(0,1)}(y)\big]^{N-1}\, dy. \quad (1)$$

where R_i and μ_i are the instantaneous capacity and throughput of user i, $E[R_i]$ and σ_i denote the statistical average and standard deviation of R_i, $M_i = E[R_i]/\mu_i$, $f_{(0,1)}(\cdot)$ and $F_{(0,1)}(\cdot)$ are the pdf and cdf of zero mean, unit variance standard normal distribution.

We can see that (1) is of great value as it has been shown that the instantaneous capacity over Rayleigh or Rician fading channels can be modeled by a Gaussian distribution with surprisingly high accuracy [14], [15].

To our knowledge, [12] is the first one that provides a analytical framework to study PFS in general fading scenario. However, like most of existing work on PFS, [12] is still for single-user scheduling, single-carrier systems where only a single node can access the channel at a given time, and the channel is assume to be flat-fading.

To improve system performance, various techniques have been proposed to schedule multiple nodes simultaneously on a number of narrowband flat-fading channels. The objective of this research is to extend our previous theoretical work on PFS from the single-user shceduling to the multi-user scheduling, from the single-carrier to the multi-carrier, and from the narrowband flat-fading channel to the wideband frequency-selective fading channel.

B. System Model

For any PFS-enabled networks, the selection of the nodes to schedule is based on a balance between the current possible rates and fairness.

For the single-user PFS in single-carrier systems, only one node can be proportionally scheduled over a single carrier at a time. Similarly, for the single-user PFS in multi-carrier systems, only one node is selected for transmission at a time, but the selected node will use all subcarriers. In both cases, PFS [4], [11], [12] performs this by comparing the ratio of the instantaneous capacity for each node to its average throughput tracked by an exponential moving average, which is defined as the preference metric (PF metric). The node with the maximum preference metric will be selected for transmission at the next scheduling slot. This is described mathematically as follows. The end of slot t is called time t. In next time slot t+1, the instantaneous capacity of node j will be $R_j[t{+}1]$. Its k-point moving average throughput up to time t is denoted by $r_{j;k}[t]$, and the preference metric by $M_{j;k}[t+1] = R_j[t+1]\,/r_{j;k}[t]$. By scheduling the node with the maximum PF metric at each time slot, the PFS algorithm maximizes the aggregate utility $\sum_j \ln[r_{j;k}]$ of all nodes [4].

For the multi-user PFS in multi-carrier systems, the PFS scheduling problem is to allocate all subcarriers to the best set of nodes at each time slot subject to proportional fairness and resource constraints.

OFDMA is a spectrum-efficient multi-access scheme widely used in multiple-channel wireless networks. By dividing the total bandwidth into multiple separate subcarriers, it allows multiple nodes to transmit simultaneously on the parallel subcarrier channels.

With the above assumption and using OFDMA as an example, the architecture of a PFS multi-carrier wireless system is depicted in Fig. 1.

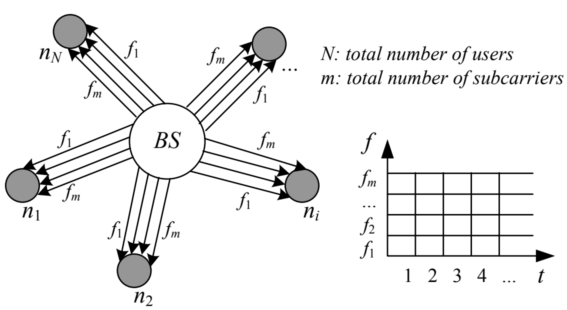


Fig. 1. PFS over a multi-carrier system

Consider the system shown in Fig. 1 where there are N nodes wishing to receive data from the base station (BS), and the rates of transmission are randomly varying due to fading. We assume that both the BS and nodes are equipped with single antenna. As in [16], the channel is assumed to be frequency-selective Rayleigh fading and is modeled as a lengh-L finite impulse-response (FIR) filter. The wide-band frequency-selective channel with W Hz bandwidth is divided into m narrowband flat-fading subcarrier channels with W/m Hz bandwidth which is less than the coherent bandwidth. Note that due to the finite-length impulse response, correlation exists between different subcarrier channels. Time is divided into small scheduling intervals called slots. We assume that channel fading keeps constant over each slot, and varies independently from slot to slot. At time slot t, the channel *SNR* of node i at subcarrier j is given by $SNR_{i,j}(t) = \overline{SNR_{i,j}} \times |h_{i,j}(t)|^2$, where $\overline{SNR_{i,j}}$ and $h_{i,j}(t)$ are the average signal-to-noise ratio and the complex channel gain of node i, respectively, both at subcarrier j. The subcarrier channels are assumed to be Rayleigh fading, *i.e.,* $h_{i,j}(t) \sim CN_{(0,1)}$. The channel gain is assumed to be known perfectly at the node, but is unknown at the BS. This means that the BS should equally split its transmit power P_T among the m subcarriers. In such case, node i will have the same *SNR* at all m subcarriers, *i.e.,* $\overline{SNR_{i,j}} = \overline{SNR_i}$, $\forall j = 1, 2, \ldots m$.

Unless otherwise specified, subcarrier bandwidth is normalized to 1 Hz. Denote $R_{i,j}[t{+}1]$ to be the instantaneous capacity of node i at subcarrier j in next time slot $t+1$. Its k-point moving average throughput up to time t is denoted by $r_{i,j;k}[t]$ (normalized, in bps/Hz). As mentioned earlier, the single-user PFS problems are basically the same for both single-carrier and multi-carrier systems, so we do not formulate the single-user PFS problem over a multi-carrier system to avoid reduplication. Instead, we formulate the multi-user PFS problem over a multi-carrier channel as follows.

$$\max \sum_{i=1}^{N} \sum_{j=1}^{m} \ln\left(r_{i,j;k}(t)\right). \tag{2}$$

s.t.

$$I_{i,j}[t+1] = \begin{cases} 1 & \text{node } i \text{ is scheduled at subcarrier } j \\ 0 & \text{else} \end{cases}. \tag{3}$$

$$\sum_{i=1}^{N} I_{i,j}[t+1] \le 1, \forall j = 1, 2, \ldots, m. \tag{4}$$

$$r_{i,j;k}[t{+}1] = \left(1 - \frac{1}{k}\right) r_{i,j;k}[t] + I_{i,j}[t{+}1] \times \frac{R_{i,j}[t+1]}{k}. \tag{5}$$

(3) and (4) indicate that at each time slot, each subcarrier can be allocated to at most one node and multiple (up to m) subcarriers can be allocated to a single node.

For the above optimization problem, we propose the following algorithm.

Multi-user PFS algorithm: At each slot t,

1) Initially, calculate PF metric for nodes at all subcarriers, $M_{i,j;k}[t+1] = R_{i,j}[t+1]\,/r_{i,j;k}[t])$, $\forall i = 1, 2, \ldots, N$, $\forall j = 1, 2, \ldots, m$, set $I_{i,j}[t+1] = 0$ to indicate that no subcarrier is allocated to any nodes yet.
2) $(i^*, j^*) = \arg\max_{(i,j)} M_{i,j;k}[t{+}1]$ is selected to indicate that node i^* is scheduled to receive at subcarrier j^* in the next time slot $t{+}1$. Update $I_{i^*,j^*}[t{+}1] = 1$;
3) $\forall i = 1, 2, \ldots, N$, set $M_{i,j*;k}[t{+}1] = 0$ so that subcarrier j^* will not be selected once again as it has already been allocated according to 2);
4) Goto 2) until all subcarriers are allocated.
5) Update the throughput of each node at each subcarrier according to (5).
6) Goto 1) to proceed to next slot.

It is easy to prove that the above algorithm is equivalent to performing single-carrier PFS for each subcarrier. Since $\sum_{i=1}^{N} \ln(r_{i,j;k}(t))$ is maximized for subcarrier j, the aggregate utility $\sum_{j=1}^{m} \sum_{i=1}^{N} \ln(r_{i,j;k}(t))$ at all subcarriers is then maximized. As $\sum_{j=1}^{m} \sum_{i=1}^{N} \ln(r_{i,j;k}(t)) = \sum_{i=1}^{N} \sum_{j=1}^{m} \ln(r_{i,j;k}(t))$, we can see that the above algorithm is obviously the optimal solution of (2). We would like to point out that though the above algorithm is proportional fair for the throughput $r_{i,j}$ of each node at each subcarrier, it does not guarantee proportional fairness for the aggregate throughput $r_i = \sum_{j=1}^{m} r_{i,j}$ of each node at all subcarriers, since

$$\sum_{i=1}^{N}\sum_{j=1}^{m} \ln\left(r_{i,j;k}(t)\right) \leq \sum_{i=1}^{N} \ln\left(r_{i;k}(t)\right) - N\ln(m). \quad (6)$$

III. Analysis of PFS over Frequency-Selective Channels

Like most existing studies, our previous work focuses on the single-user PFS problem for single-carrier systems over a flat fading channel. In this section, we analyze both the single-user and multi-user PFS problems for multi-carrier systems over a frequency-selective channel, and provide closed-form theoretical results for such cases.

A. Single-user PFS

Refer to Fig. 1, the wideband frequency-selective channel is divided into multiple narrowband flat-fading subcarrier channels. We assume that correlation exists between different subcarrier channels due to the finite-length impulse response.

The ergodic capacity (average mutual information) of a frequency-selective channel was found to be easily obtained by summing the equivalent flat-fading ergodic capacity of each individual OFDM subcarrier, *i.e.,*

$$E[R_i] = \sum_{j=1}^{m} E[R_{i,j}]. \quad (7)$$

where R_i is the instantaneous capacity of node i over the frequency-selective channel, $R_{i,j}$ is the instantaneous capacity of node i over the equivalent j-th flat-fading subcarrier channel, $E[\cdot]$ denotes the statistical average. According to [17], $E[R_{i,j}]$ in Rayleigh flat-fading channel can be written as

$$E[R_{i,j}] = \int_0^{\infty} e^{-\lambda} \times \log_2\left(1 + \overline{SNR_{i,j}} \times \lambda\right) d\lambda. \quad (8)$$

As stated in Section II, the BS equally split its transmit power among the m subcarriers, *i.e.,* all subcarriers of node i will have the same average *SNR*, denoted as $\overline{SNR_i}$. We have

$$E[R_i] = m\int_0^{\infty} e^{-\lambda} \times \log_2\left(1 + \overline{SNR_i} \times \lambda\right) d\lambda. \quad (9)$$

We use a lengh-L FIR filter to model the frequency-selective Rayleigh fading channel. Assuming uniform power delay profile for all L branches, we can write as follows the frequency correlation coefficients between two subcarriers of a distance of $d = 0, 1, \ldots, m-1$ subcarriers [18].

$$\rho_d = \frac{\sin\left(\pi d L/m\right)}{L \times \sin\left(\pi d/m\right)} \times e^{j\pi d/m} \quad (10)$$

Let $G_{p,q}^{m,n}$ be the Meijer-G function, $\mathbf{E}_h$ be the exponential integral function [19]. With these assumptions, [20] proved that the instantaneous capacity R_i over a frequency-selective channel can be approximated with extremely accuracy by a Gaussian distribution with a variance as follows,

$$\sigma_i^2 = (\log_2 e)^2 \times \left(\begin{array}{l} \frac{2}{m^2}\sum_{d=1}^{m-1}(m-d)\varphi(\rho_d) - e^{2/\overline{SNR_i}} \times \left(g_1(1)\right)^2 \\ + \dfrac{2e^{1/\overline{SNR_i}}}{\overline{SNR_i} \times m} \times G_{3,4}^{4,0}\left(\frac{1}{\overline{SNR_i}}\middle|\begin{smallmatrix}0,0,0\\0,-1,-1,-1\end{smallmatrix}\right)\end{array}\right). \quad (11)$$

where

$$\varphi(\rho_d) = \begin{cases} e^{2/\overline{SNR_i}} \times \left(g_1(1)\right)^2, & \text{for } |\rho_d| = 0 \\ \left(1-|\rho_d|^2\right) e^{2/\left(\overline{SNR_i}\times\left(1-|\rho_d|^2\right)\right)} \\ \quad \times \sum_{t=0}^{\infty} |\rho_d|^{2t} \left(g_2(1+t)\right)^2, & \text{for } 0<|\rho_d|<1 \\ \frac{2e^{1/\overline{SNR_i}}}{\overline{SNR_i}} \times G_{3,4}^{4,0}\left(\frac{1}{\overline{SNR_i}}\middle|\begin{smallmatrix}0,0,0\\0,-1,-1,-1\end{smallmatrix}\right), & \text{for } |\rho_d| = 0 \end{cases} \quad (12)$$

$$g_1(z) = \sum_{h=1}^{z} \mathbf{E}_h\left(1/\overline{SNR_i}\right). \quad (13)$$

$$g_2(z) = \sum_{h=1}^{z} \mathbf{E}_h\left(\frac{1}{\overline{SNR_i} \times (1-|\rho_d|^2)}\right). \quad (14)$$

Using (1) and (9), we have the closed-form expression for the throughput of node i under single-user PFS over a multi-carrier frequency-selective channel,

$$\begin{aligned} E[r_i]|_{\text{su}} \approx\ & \frac{m\int_0^{\infty} e^{-\lambda} \times \log_2\left(1 + \overline{SNR_i} \times \lambda\right) d\lambda}{N} \\ & \times \left(1 - \left[F_{(0,1)}(-M_i)\right]^N\right) \\ & + \sqrt{\left(\begin{array}{l} \frac{2}{m^2}\sum_{d=1}^{m-1}(m-d)\varphi(\rho_d) - e^{2/\overline{SNR_i}} \times \left(g_1(1)\right)^2 \\ + \dfrac{2e^{1/\overline{SNR_i}}}{\overline{SNR_i} \times m} \times G_{3,4}^{4,0}\left(\frac{1}{\overline{SNR_i}}\middle|\begin{smallmatrix}0,0,0\\0,-1,-1,-1\end{smallmatrix}\right)\end{array}\right)} \\ & \times \log_2 e \times \int_{-M_i}^{\infty} y f_{(0,1)}(y) \times \left[F_{(0,1)}(y)\right]^{N-1} dy. \end{aligned} \quad (15)$$

We now turn to the multi-user scheduling case.

B. Multi-user PFS

In multi-user PFS, each subcarrier can be independently scheduled. For the algorithm presented in section II, it is equivalent to performing single-user PFS for each subcarrier, i.e., we can think of the proposed multi-user scheduler as m PFS schedulers each of which independently allocates a separate subcarrier to one of N users at a time. Though correlation exists between different subcarrier channels, the instantaneous capacities of different nodes at the same subcarrier are basically independent. Using (1), we then have

$$\begin{aligned} E[r_{i,j}] \approx\ & \frac{E[R_{i,j}]}{N} \times \left(1 - \left[F_{(0,1)}(-M_j)\right]^N\right) \\ & + \sigma_{i,j}\int_{-M_{i,j}}^{\infty} y f_{(0,1)}(y) \times \left[F_{(0,1)}(y)\right]^{N-1} dy. \end{aligned} \quad (16)$$

where $M_{i,j} = E[R_{i,j}]/\sigma_{i,j}$, $\sigma_{i,j}$ is the standard deviation of instantaneous capacity $R_{i,j}$.

According to [15], we can write the variance of $R_{i,j}$ as,

$$\sigma_{i,j}^2 = \int_0^\infty e^{-\lambda} \times \left(\log_2\left(1+\overline{SNR_{i,j}}\times\lambda\right)\right)^2 d\lambda - \left(E[R_{i,j}]\right)^2. \tag{17}$$

As stated earlier, $\forall j, \overline{SNR_{i,j}} = \overline{SNR_i}$ under equal subcarrier power allocation. From (8) and (17), we have the same $M_{i,j}$ at all subcarrier j for a node i, denoted by M_i. We then have the throughput of node i under multi-user PFS over a multi-carrier frequency-selective channel,

$$\begin{aligned}
E[r_i]|_{\text{mu}} &= \sum_{j=1}^{m} E[r_{i,j}] \\
&\approx \frac{\sum_{j=1}^{m} E[R_{i,j}] \times \left(1-\left[F_{(0,1)}(-M_{i,j})\right]^N\right)}{N} \\
&\quad + \sum_{j=1}^{m} \sigma_{i,j} \int_{-M_{i,j}}^{\infty} y f_{(0,1)}(y) \times \left[F_{(0.1)}(y)\right]^{N-1} dy \\
&= \frac{m\int_0^\infty e^{-\lambda}\times\log_2\left(1+\overline{SNR_i}\times\lambda\right)d\lambda}{N} \\
&\quad \times \left(1-\left[F_{(0,1)}(-M_i)\right]^N\right) \\
&\quad + \sqrt{\left(\begin{array}{c}\int_0^\infty e^{-\lambda}\times\left(\log_2\left(1+\overline{SNR_i}\times\lambda\right)\right)^2 d\lambda \\ -\left(\int_0^\infty e^{-\lambda}\times\log_2\left(1+\overline{SNR_i}\times\lambda\right)d\lambda\right)^2\end{array}\right)} \\
&\quad \times m \times \int_{-M_i}^{\infty} y f_{(0,1)}(y) \times \left[F_{(0,1)}(y)\right]^{N-1} dy.
\end{aligned} \tag{18}$$

Using (15) and (18), we have the closed-form expression for the multi-user PFS gain over the single-user PFS,

$$\begin{aligned}
MUG_i &= \frac{E[r_i]|_{\text{mu}} - E[r_i]|_{\text{su}}}{E[r_i]|_{\text{su}}} \\
&= \frac{\int_{-M_i}^{\infty} y f_{(0,1)}(y) \times \left[F_{(0,1)}(y)\right]^{N-1} dy}{E[r_i]|_{\text{su}}} \\
&\times \left(\begin{array}{l} m \times \sqrt{\left(\begin{array}{c}\int_0^\infty e^{-\lambda}\times\left(\log_2\left(1+\overline{SNR_i}\times\lambda\right)\right)^2 d\lambda \\ -\left(\int_0^\infty e^{-\lambda}\times\log_2\left(1+\overline{SNR_i}\times\lambda\right)d\lambda\right)^2\end{array}\right)} \\ -\log_2 e \times \sqrt{\left(\begin{array}{c}\frac{2}{m^2}\sum_{d=1}^{m-1}(m-d)\varphi(\rho_d) - e^{2/\overline{SNR_i}}\times\left(g_1(1)\right)^2 \\ +\dfrac{2e^{1/\overline{SNR_i}}}{\overline{SNR_i}\times m}\times G_{3,4}^{4,0}\left(\frac{1}{\overline{SNR_i}}\middle|\begin{smallmatrix}0,0,0\\0,-1,-1,-1\end{smallmatrix}\right)\end{array}\right)}\end{array}\right).
\end{aligned} \tag{19}$$

We plot in Fig. 2 the multi-user PFS gain for various *SNRs*, m and N. As shown in Fig. 2, the multi-user PFS gain increased with the total number of subcarriers. Also, one can expect higher multi-user PFS gain at lower *SNR* and/or larger number of nodes. For implementation reference, Fig. 3 depicts the multi-user PFS gain for various N and FIR filter lengh L at $SNR = 10$ dB.

IV. Simulation

We conduct simulations to validate (15) and (18). Throughout, we assume single antenna at both the BS and the nodes. Each node i at each subcarrier j channel experiences Rayleigh fading, *i.e.,* the channel gain $h_{i,j} \sim CN_{(0,1)}$. We use a $L=8$ FIR filter to model the frequency-selective fading channel. We assume uniform power delay profile for all 8 branches. Simulation parameters are: 4000 scheduling slots, moving average factor $k = 500$, overall bandwidth $W = 16$ MHz, total number of subcarriers $m = 16$, there are 10 nodes with average $SNR = 1,2,3,4,5,6,7,8,9,10$ dB, respectively. For the single-user PFS over multi-carrier, we use the same procedure proposed for the single-user PFS over single-carrier, except that each node will use all subcarriers once it is scheduled. For the multi-user PFS over multi-carrier, we use the algorithm presented in section II.

The validities of our theoretical results are illustrated in Fig. 4. Fig. 4(a) depicts the instantaneous throughput from simulation and the average throughput from analysis for various nodes under the single-user PFS policy, while Fig. 4(b) is for the multi-user PFS case.

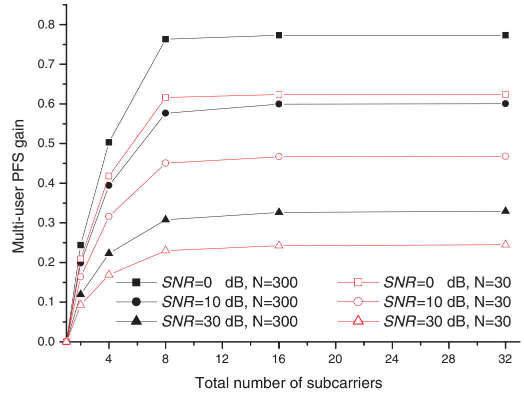


Fig. 2. Multi-user PFS gain over single-user PFS (FIR filter lengh=8)

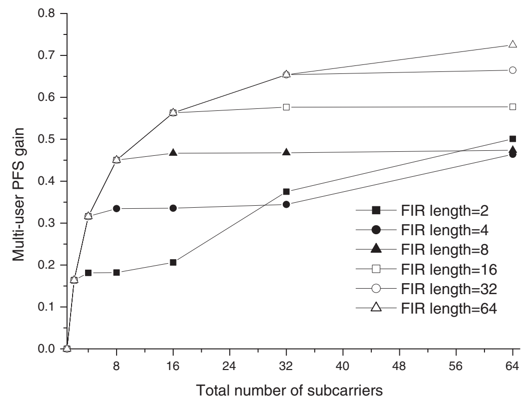
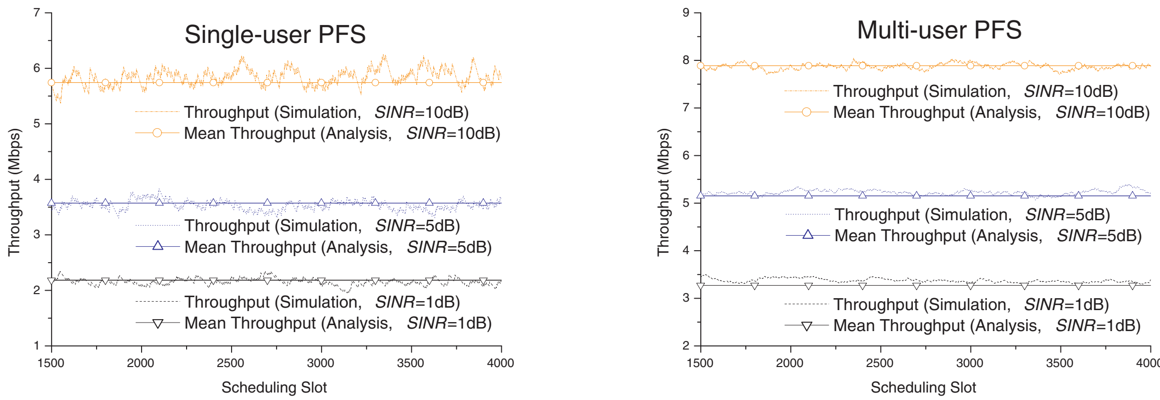


Fig. 3. Multi-user PFS gain for various FIR length ($SNR = 10$ dB)

V. Conclusion

We have extended previous work on PFS from the single-user scheduling to the multi-user scheduling, from the single-carrier to the multi-carrier, and from the narrowband flat-fading channel to the wideband frequency-selective fading



(a) Single-user PFS over multi-carrier

(b) Multi-user PFS over multi-carrier

Fig. 4. PFS throughput: Simulation vs Analysis ($L = 8$, $W = 16$ MHz, 16 subcarriers, 10 nodes)

channel. We also provide a closed-form expression to quantify the multi-user PFS gain over the single-user PFS for a multi-carrier, frequency-selective system. Together with the Gaussian approximation method [14], [20], our results have the great practical and theoretical interests and provide guideline and analytical support on system design, simulation-based modeling and performance analysis of the PFS algorithm.

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