Analogue Network Coding For Multi-Pair, Bidirectional Relay Channels

Chee Yen Leow, Zhiguo Ding, Kin K. Leung, Dennis L. Goeckel

***Abstract*—We consider a scenario where multiple pairs of users exchange information within pair, with the help of a dedicated multi-antenna relay. The proposed protocol integrates the idea of analogue network coding in mixing two data streams originating from the same user pair, together with the spatial multiplexing of the data streams originating from different user pairs. We propose several beamforming schemes and evaluate the performance using information theoretical metrics such as ergodic capacity and outage probability. Simulation results justify that the ergodic capacity and the outage probability of the proposed beamforming schemes outperforms comparable schemes.**

I. INTRODUCTION

Bidirectional relaying is a promising technique to enhance the throughput in wireless networks by reducing the channel resources used in the information exchange between users. Bidirectional relaying schemes such as strategies summarised in [1] based on decode-and-forward (DF) relaying, analogue network coding [2] based on amplify-and-forward (AF) relaying, physical network coding [3] based on estimate-and-forward (EF) relaying, etc are able to complete the two way information passing in only 2 channel uses, if compared to conventional time division protocol which requires 4 channel uses.

Attracted by the benefits of multi-antenna in enhancing the system capacity and reliability, bidirectional relaying has been generalised to the multi-antenna case. [4] generalises the DF based bidirectional relaying to multi-antenna setting using multiple access (MAC) and broadcast (BC) capacity regions. [5] proposes an AF based protocol which uses zero-forcing beamformer to eliminate the co-channel interference between users. However, no network coding principle is used to mix the data streams from two users. The special case where only the relay is equipped with multi-antenna is considered in [6]. The beamformers based on AF in [6] is designed to maximise the sum rate of single pair of users. On the other hand, the extension of the bidirectional relaying to multi-pair introduces the problem of multi-user interference. [7] proposes a scheme for CDMA system where each pair of users share a common spreading code as a means to reduce the multi-user interference. The proposed demodulate-and-forward based scheme in [7] uses multi-user receiver which requires high computational complexity at the relay. The suboptimal AF based scheme proposed in [7] suffers from poor BER performance when the number of users is low, because of noise domination. [8] proposes a scheme with DF relay for narrowband system which uses block-diagonalisation to mitigate the interference caused by multi-pair. However, this scheme requires higher complexity for decoding/encoding at the relay if compared to the AF based scheme. [9] and [10] consider the multi-pair scenario with an AF based multi-antenna relay and propose block-diagonalisation based schemes which are shown to deliver higher sum-rate than conventional multi-user zero-forcing scheme. Nevertheless, the potential benefit of block-diagonalisation in the scenario where the relay does not have enough degrees-of-freedom to spatially separate and decode each independent message is not covered in [8], [9], [10]. Furthermore, the spatial diversity gain offered by block-diagonalisation in the multi-pair scenario has not been studied.

In this paper, we consider a scenario where multiple pairs of users exchange information within pair with the help of a dedicated AF based, multi-antenna relay. The transmission protocol employed in this paper utilises the principal concept of network coding in mixing two data streams originating from the same user pair, coupled with the spatial multiplexing of the data streams originating from different user pairs. We propose several low complexity beamforming schemes based on the idea of block-diagonalisation, and evaluate their performance using information theoretical metrics, such as the ergodic capacity and outage probability. Two cases are considered. First, the case where the number of antennas at the relay is less than the total single-antenna users. Second, the case where the number of antennas at the relay is at least the total single-antenna users. In the first case, simulation results show that our proposed beamforming scheme is able to deliver significant ergodic capacity improvement and higher multiplexing gain if compared to existing schemes based on time sharing between pairs. In the second case, we propose two beamforming schemes and show that appropriate selection or coherent combining of null-space vectors is able to achieve all the available diversity gain offered by block-diagonalisation. The proposed beamforming with coherent combining of null-space vectors achieves highest ergodic capacity and lowest outage probability among all comparable schemes, while the proposed beamforming with null-space vector selection performs close to the former. Simulation results show that the proposed beamforming schemes deliver higher diversity gain if compared to existing zero-forcing scheme [5] and several comparable schemes based on block-diagonalisation [9], [10].

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

We consider a scenario where there are M pairs of single-antenna users wish to exchange information with their partners, with the help of a dedicated AF based relay equipped with N antennas.We assume the symmetric case, where all nodes subject to unit average power constraint and have same channel statistics. All channels undergo i.i.d. quasi-static Rayleigh fading and channel reciprocity is assumed. The receiver is corrupted by circularly symmetric additive white

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Gaussian noise with distribution $\mathcal{CN} \sim (0, \sigma^2)$. Half duplex constraint is assumed throughout the paper and it is realised using time division duplexing. Every user knows his and his partner's effective user-to-relay channel state information (CSI) while the relay has the CSI of all user-to-relay links.

We use (i, j) to represent the pair of user i and user j who exchange information with each other, such that the mth user pair is denoted as $(2m-1, 2m)$. The channel from user i to the relay is $\mathbf{h}_i \in \mathbb{C}^{N\times 1}$, the message transmitted from user i is $x_i \in \mathbb{C}$, the combined channels of user pair (i, j) is $\mathbf{H}_{i,j} = [\begin{array}{cc} \mathbf{h}_i & \mathbf{h}_j \end{array}] \in \mathbb{C}^{N\times 2}$ and the the message vector of user pair (i, j) is $\mathbf{x}_{i,j} = [\begin{array}{cc} x_i & x_j \end{array}]^T \in \mathbb{C}^{2\times 1}$ where $[.]^T$ denotes the transpose operation. The noise observed by the relay and user i is $\mathbf{n} \in \mathbb{C}^{N\times 1}$ and $n_i \in \mathbb{C}$ respectively. We define the multi-pair interference[1] channel seen by user pair (i, j) as $\tilde{\mathbf{H}}_{i,j} \in \mathbb{C}^{N\times 2(M-1)}$ by stacking all user channels other than $\mathbf{H}_{i,j}$. Similarly, we define the message vector conveyed through $\tilde{\mathbf{H}}_{i,j}$ as $\tilde{\mathbf{x}}_{i,j} \in \mathbb{C}^{2(M-1)\times 1}$ by stacking all messages other than $\mathbf{x}_{i,j}$.

The proposed protocol combines the application of analogue network coding within pair and spatial multiplexing between pairs, thus we name it as network coding with spatial multiplexing (NC-SM) protocol. The NC-SM protocol can be described in two time slots. In the first time slot, M pairs of users transmit simultaneously in the same channel with unit power. The relay observes a mixture of all messages from the users, which can be expressed as

$$\mathbf{r} = \sum_{m=1,}^{M} \mathbf{H}_{i,j}\mathbf{x}_{i,j} + \mathbf{n}, \tag{1}$$

where $i = 2m-1$ and $j = 2m$. In the second time slot, the AF based relay broadcasts the linearly processed observation, i.e. $\mathbf{Fr}$, where $\mathbf{F} \in \mathbb{C}^{N\times N}$ is the beamforming matrix at the relay. The signal received by user i can be expressed as

$$y_i = \mathbf{h}_i^T\mathbf{F}\mathbf{H}_{i,j}\mathbf{x}_{i,j} + \mathbf{h}_i^T\mathbf{F}\tilde{\mathbf{H}}_{i,j}\tilde{\mathbf{x}}_{i,j} + \mathbf{h}_i^T\mathbf{F}\mathbf{n} + n_i. \tag{2}$$

The first term on the RHS of the equation contains the mixture of messages from user pair (i, j), the second term contains the multi-pair interference while the last two terms contain the relay propagated noise and the receiver noise of user i. The unique feature of the NC-SM protocol is to allow both the relay and the users to participate in the interference cancellation. The relay eliminates the multi-pair interference, $\mathbf{h}_i^T\mathbf{F}\tilde{\mathbf{H}}_{i,j}\tilde{\mathbf{x}}_{i,j}$, while user i remove the self-interference, $\mathbf{h}_i^T\mathbf{F}\mathbf{h}_i x_i$. The design of beamforming matrix $\mathbf{F}$ is discussed in the following section.

III. JOINT RECEIVE AND TRANSMIT BEAMFORMING DESIGN

In this section, we present the low complexity beamforming design at the relay, for two cases. Case I, when the number of antennas at the relay is less than total number of users, i.e. $N = 2M-1$, and case II, when the number of antennas at the relay is at least the total number of users, i.e. $N \geq 2M$.

A. Case I: $N = 2M-1$

This case corresponds to the situation where conventional multi-antenna receiver or transmitter is not able to spatially support $2M$ independent data streams, due to the limitation of the available degrees-of-freedom [11], i.e. $\min(N, 2M)$. The scheme proposed in [8] does not work under this case due to insufficient antennas at the relay, while [9] and [10] have not explored this specific setting. It will be shown in the following paragraph that the proposed beamforming structure is able to support $2M$ independent data streams (from $2M$ users) simultaneously, given only $N = 2M-1$, by aligning the data streams of each user pair to occupy only one spatial dimension. Hence, a higher multiplexing gain can be achieved.

The proposed beamforming matrix $\mathbf{F}$ consists of the receive beamforming matrix $\mathbf{W}_R$ and transmit beamforming matrix $\mathbf{W}_T$, which are directly cascaded as following,

$$\mathbf{F} = \mathbf{W}_T\mathbf{A}\mathbf{W}_R, \tag{3}$$

where the receive beamforming matrix $\mathbf{W}_R \in \mathbb{C}^{M\times N}$ and the transmit beamforming matrix $\mathbf{W}_T \in \mathbb{C}^{N\times M}$ while the diagonal matrix $\mathbf{A} \in \mathbb{R}^{M\times M}$ is the power allocation matrix. Due to channel reciprocity, $\mathbf{W}_T = \mathbf{W}_R^T$. This allows us to concentrate on the design of the transmit beamforming matrix. For simplicity, we omit the subscript and let $\mathbf{W}_T = \mathbf{W}$. Represent $\mathbf{W} = [\begin{array}{ccc} \mathbf{w}_{1,2} & \ldots & \mathbf{w}_{2M-1,2M} \end{array}]$ where $\mathbf{w}_{i,j} \in \mathbb{C}^{N\times 1}$ is the transmit beamforming vector for user pair (i, j), $\mathbf{A} = diag\ (\begin{array}{ccc} \alpha_{1,2} & \ldots & \alpha_{2M-1,2M} \end{array})$, and $\mathbf{F}_{i,j} \in \mathbb{C}^{N\times N} = \mathbf{w}_{i,j}\mathbf{w}_{i,j}^T$ as the effective beamforming matrix for pair (i, j). We can rewrite (3) as $\mathbf{F} = \sum_{m=1}^{M} \alpha_{i,j}\mathbf{F}_{i,j}$ where $i = 2m-1,\ j = 2m$. The design objective of $\mathbf{w}_{i,j}$ is to ensure that each user pair is free from the multi-pair interference. In other words, the zero-forcing criterion $\tilde{\mathbf{H}}_{i,j}^T\mathbf{w}_{i,j} = \mathbf{0}$ has to be satisfied for all pair (i, j), where $\mathbf{0}$ is a column vector of all zeros. This criterion coincides with the block-diagonalisation[2] for the MIMO broadcast channels in [12]. To satisfy this criterion, we choose $\mathbf{w}_{i,j}$ to lie in the null-space of interference, i.e. $\mathbf{w}_{i,j} = null(\tilde{\mathbf{H}}_{i,j}^T)$, which exists as a non-zero vector when $N = 2M-1$. Note that $rank(\mathbf{H}_{i,j}^T\mathbf{w}_{i,j}) = 1$, i.e. each user pair only occupies one spatial dimension. This enables the relay to spatially multiplex $2M$ independent streams by using only $N = 2M-1$ antennas.

The transmission from the relay is subjected to unit average power constraint. The power constraint can be expressed as

$$\sum_{m=1}^{M} \alpha_{i,j}^2 \left(||\mathbf{F}_{i,j}\mathbf{H}_{i,j}||_F^2 + \sigma^2||\mathbf{F}_{i,j}||_F^2\right) \leq 1, \tag{4}$$

where $i = 2m-1$ and $j = 2m$. Note that $E[\mathbf{n}\mathbf{n}^H] = \mathbf{I}$. Since we are interested in the high SNR performance, i.e. diversity and multiplexing gains, equal power allocation across M data streams from M user pairs is sufficient. Although optimal power allocation among user pairs is able to further improve the sum-rate performance, it improves neither the diversity gain nor the multiplexing gain. Using equal power

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[1] The multi-pair interference is different from the multi-user interference defined in the literature. The multi-user interference is the interference observed by each user, not by each pair.

[2] Different from the MIMO broadcast channels, the users in each pair are not able to cooperate with each other, i.e. linear postprocessing within user pair is not possible. Hence, the block-diagonalisation proposed in [12] cannot be directly applied in the multi-pair scenario.

allocation, the equation above is satisfied in equality by choosing $\alpha_{i,j} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{||\mathbf{F}_{i,j}\mathbf{H}_{i,j}||_F^2+\sigma^2}}$. Note that $||\mathbf{F}_{i,j}||_F^2 = ||\mathbf{w}_{i,j}\mathbf{w}_{i,j}^T||_F^2 = 1$. The signal received by user i can be expressed as

$$y_i = \alpha_{i,j}\mathbf{h}_i^T\mathbf{F}_{i,j}\left(\mathbf{h}_i x_i + \mathbf{h}_j x_j + \mathbf{n}\right) + n_i, \tag{5}$$

while the signal received by user j is

$$y_j = \alpha_{i,j}\mathbf{h}_j^T\mathbf{F}_{i,j}\left(\mathbf{h}_i x_i + \mathbf{h}_j x_j + \mathbf{n}\right) + n_j. \tag{6}$$

Note that for all $p \neq i$ and $q \neq j$, we have $\mathbf{h}_i^T\mathbf{F}_{p,q}\mathbf{h}_i = 0$, $\mathbf{h}_i^T\mathbf{F}_{p,q}\mathbf{h}_j = 0$, $\mathbf{h}_j^T\mathbf{F}_{p,q}\mathbf{h}_i = 0$ and $\mathbf{h}_j^T\mathbf{F}_{p,q}\mathbf{h}_j = 0$. Since user i has the knowledge of x_i, and the knowledge of the effective channels, $\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_i$ and $\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_j$, he can decode the desired message, x_j, by subtracting the self-interference, $\alpha_{i,j}\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_i x_i$, from the received mixture. Similar strategy is used by user j to decode the desired message, x_i. Notice that the effective channels are scalars. The effective scalar channels carrying self-interference, $\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_i$ and $\mathbf{h}_j^T\mathbf{F}_{i,j}\mathbf{h}_j$, can be fed back from the relay to user i and j respectively using orthogonal feedback channels, while the effective scalar channel carrying desired message, $\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_j$, can be fed back from the relay to user pair (i,j) simultaneously using a common feedback channel, with low overhead. Note that $\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_j = \mathbf{h}_j^T\mathbf{F}_{i,j}\mathbf{h}_i$. User pair (i,j) do not need to know the exact channel vectors, $\mathbf{h}_i$ and $\mathbf{h}_j$.

Assuming Gaussian channel coding, the mutual information of user i can be described as

$$\mathcal{I}_i = \beta \log_2\left(1 + \frac{\alpha_{i,j}^2|\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_j|^2}{\sigma^2\left(\alpha_{i,j}^2||\mathbf{h}_i^T\mathbf{F}_{i,j}||^2+1\right)}\right). \tag{7}$$

where the pre-log, $\beta = \frac{1}{2}$, reflects the two time slots used. The mutual information of user j, $\mathcal{I}_j$ can be obtained by interchanging $\mathbf{h}_i$ and $\mathbf{h}_j$ in (7).

B. Case II: $N \geq 2M$

Similar to the previous case, the design objective of the beamforming matrix $\mathbf{F}$ is to ensure that the multi-pair interference is nullified. However, different from the previous case, the dimension of $null(\tilde{\mathbf{H}}_{i,j}^T)$ is greater than 1, indicating that the null-space consists multiple vectors. The transmit beamforming matrix for pair (i,j) is now $\mathbf{W}_{i,j} \in \mathbb{C}^{N\times(N-2(M-1))}$, where $\mathbf{W}_{i,j} = null(\tilde{\mathbf{H}}_{i,j}^T)$. Multiple null-space vectors are able to improve the diversity gain, by providing multiple statistically independent paths for the messages to travel through. In order to benefit from the additional diversity gain, the beamforming structure need to be carefully designed. A trivial choice of directly cascading the the receive and transmit beamforming matrices as in (3), destroys the diversity gain offered by multiple null-space vectors. This is due to the fact that the superposition of multiple diversity streams can either add up constructively or destructively at the destinations. It will be shown that appropriate selection or coherent combining of null-space vectors is important to achieve the available diversity gain offered by block-diagonalisation based beamforming in comparison with the zero-forcing scheme [5]. The block-diagonalisation with singular value decomposition (BD-SVD) [10] and pair-aware matched filter (PA-MF) [9] fail to achieve the available diversity gain because the diversity streams are not coherently combined. In this paper, two beamforming schemes which are able to achieve all diversity gain offered by block-diagonalisation, are proposed. One is based on null-space vector selection and the other is based on coherent combining of all null-space vectors.

1) Null-Space Vector Selection : In this subsection, we propose a beamformer with null-space vector selection. The relay performs selection to determine the null-space vector that can deliver the best performance in maximising the sum rate of each user pair (i,j). The overall beamforming structure $\mathbf{F}$ is similar to (3). Since we are interested in the high SNR performance, i.e. diversity and multiplexing gains, equal power allocation among user pairs is sufficient. Denote the kth null-space vector for pair (i,j) (obtained from the kth column of $\mathbf{W}_{i,j}$) as $\mathbf{w}_{i,j}(k)$. We have the following null-space vector selection criterion for user pair (i,j),

$$\arg \max_{k=1,\ldots,N-2(M-1)} \mathcal{I}_i(k) + \mathcal{I}_j(k), \tag{8}$$

where $\mathcal{I}_i(k)$ and $\mathcal{I}_j(k)$ are the mutual information of user i and user j respectively, when $\mathbf{w}_{i,j}(k)$ is used. $\mathcal{I}_i(k)$ can be obtained by replacing $\mathbf{F}_{i,j} = \mathbf{w}_{i,j}(k)\mathbf{w}_{i,j}^T(k)$ in (7) while $\mathcal{I}_j(k)$ can be derived similarly. The best null-space vector, denoted as $\mathbf{w}_{i,j}(k_{best})$ is able to maximise the sum-rate of user pair (i,j) and is used as the receive and transmit beamforming vectors for pair (i,j). The sum-rate of user pair (i,j) is used instead of the individual rate because the best null-space vector, $\mathbf{w}_{i,j}(k_{best})$ affects both user i and user j simultaneously. In other words, each beam carries the mixture of the messages of user pair (i,j). The received signal and the mutual information of user i can be expressed in similar way as (5) and (7) by substituting $\mathbf{F}_{i,j} = \mathbf{w}_{i,j}(k_{best})\mathbf{w}_{i,j}(k_{best})^T$.

2) Coherent Combining of Null-space Vectors: In contrast to the null-space vector selection scheme, the beamformer proposed in this subsection utilises all the available null-space vectors. In order to guarantee that the superposition of multiple diversity streams at the target destination is constructive, we propose the following beamforming structure,

$$\mathbf{F} = \sum_{m=1}^{M} \mathbf{W}_{i,j}\mathbf{B}_{i,j}\mathbf{A}_{i,j}\mathbf{P}_{\pi}\mathbf{B}_{i,j}^T\mathbf{W}_{i,j}^T, \tag{9}$$

where $i = 2m-1$ and $j = 2m$. The matrix $\mathbf{W}_{i,j} \in \mathbb{C}^{N\times(N-2(M-1))}$ is the transmit beamforming matrix for pair (i,j), the matrix $\mathbf{B}_{i,j} \in \mathbb{C}^{(N-2(M-1))\times 2}$ is the channel matching matrix for pair (i,j), the diagonal matrix $\mathbf{A}_{i,j} \in \mathbb{R}^{2\times 2}$ is the power allocation matrix for pair (i,j) while the matrix $\mathbf{P}_{\pi} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the permutation matrix. The channel matching matrix for pair (i,j) is designed as $\mathbf{B}_{i,j} = \mathbf{W}_{i,j}^H\mathbf{H}_{i,j}^*$ where $[.]^H$ and $[.]^*$ denote the Hermitian transpose and complex conjugate operations respectively. We can express the effective relay to user (i,j) channel as $\mathbf{H}_{i,j}^T\mathbf{W}_{i,j}\mathbf{B}_{i,j} = \begin{bmatrix} \Phi_i & \Psi \\ \Psi^* & \Phi_j \end{bmatrix}$, where $\Phi_i = \sum_{k=1}^{N-2(M-1)} |\mathbf{h}_i^T\mathbf{w}(k)|^2$, $\Phi_j = \sum_{k=1}^{N-2(M-1)} \left|\mathbf{h}_j^T\mathbf{w}(k)\right|^2$ and $\Psi = \sum_{k=1}^{N-2(M-1)} \mathbf{h}_i^T\mathbf{w}(k)\mathbf{w}^H(k)\mathbf{h}_j^*$. The channel matching matrix $\mathbf{B}_{i,j}$ ensures that main digonal elements Φ_i and Φ_j

contain the coherently combined (at zero phase) diversity streams of user i and user j respectively, while the off-diagonal element, Ψ, contains the non-coherent superposition of the correlated streams for user i and j. $\mathbf{P}_\pi$ plays an important role to ensure that diversity gain is preserved when the transmit beamforming matrix $\mathbf{W}_{i,j}\mathbf{B}_{i,j}\mathbf{A}_{i,j}$ and receive beamforming matrix $\mathbf{B}_{i,j}^T\mathbf{W}_{i,j}^T$ are cascaded. Under equal power allocation, we can express $\mathbf{A}_{i,j} = \alpha_{i,j}\mathbf{I}$ and further denote $\mathbf{F}_{i,j} \in \mathbb{C}^{N\times N} = \mathbf{W}_{i,j}\mathbf{B}_{i,j}\mathbf{P}_\pi\mathbf{B}_{i,j}^T\mathbf{W}_{i,j}^T$ such that $\mathbf{F} = \sum_{m=1}^{M} \alpha_{i,j}\mathbf{F}_{i,j}$ where $i = 2M - 1$ and $j = 2M$. The power constraint can be expressed similarly as in (4), which is satisfied in equality by choosing $\alpha_{i,j} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{||\mathbf{F}_{i,j}\mathbf{H}_{i,j}||_F^2 + \sigma^2||\mathbf{F}_{i,j}||_F^2}}$. The signal received by user i can be written in the same way as in (5). User i is able to decode the desired message, x_j, by subtracting the self-interference from the observation. Expand the effective channel carrying the desired message of user i,

$$\alpha_{i,j}\mathbf{h}_i^T\mathbf{F}_{i,j}\mathbf{h}_j \quad = \quad \alpha_{i,j}\Phi_i\Phi_j + \alpha_{i,j}|\Psi|^2. \qquad (10)$$

The first term on the RHS of (10) contains the multiplication of the coherently combined diversity streams of user i, i.e. Φ_i, and coherently combined diversity streams of user j, i.e. Φ_j, while the last term contains the magnitude square of the non-coherent combination of the correlated streams of user i and user j, i.e. Ψ. Recall that the diversity gain is obtained when statistically independent (uncorrelated) streams are used. Hence, only the first term in (10) contributes to the diversity gain. The correlated streams in the last term of (10) is allowed to combine non-coherently as it does not contribute to the diversity gain. Note that the last term does not affect the diversity gain contributed by the first term, as it only has magnitude (with zero phase). The mutual information of user i can be written in similar form as in (7). The received signal and the mutual information of user j can be derived easily.

IV. NUMERICAL RESULTS

In this section, we present Monte Carlo simulation results to assess the performance of the proposed protocol in comparison with existing AF based schemes in terms of single-user ergodic capacity and single-user outage probability. Since we consider the symmetrical channels, i.e. all the users have the same channel statistics, it is sufficient to study the single-user performance.

Fig. 1 compares the ergodic capacity versus SNR of the proposed protocol and three baseline schemes, under case I, i.e. $N = 2M - 1$. The fixed parameters: $M = 2$ and $N = 3$. The baseline schemes: 1.pure AF (where the relay only forwards power normalised mixture without beamforming), 2. maximal ratio reception-transmission (MRR-MRT) [6] and 3. zero-forcing [5], are extended to multi-pair using time sharing between pairs. Note that the zero-forcing [5] cannot support all user pairs simultaneously because the zero-forcing criterion requires $N \geq 2M$. It can be observed that from medium to high SNR, i.e. SNR>17dB, the ergodic capacity of the proposed NC-SM protocol outperforms all comparable schemes. At SNR=30dB, gain of 36%, 45% and 75% are obtained by the proposed NC-SM scheme in comparison with the MRR-MRT, zero-forcing, and the pure AF schemes respectively. This reveals that allowing both the relay and the user to participate in the interference cancellation, i.e. users eliminate self-interference while relay eliminates multi-pair interference, is spectrally more efficient. The slope of the ergodic capacity curve characterises the multiplexing gain. It is obvious that the proposed NC-SM protocol delivers the highest multiplexing gain among all schemes.

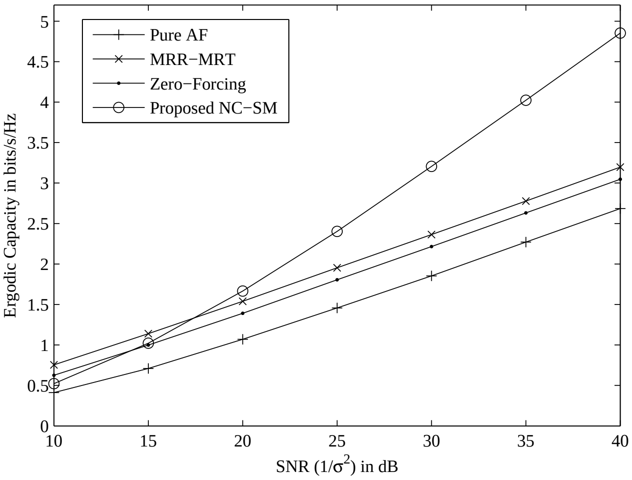

Figure 1. Single-user ergodic capacity versus SNR when $M = 2$ and $N_r = 3$

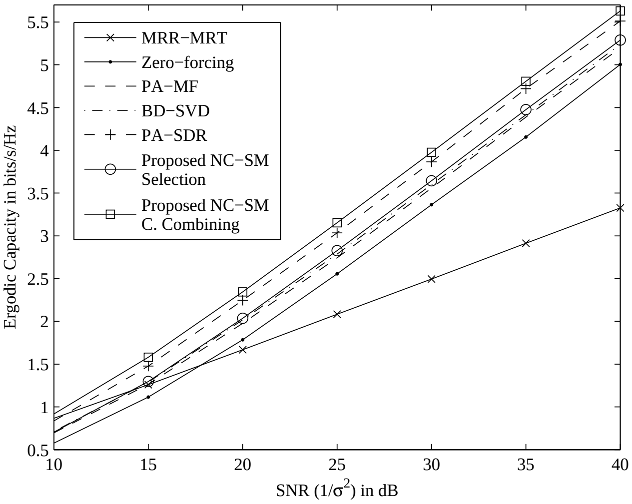

Figure 2. Single-user ergodic capacity versus SNR when $M = 2$ and $N_r = 4$

Fig. 2 shows the ergodic capacity versus SNR for various schemes under case II, i.e. $N \geq 2M$. The fixed parameters are $M = 2$ and $N = 4$. The baseline schemes are MRR-MRT [6], zero-forcing [5], BD-SVD [10], PA-MF and pair-aware with semi-definite relaxation (PA-SDR) [9]. All baseline schemes are able to support all user pairs simultaneously (spatial multiplexing), except the MRR-MRT which uses time-sharing between pairs. From fig. 2, it can be observed that all schemes supporting spatial multiplexing achieve higher ergodic capacity and higher multiplexing gain if compared to the scheme based on time sharing between pairs (MRR-MRT scheme). All spatial multiplexing schemes achieve the same multiplexing gain, evident from the slope of ergodic capacity curves. It can be observed that block-diagonalisation

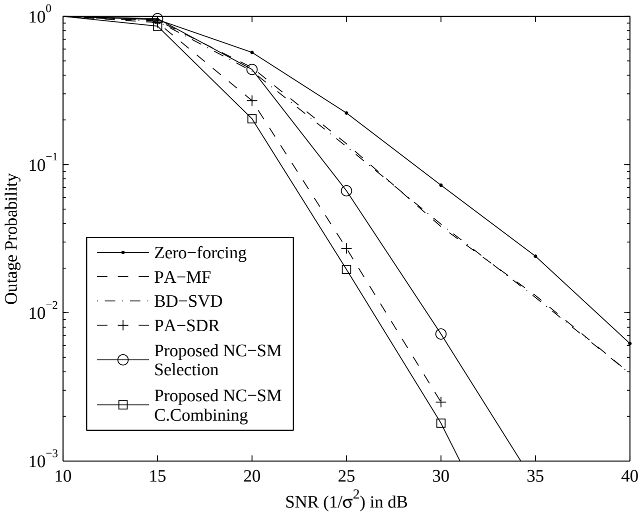

Figure 3. Single-user outage probability versus SNR when $M = 2$ and $N_r = 4$ at different target data rate R

based schemes (including proposed NC-SM schemes, BD-SVD, PA-MF and PA-SDR) are able to achieve higher ergodic capacity for any fixed SNR, if compared to zero-forcing scheme. Among all block-diagonalisation based schemes, the proposed NC-SM with coherent combining delivers the best performance. The PA-SDR scheme does not perform better than the proposed NC-SM with coherent combining, although it consumes higher computational complexity. The simpler scheme, NC-SM with null-space vector selection, is about 2 dB away from the coherent combining scheme. Notice that the PA-MF and BD-SVD schemes do not perform better than the proposed null-space vector selection scheme.

Besides providing ergodic capacity improvement, block-diagonalisation offers higher diversity gain if compared to zero-forcing scheme. The diversity gain achieved by the proposed NC-SM schemes in comparison with the existing block-diagonalisation based schemes and zero-forcing scheme can be verified from the outage probability versus SNR curves shown in fig. 3. The fixed parameters are $M = 2$, $N = 4$, and R=2 bits/s/Hz. Generally, the proposed NC-SM scheme with coherent combining of null-space vectors achieves the lowest outage probability. The PA-SDR scheme performs close to the proposed NC-SM with coherent combining while the simpler scheme, NC-SM with null-space vector selection is about 2.5dB away from the proposed NC-SM with coherent combining. Recall that the slope of the outage probability curve characterises the diversity gain. The steeper the outage probability curve, the higher the diversity gain is. The proposed NC-SM schemes and the PA-SDR are able to achieve a higher diversity gain if compared to the BD-SVD, PA-MF and zero-forcing schemes. The BD-SVD and PA-MF schemes are not able to extract the additional diversity gain offered by block-diagonalisation, due to the fact that the diversity streams add up either constructively or destructively at the target destinations. Although the PA-SDR scheme is able to extract all the diversity gain offered by block-diagonalisation, it requires higher computational complexity, occured in solving the max-min optimisation problem [9]. The proposed NC-SM schemes have lower complexity while being able to achieve all the diversity gain offered by block-diagonalisation.

V. CONCLUSIONS

The transmission protocol employed in this paper combines analogue network coding and spatial multiplexing, which allows both the relay and the users to participate in interference cancellation. The proposed low complexity beamforming schemes yield significant improvement in terms of ergodic capacity and outage probability. Simulation shows that the proposed beamforming scheme achieves higher multiplexing gain than existing schemes, when the number of antennas at the relay is less than the total users. When the number of antennas at the relay is at least the total users, simulation results show that the proposed beamforming schemes not only deliver capacity improvement if compared to zero-forcing scheme, but also able to extract all additional diversity gain offered by block-diagonalisation, whereas several existing block-diagonalisation based schemes fail to do so.

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