E2.5 Signals & Linear Systems

Tutorial Sheet 8 – DFT and z-transform

(Lectures 14 - 15)

1.* For a signal *f(t)* that is time-limited to 10 ms and has an essential bandwidth of 10 kHz, determine N_0, the number of signal samples necessary to compute a power of 2 DFT with a frequency resolution f_0 of at least 50 Hz. Explain if any zero padding is necessary.

2.* Choose appropriate values for N_0 and T and compute the DFT of the signal $e^{-t}\,u(t)$. (Note that the choice of N_0 and T is not unique; it will depend on your assumptions. What is important here is the reasoning that you use to arrive at your answer.)

3. Using the definition of z-transform, show that

(a)* $\gamma^{k-1}u[k-1] \Leftrightarrow \dfrac{1}{z-\gamma}$

(b)** $u[k-m] \Leftrightarrow \dfrac{z}{z^m(z-1)}$

(c)** $\dfrac{\gamma^k}{k!}u[k] \Leftrightarrow e^{\frac{\gamma}{z}}$

4. Using z-transform table given in the lecture notes, show that

(a)* $2^{k+1}u[k-1] + e^{k-1}u[k] \Leftrightarrow \dfrac{4}{z-2} + \dfrac{z}{e(z-e)}$

(b)** $k\gamma^k u[k-1] \Leftrightarrow \dfrac{\gamma z}{(z-\gamma)^2}$ (Hint: $u[k-1] = u[k] - \delta[k]$, $f[k]\delta[k] = f[0]\delta[k]$.)

(c)** $[2^{-k}\cos\frac{\pi}{3}k]u[k-1] \Leftrightarrow \dfrac{0.25(z-1)}{z^2-0.5z+0.25}$

5. Find the causal inverse z-transform of

(a)* $X[z]=\dfrac{z(z-4)}{z^2-5z+6}$

(b)** $X[z]=\dfrac{z(e^{-2}-2)}{(e^{-2}-z)(z-2)}$

(c)** $X[z]=\dfrac{z(-5z+22)}{(z+1)(z-2)^2}$

6. MATLAB exercise. Using the m-file sinegen of the previous class problem. Compute and plot the amplitude of the DFT of a sine wave of freq=11Hz sampled with sampf=31Hz with a window size of T=0.25 sec. Use zero padding to increase of 4 the number of frequencies you can see. Finally compute the DFT of the same signal but using a window of size T=2sec. Use the command stem to plor the 3 different DFT, do you understand the differences? Also use the help on line to learn about ‘fft’ and ‘fftshift’.