

E 2.5 Signals & Linear Systems (1)

Tutorial sheet 8 Solutions

- 1) Given that bandwidth of $f(t)$ is 10kHz,
sampling frequency $F_s \geq 2 \times 10\text{kHz}$
 $\geq 20,000$.

If we have frequency resolution $\Delta f = 50\text{Hz}$,
the time window T_w required to ~~provide~~ provide
the DFT is $T_w = \frac{1}{\Delta f} = 20\text{ms}$.

$$\therefore N_0 \geq \frac{1/F_s}{T_w} \geq 400$$

Since N_0 must be a power of 2,
choose $N_0 = 512$. //

Now if we have 512 samples ~~at~~ at $T_s = 50\mu\text{s}$
we need a signal of duration
 $512 \times 50\mu\text{s} = 25.6\text{ms}$.

Since we only have a signal duration
of 10ms, we need to zero padding
over 15.6ms //

$$2) f(t) = e^{-t} u(t) \iff F(j\omega) = \frac{1}{j\omega + 1} \quad (2)$$

$$|F(\omega)| = \frac{1}{\sqrt{\omega^2 + 1}} \approx \frac{1}{\omega} \quad \text{for } \omega \gg 1.$$

From results of sheet 7 Q4, we know that the essential bandwidth of

$f(t)$ is 10.13 aHz ($a=1$), ~~or 20.13 Hz.~~

or ~ 10 Hz.

\therefore Choose $F_s = 20$ Hz + Sampling interval
 $T_s = 0.05$ sec.

We assume that $e^{-t} u(t)$ becomes negligible after, say, 6 time constants ($e^{-6} \approx 0$).

Choose time window $T_0 = 6$ sec.

$$N_0 = \frac{6}{0.05} = 120. \quad \text{Take } N_0 = 128.$$

$$T = 0.05 \quad T_0 = 128 \times 0.05 = 6.4.$$

$$\therefore F_0 \text{ (frequency resolution)} = \frac{1}{6.4} = 0.15625 \text{ Hz}$$

//

$$3) (a) F[z] = \sum_{k=1}^{\infty} \gamma^{k-1} \cancel{\gamma^k} z^{-k}$$

$$= \frac{1}{\gamma} \sum_{k=1}^{\infty} \left(\frac{\gamma}{z}\right)^k$$

$$= \frac{1}{\gamma} \left[\frac{\gamma}{z} + \left(\frac{\gamma}{z}\right)^2 + \left(\frac{\gamma}{z}\right)^3 + \dots \right]$$

$$= \frac{1}{\gamma} \left[-1 + \left(1 + \frac{\gamma}{z} + \left(\frac{\gamma}{z}\right)^2 + \dots\right) \right]$$

$$= \frac{1}{\gamma} \left[-1 + \frac{1}{1 - \frac{\gamma}{z}} \right] = \frac{1}{z - \gamma} //$$

$$(b) F[z] = \sum_{k=m}^{\infty} z^{-k} = z^{-m} + z^{-(m+1)} + z^{-(m+2)} + \dots$$

$$= z^{-m} \left[1 + z^{-1} + z^{-2} + \dots \right]$$

$$= z^{-m} \left(\frac{1}{1 - \frac{1}{z}} \right) = \frac{z}{z^m (z-1)} //$$

$$(c) F[z] = \sum_{k=0}^{\infty} \frac{\gamma^k}{k!} z^{-k} = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\gamma}{z}\right)^k //$$

$$\text{Since } e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

$$\therefore F[z] = e^{\frac{\gamma}{z}} //$$

3

④

(a) $f[k] = 2^{k+1} u[k-1] + e^{k-1} u[k]$
 $= 4 \times 2^{k-1} u[k-1] + \frac{1}{e} e^k u[k]$
 $\quad \quad \quad \updownarrow \quad \quad \quad \updownarrow$
 $\quad \quad \quad \frac{4}{z-2} \quad \quad \quad \frac{1}{e} \frac{z}{z-e}$
 $\therefore F[z] = \frac{4}{z-2} + \frac{1}{e} \frac{z}{z-e} //$

(b) $f[k] = k \gamma^k u[k-1]$
 $= k \gamma^k (u[k] - \delta[k])$
 $= k \gamma^k u[k] - 0$
 $= k \gamma^k u[k]$

$\therefore F[z] = \frac{\gamma z}{(z-\gamma)^2} //$

(c) $f[k] = \left[2^{-k} \cos \frac{\pi}{3} k \right] u[k-1]$
 $= 2^{-k} \cos \frac{\pi}{3} k \{ u[k] - \delta[k] \}$
 $= \cancel{2^{-k} \cos \frac{\pi}{3} k}$

$\therefore F(z) = \frac{z(z-0.25)}{z^2 - 0.5z + 0.25} - 1$
 $= \frac{0.25(z-1)}{z^2 - 0.5z + 0.25} //$

5

$$5). (a) \frac{F(z)}{z} = \frac{z-4}{(z-2)(z-3)}$$

$$= \frac{2}{z-2} - \frac{1}{z-3}$$

$$\therefore F(z) = \frac{2z}{z-2} - \frac{z}{z-3}$$

$$\therefore f[k] = [2 \times 2^k - 3^k] u[k] //$$

$$(b) \frac{F[z]}{z} = \frac{e^{-2} - 2}{(z - e^{-2})(z-2)}$$

$$= \frac{1}{z - e^{-2}} - \frac{1}{z-2}$$

$$F[z] = \frac{z}{z - e^{-2}} - \frac{z}{z-2}$$

$$f[k] = [e^{-2k} - 2^k] u[k] //$$

$$(c) \frac{F(z)}{z} = \frac{-5z + 22}{(z+1)(z-2)^2}$$

$$= \frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}$$

Multiply both sides by z & let $z \rightarrow \infty$
yields $0 = 3 + k + 0 \Rightarrow k = -3$

$$\therefore F[z] = 3 \frac{z}{z+1} - 3 \frac{z}{z-2} + 4 \frac{z}{(z-2)^2}$$

$$f[k] = [3(-1)^k - 3(2)^k + 2k(2)^k] u[k] //$$

⑥

$$5) (d) \frac{F(z)}{z} = \frac{2z^2 - 0.3z + 0.25}{z(z^2 + 0.6z + 0.25)}$$

$$= \frac{1}{z} + \frac{Az + B}{z^2 + 0.6z + 0.25}$$

Multiplying both sides by z & let $z \rightarrow \infty$.

This yields

$$2 = 1 + A \Rightarrow A = 1.$$

Setting $z = 1$ on both sides yields

$$\frac{1.95}{1.85} = 1 + \frac{1+B}{1.85}$$

$$\Rightarrow B = -0.9.$$

$$\therefore F[z] = 1 + \frac{z(z - 0.9)}{z^2 + 0.6z + 0.25}$$

For the second term on the right side, we use pair #12c in z-transform table:

$$A=1, B=-0.9, a=0.3, |r|=0.5.$$

$$r = \sqrt{10} \quad \beta = \cos^{-1}\left(\frac{-0.3}{0.5}\right) = 2.214$$

$$\theta = \tan^{-1} \frac{1.2}{0.4} = 1.249.$$

$$\therefore f[k] = \delta[k] + \sqrt{10} 0.5^k \cos(2.214k + 1.249)^u[k]$$

6. Matlab m-script

```
%---DFT, windowing and zeropadding
% this scripts generate three plots representing the amplitude of a sine
% wave of a fixed frequency and fixed sampling rate, but changing the
% window size and with ot without zero-padding.

%Create sampled sine wave at frequenct 21Hz, with fs=127 herz and windowing
%with a recangular window of width 0.5sec
fsig=11;
fsamp=31;
T=0.25;
[t sine]=sinegen(fsig, fsamp, T);

%Compute the DFT and make sure it is symmetric around zero
N_0=length(t);
Y=fftshift(fft(sine));

%plot the amplitude of Y
figure(2)
n=-floor((N_0)/2): 1: floor((N_0-1)/2);
%n=0: 1: N_0-1;
freq=n/T;
stem(freq,abs(Y),'.');

%Increase the size of the window to 1sec
T=1;
[t2 sine2]=sinegen(fsig, fsamp, T);
%plot the amplitude of Y
%Compute the DFT and make sure it is symmetric around zero
N_0=length(t2);
Y2=fftshift(fft(sine2));

figure(3)
n=-floor((N_0)/2): 1: floor((N_0-1)/2);
%n=0: 1: N_0-1;
freq=n/T;
stem(freq,abs(Y2),'.');

%Go back to case 1 i.e., T=0.5sec but increase the number of DFT elements
%using zero padding by a factor 4
%Zero padding helps you to get a better estimate of the spectrum of case
%one but does not increase the resolution of the spectrum

%zero pad the signal sine
sine_zp=zeros(1,4*length(t));
sine_zp(1:length(t))=sine;

Y_z=fftshift(fft(sine_zp));

%plot the amplitude of Y
figure(4)
n=-floor((N_0)/2): 1: floor((N_0-1)/2);
%n=0: 1: N_0-1;
freq=n/T;
stem(freq,abs(Y_z),'.');
title('zero padding of signal in figure 2')
```

