E2.5 Signals & Linear Systems

Tutorial Sheet 7 – Sampling

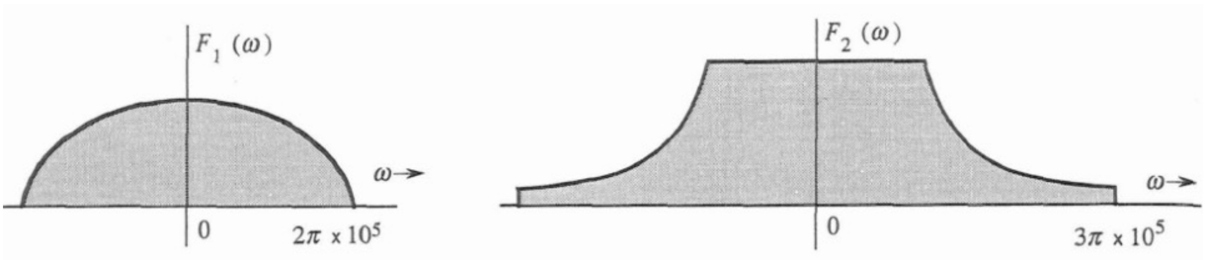
1.* By applying the Parseval's theorem, show that

$$\int_{-\infty}^{\infty} \operatorname{sinc}^2(kx)dx = \frac{\pi}{k}.$$

2.* Fig. Q2 (a) and (b) shows Fourier spectra of signals $f_1(t)$ and $f_2(t)$. Determine the Nyquist sampling rates for the following signals. (Hint: Use the frequency convolution and the width property of the convolution.)

a) $f_1(t)$ b) $f_2(t)$ c) $f_1^2(t)$

d) $f_2^3(t)$ e) $f_1(t)f_2(t)$



(a) (b)

Figure Q2

3.* Signals $f_1(t) = 10^4 rect(10^4 t)$ and $f_2(t) = \delta(t)$ are applied at the inputs of ideal lowpass filters $H_1(\omega) = rect(\frac{\omega}{40,000\pi})$ and $H_2(\omega) = rect(\frac{\omega}{20,000\pi})$. The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$ as shown in Figure Q3.

a) Sketch $F_1(\omega)$ and $F_2(\omega)$.

b) Sketch $H_1(\omega)$ and $H_2(\omega)$.

c) Sketch $Y_1(\omega)$ and $Y_2(\omega)$.

d) Find the Nyquist sampling rate of $y_1(t)$, $y_2(t)$ and $y(t)$.

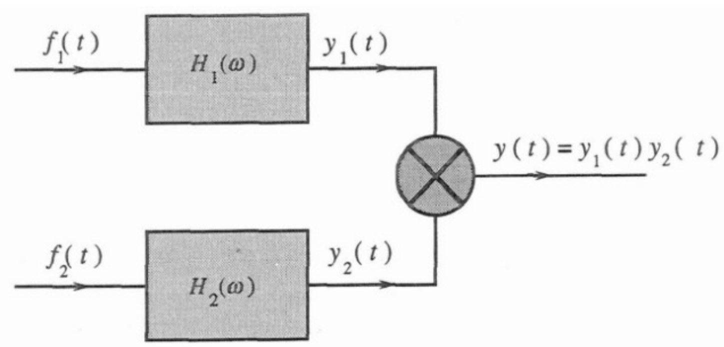


Figure Q3

4.** For the signal $e^{-at}u(t)$, determine the bandwidth of an anti-aliasing filter if the essential bandwidth of the signal contains 99% of the signal energy.

5.** A zero-order hold circuit shown in Fig. Q5 is often used to reconstruct a signal *f(t)* from its samples.

a) Find the unit impulse response of this circuit.

b) Find the transfer function $H(\omega)$, and sketch $|H(\omega)|$.

c) Sketch the output of this circuit for an input f[n] which is the sampled version of *f(t)* which is ¼ cycle of a sinewave. The sampling period is T.

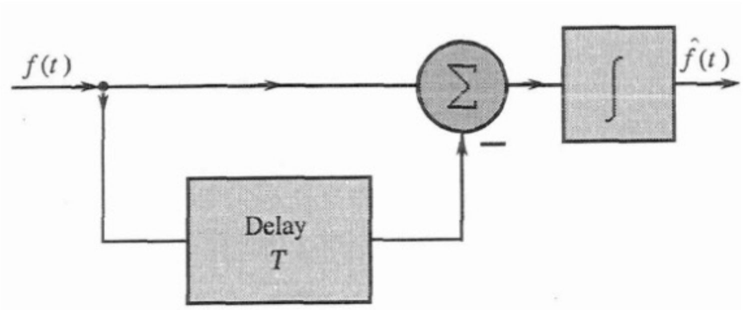


Figure Q5

6. MATLAB exercise: Write a matlab function of the form:

function [t, sinewave] = sinegen(fsig,fsamp,T)

that is able to generate a sampled sinewave of frequency fsig sampled at fsamp on the interval [0,T). Using this function reproduce the figures in slides 14 of lecture 13 which shows aliasing when sampling sinewaves of frequency 1Hz and 6Hz using the sampling rate fs=5Hz. Plot the sampled signals using the command ‘stem’.