E2.5 Signals & Linear Systems

Tutorial Sheet 5 – Laplace Transform & Frequency Response

(Lectures 7 - 9)

1.* Using Laplace transform, solve the following differential equations:

a) $(D^2+3D+2)y(t) = Df(t)$ if $y(0^-) = \dot{y}(0^-) = 0$ and $f(t) = u(t)$

b) $(D^2+4D+4)y(t) = (D+1)f(t)$ if $y(0^-) = 2, \dot{y}(0^-) = 1$ and $f(t) = e^{-t}u(t)$

c) $(D^2+6D+25)y(t) = (D+2)f(t)$ if $y(0^-) = \dot{y}(0^-) = 1$ and $f(t) = 25u(t)$.

2.* For each of the system described by the following differential equations, find the system transfer function.

a) $\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 24y(t) = 5\frac{df}{dt} + 3f(t)$

b) $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} - 11\frac{dy}{dt} + 6y(t) = 3\frac{d^2f}{dt^2} + 7\frac{df}{dt} + 5f(t)$

c) $\frac{d^4y}{dt^4} + 4\frac{dy}{dt} = 3\frac{df}{dt} + 2f(t)$.

3.** For a system with transfer function

$$H(s) = \frac{s+5}{s^2+5s+6}$$

a) Find the zero-state response if the input *f(t)* is

(i) $e^{-4t}u(t)$ (ii) $e^{-3t}u(t)$ (iii) $e^{-4(t-5)}u(t-5)$

b) For this system write the differential equation relating the output *y(t)* to the input *f(t)*.

4.** For the circuit shown in Figure Q4, the switch is in open position for a long time before t = 0, when it is closed instantaneously.

a) Write loop equations in time domain for t ≥ 0.

b) Solve for $y_1(t)$ and $y_2(t)$ by taking the Laplace transform of loop equations found in part a).

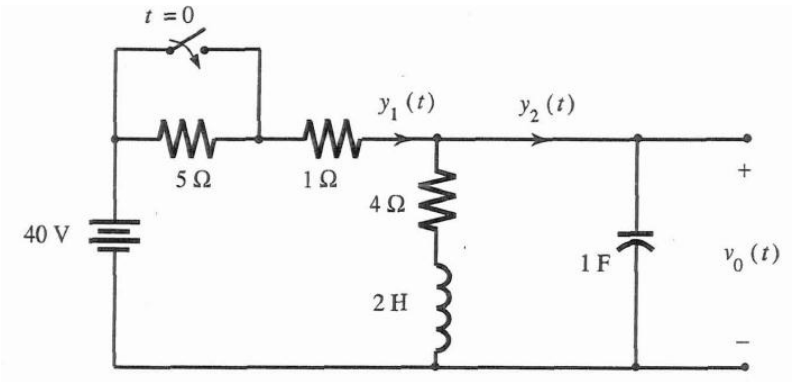

Fig. Q4

5.* Using the initial and final value theorems, find the initial and final values of the zero-state response of a system with the transfer function

$$H(s)=\frac{6s^2+3s+10}{2s^2+6s+5}$$

and the input is

a) $u(t)$

b) $e^{-t}u(t)$.

6.** For a LTI system described by the transfer function

$$H(s)=\frac{s+3}{(s+2)^2}$$

Find the system response to the following inputs:

a) $\cos(2t+60^\circ)$

b) $\sin(3t-45^\circ)$

c) e^{j3t}

7.** Using graphical method, draw a rough sketch of the amplitude and phase response of LTI systems whose pole-zero plots are shown in Fig. Q7(a) & (b).

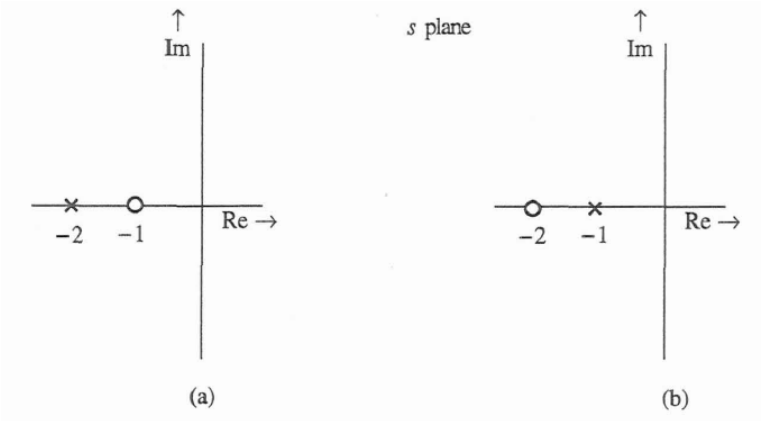

Fig. Q7