E2.5 Signals & Linear Systems

Tutorial Sheet 4 – Laplace Transform

(Support Lecture 6)

1.* By direct integration, find the one-sided Laplace transforms of the following functions (please note that if not otherwise stated, we will always consider the unilateral Laplace transform):

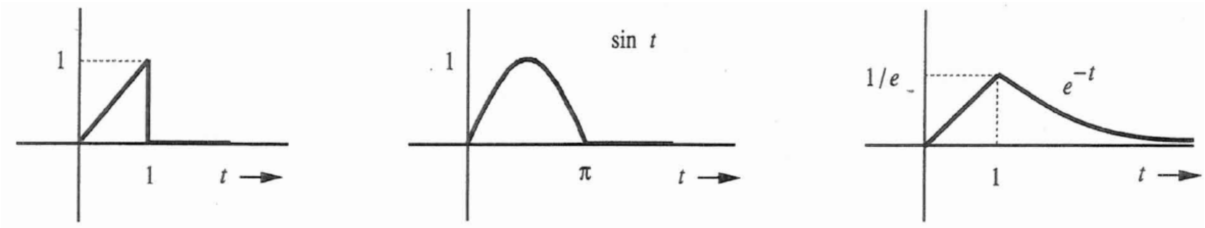
a) $u(t) - u(t-1)$

b) $te^{-t}u(t)$

c) $t\cos\omega_0 t\, u(t)$

d) $e^{-2t}\cos(5t+\theta)\, u(t)$

2.* By direct integration, find the Laplace transforms of the following signals:



(a) (b) (c)

3.* Find the inverse (one-sided) Laplace transforms of the following functions (from now on, if not stated otherwise, we always look for a causal solution):

a) $\dfrac{2s+5}{s^2+5s+6}$

b) $\dfrac{3s+5}{s^2+4s+13}$

c) $\dfrac{(s+1)^2}{s^2-s-6}$

d) $\dfrac{2s+1}{(s+1)(s^2+2s+2)}$

e) $\dfrac{s+3}{(s+2)(s+1)^2}$

4.** Find the Laplace transforms of the following function using the Laplace Transform Table and the time-shifting property where appropriate.

a) $u(t) - u(t-1)$

b) $e^{-(t-\tau)}u(t)$

c) $e^{-t}u(t-\tau)$

d) $\sin[\omega_0(t-\tau)]\,u(t-\tau)$

e) $\sin[\omega_0(t-\tau)]\,u(t)$

5.** Find the inverse Laplace transform of the function:

$$\frac{2s+5}{s^2+5s+6}e^{-2s}.$$

6.*** The Laplace transform of a causal periodic signal can be found from the knowledge of the Laplace transform of its first cycle alone.

a) If the Laplace transform of *f(t)* shown in Fig. 6 a) is *F(s),* show that G(s), the Laplace transform of *g(t)* shown in Fig. 6 b) is given by:

$$G(s)=\frac{F(s)}{1-e^{-sT_0}} \qquad \operatorname{Re} s>0$$

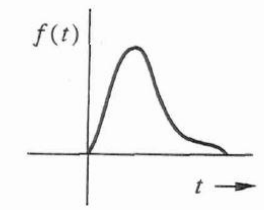


Fig 6 a)

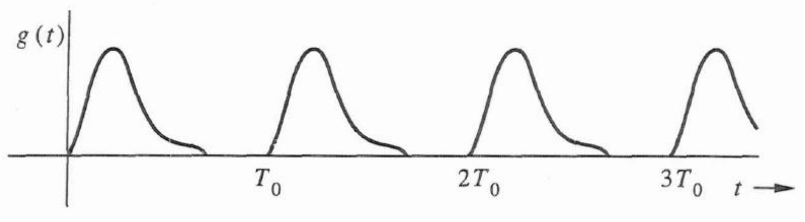
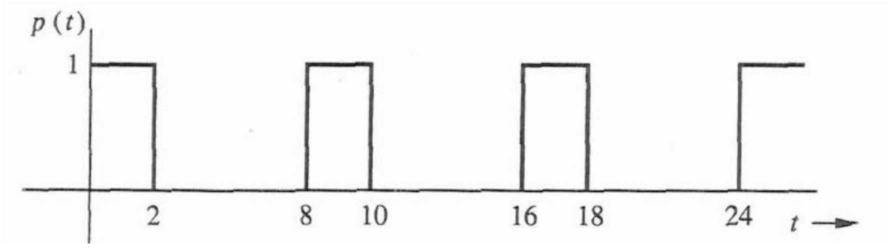


Fig 6 b)

b) Using the results in a), find the Laplace transform of the signal p(t) shown in Fig. 6 c).



:*Fig 6 c)*

(Hint: Remember that $1+x+x^2+x^3+\ldots=\dfrac{1}{1-x}$ for $|x|<1$.)