E2.5 Signals & Linear Systems

Tutorial Sheet 3 – Zero-state Responses & Convolution

1.* Using direct integration, find the expression for:

a) $y(t) = u(t) * u(t)$

b) $y(t) = e^{-at}u(t) * e^{-bt}u(t)$

c) $y(t) = tu(t) * u(t)$.

2.* Using direct integration, find:

a) $y(t) = \sin t\, u(t) * u(t)$

b) $y(t) = \cos t\, u(t) * u(t)$.

3.* The unit impulse response of an LTI system is $h(t) = e^{-t}u(t)$. Use the convolution table to find this system's zero-state response $y(t)$ if the input $f(t)$ is:

a) $u(t)$

b) $e^{-2t}u(t)$

c) $\sin 3t\, u(t)$

4.** By applying the shift property of convolution, find the system's response (i.e. zero-state response) given that $h(t) = e^{-t}u(t)$ and that the input $f(t)$ is as shown in Fig 4.1.

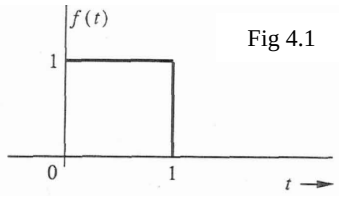


Fig 4.1

5.** A first-order allpass filter impulse response is given by

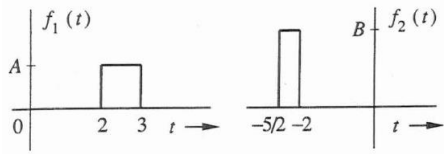
$$h(t) = -\delta(t) + 2e^{-t}u(t).$$

a) Find the zero-state response of this filter for the input $e^{t}u(-t)$.

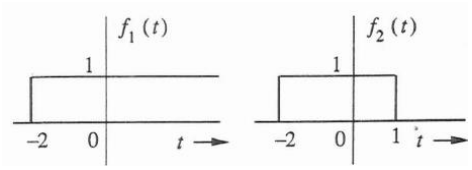
b) Sketch the input and the corresponding zero-state response.

6.** Find and sketch $c(t) = f_1(t) * f_2(t)$ for the pairs of functions shown as follow:

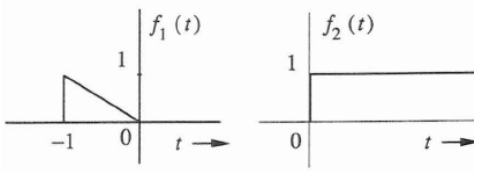
a)



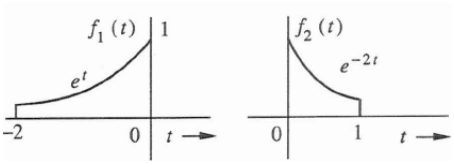
b)



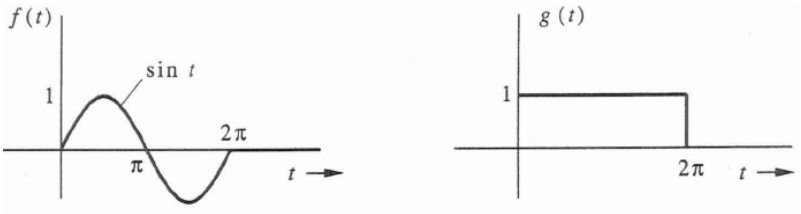
c)



d)



7.*** Find and sketch $c(t) = f(t) * g(t)$ for the pairs of functions shown below.



8. Matlab exercise. Write a routine in matlab that, given two functions x(t) and h(t), computes y(t)=h(t)*x(t). You may use this routine to verify the convolutions you have computed in 6 and 7.

Also remember that to implement an integral in matlab you have to replace it using trapezoidal approximation of integral. If you have two discrete-time sequences x[n],h[n] the discrete-time convolution is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

This is slightly different from the formula that the trapezoidal rule would give you. Can you appreciate the difference? If you are lost you can always use and modify the m-files 'graphical_conv.m' which has been provided to you.