E2.5 Signals & Linear Systems

Tutorial Sheet 1 SOLUTIONS

1. (i) Periodic with period 1. Odd because $x(-t) = -x(t)$.

(ii) A-periodic. Neither odd nor even.

(iii) A-periodic. Even because $x(-t) = x(t)$.

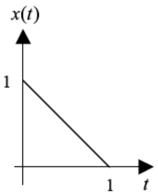
(iv) Periodic with period 15, odd.

(v) A-periodic, odd.

Note:
Suppose f, g are both non-constant periodic functions within the domain R, the real numbers. If f has period p, and g has period q, and p and q are incommensurable then f + g will not be periodic. On the other hand, if there exist integers m and n so that mp = nq then f + g will be periodic and pq will be a period of f + g.

f(x) = sin(x), g(x) = sin(sqrt(2) x) is an example where f + g is not periodic. f has period 2π, g has period sqrt(2)π, and the ratio of the periods is sqrt(2) which is irrational.

2.



(i) Left shift by 3.

(ii) Linearly expand by factor of 3.

(iii) $x(t/3+1) = x[(t+3)/3]$. Linearly stress (expand) by factor of 3 and shift left by 3.

(iv) Time reverse and shift right by 2.

(v) $x(-2t+1) = x[-2(t-1/2)]$. Time reverse, linearly compress by factor of 2 and shift right by ½.

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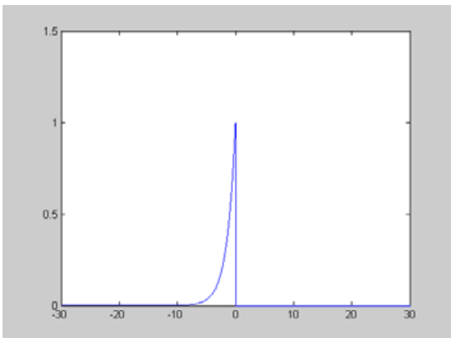
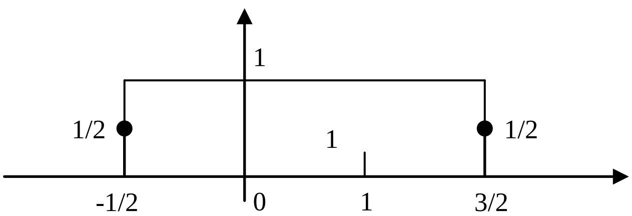
3. (i) Periodic with period 2. Even because $x[-n] = x[n]$. We all know how it looks like.

(ii) Non-periodic. Neither odd nor even.



(iii) The only possible frequencies of $e^{j\omega_0 n}$ are $f_0 \in [0,1)$. To show this fact, assume f0>1 for example f0=1+δ with 0<δ<1, then $e^{j\omega_0 n} = e^{j2\pi f_0 n} = e^{j2\pi n}e^{j2\pi\delta} = e^{j2\pi\delta}$, since n in an integer. This is a fundamental difference with the continuous-time case. You may use the matlab routine of exercise 8, to further convince yourself of this result. We will revisit this fact when talking about sampling and aliasing in lectures 13-14.

4.



(i) $x(t)=\begin{cases}1, & -1/2<t<3/2\\ 1/2, & t=-1/2, \text{ and } t=3/2\\ 0, & \text{otherwise}\end{cases}$

(ii) $x(t)=1$

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5. (i) It is memoryless since the output at time instant n depends on the input only at time instant n and not past or future time instants.

(ii) It is causal since the output at time instant n depends on the input only at time instant n and not future time instants.

(iii). No. If the output at time instant n depends on the input at time instant n **and** past time instants the system is causal but not memoryless.

(iv) $y[n] = \dfrac{x[n] + (-1)^n x[n]}{2}$.

From this we see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$ then the input signal $a_1x_1[n] + a_2x_2[n]$ produces the output

$$y_3[n] = \frac{(a_1x_1[n] + a_2x_2[n]) + (-1)^n (a_1x_1[n] + a_2x_2[n])}{2} = a_1y_1[n] + a_2y_2[n]).$$

Therefore, the system is linear.

However, if the input signal $x[n]$ produces an output signal $y[n]$ then the input signal $x[n-n_o]$ produces the output $y_1[n] = \dfrac{x[n-n_o] + (-1)^n x[n-n_o]}{2}$.

We see that $y[n-n_o] = \dfrac{x[n-n_o] + (-1)^{n-n_o} x[n-n_o]}{2} \neq y_1[n]$

Therefore, the system is time varying.

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6. (i) Linear, causal, time invariant.

(ii) Non-linear, causal, time invariant.

(iii) Linear, non-causal, time varying.

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7. (i) Linear, causal, time varying.

(ii) Non-linear, causal, time varying.

(iii) Linear, causal, time invariant.

(iv) Linear, non-causal, time varying.

(v) Linear, non-causal, time varying.

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8. Matlab exercise

M-file:

```
function [ y n] = discretecosine( f_0 )
%The function [y n]=discretecosine(f0) generates a discrete-time
%cosinewave of frequency f0.

n=0:1:50;
y=cos(2*pi*f_0*n);


end
```

to plot the function:

```
plot(n,y)
% scale axis for suitable max and min values
axis([0 50 -1 1]);

% label axes
xlabel('n');
ylabel('Amplitude');
```

or use

```
stem(n,y,'.')
% scale axis for suitable max and min values
axis([0 50 -1 1]);

% label axes
xlabel('n');
ylabel('Amplitude');
```