E2.5 Signals & Linear Systems

Tutorial Sheet 1 – Introduction to Signals & Systems

1. Sketch each of the following continuous-time signals, specify if the signal is periodic/non-periodic, odd/even. If the signal is periodic specify its period.

   (i) $x(t) = 2\sin(2\pi t)$

   (ii) $x(t) = \begin{cases} 3e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

   (iii) $x(t) = 1/|t|$

   (iv) $x(t) = \sin(\frac{2\pi}{5}t) + \sin(\frac{2\pi}{3}t)$

   (v) $x(t) = \sin(2\pi t) + \sin(\sqrt{2}\pi t)$

2. Sketch the signal

$$x(t) = \begin{cases} 1-t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   Now sketch each of the following and describe briefly in words how each of the signals can be derived from the original signal $x(t)$.

   (i) $x(t+3)$

   (ii) $x(t/3)$

   (iii) $x(t/3+1)$

   (iv) $x(-t+2)$

   (v) $x(-2t+1)$

3. Sketch each of the following signals. For each case, specify if the signal is periodic/non-periodic, odd/even. If the signal is periodic specify its period.

   (i) $x[n] = \cos(n\pi)$

   (ii) $x[n] = \begin{cases} 0.5^{-n}, & n \leq 0 \\ 0, & n > 0 \end{cases}$

   (iii) What is the maximum possible frequency of $e^{j\omega_0 n}$ (compare this result with the case $e^{j\omega_0 t}$)?

4. Consider the rectangular function

$$\Pi(t) = \begin{cases} 1, & |t| < 1/2 \\ 1/2, & |t| = 1/2 \\ 0, & \text{otherwise} \end{cases}$$

   (i) Sketch $x(t) = \sum_{k=0}^{1} \Pi(t-k)$

   (ii) Sketch $x(t) = \sum_{k=-\infty}^{+\infty} \Pi(t-k)$. (Hint: there is a simple way to express this signal.)

5. Consider a discrete-time signal $x[n]$, fed as input into a system. The system produces the discrete-time output $y[n]$ such that

$$y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

   (i) Is the system described above memoryless? Explain.

(ii) Is the system described above causal? Explain.
(iii) Are causal systems in general memoryless? Explain.
(iv) Is the system described above linear and time-invariant? Explain.

6. State with a brief explanation if the following systems are linear/non-linear, causal/non-causal, time-invariant/time-varying.

(i) $y[n] = x[n] - x[n-1]$

(ii) $y[n] = \mathrm{sgn}(x[n])$

(iii) $y[n] = n^2 x[n+2]$

7. State with a brief explanation if the following systems are linear/non-linear, causal/non-causal, time-invariant/time-varying.

(i) $y(t) = x(t)\cos(2\pi f_o t + \phi)$

(ii) $y(t) = A\cos(2\pi f_o t + x(t))$

(iii) $y(t) = \int_{-\infty}^{t} x(\delta) d\delta$

(iv) $y(t) = x(2t)$

(v) $y(t) = x(-t)$

8. To understand better the periodicity properties of a discrete-time sine function, write an M-file in Matlab that generates the function $y = \cos(2\pi f_0 n)$ where $n = 0,1,\ldots,50$ and f_0 is arbitrary, then plot the function using either 'plot' or 'stem' commands.