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Signals and Systems

Lecture 1

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Miscellanea

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Logistics of the course

- Lectures - 15 hours over 8-9 weeks
- Problem Classes – 7-8 hours over 8-9 weeks
- Assessment – 100% by examination in June
- Handouts in the form of pdf slides are available at http://www.commsp.ee.ic.ac.uk/~tania/
- Text Books:
  - B.P. Lathi, “Linear Systems and Signals”, 2nd Ed., Oxford University Press
  - A.V. Oppenheim & A.S. Willsky “Signals and Systems”, Prentice Hall

Aims and Objectives of Signals and Systems

- The concepts of signals and systems arise in a variety of fields such as, communications, aeronautics, bio-engineering, energy, circuit design and others.
- Although the physical attributes of the signals and systems involved in the above disciplines are different, all signal and systems have basic features in common.
- The aim of this course is to provide the fundamental and universal tools for the analysis of signals.
- Furthermore, the course aims at teaching the analysis and design of basic systems independently of the domain of application.

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Aims and Objectives cont.

By the end of the course, you will have understood:

- Basic signal analysis
- Basic system analysis
- Time-domain system analysis including convolution
- Laplace and Fourier Transform
- System analysis in Laplace and Fourier domains
- Filter design
- Sampling Theorem and signal reconstructions
- Basics on z-transform

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Today's Lecture

In this lecture we will talk about:

- Examples of signals
- Some useful signal operations
- Classification of signals
- Some specific widely used signals:
  - Unit step function
  - Unit impulse function
  - The exponential function

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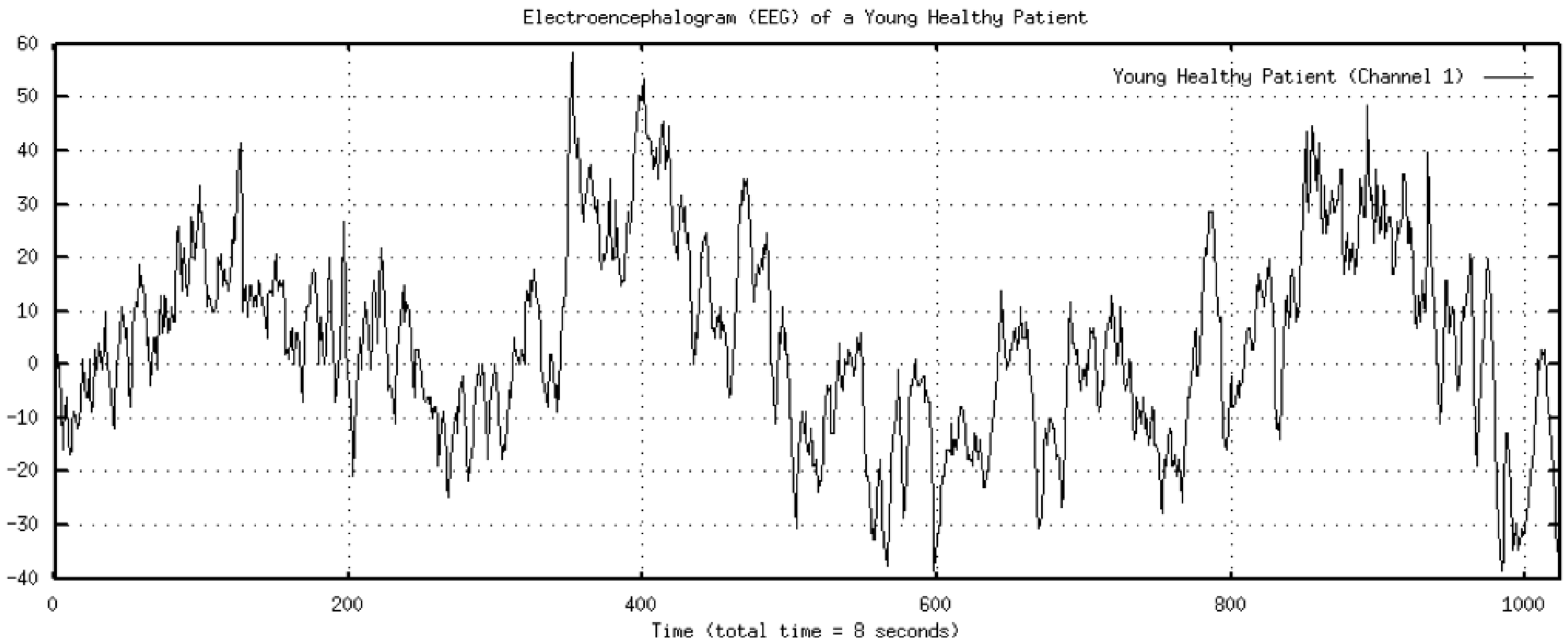
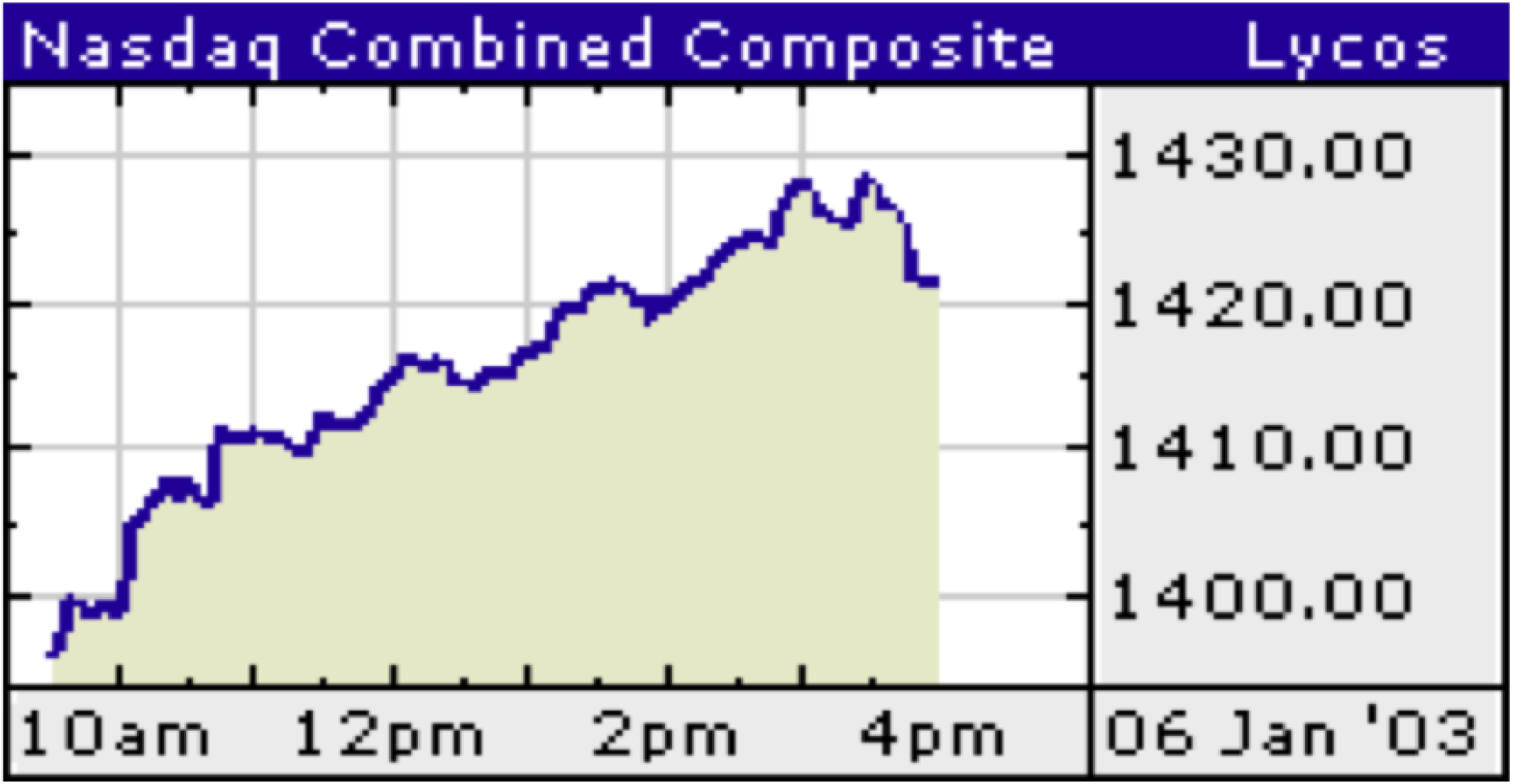
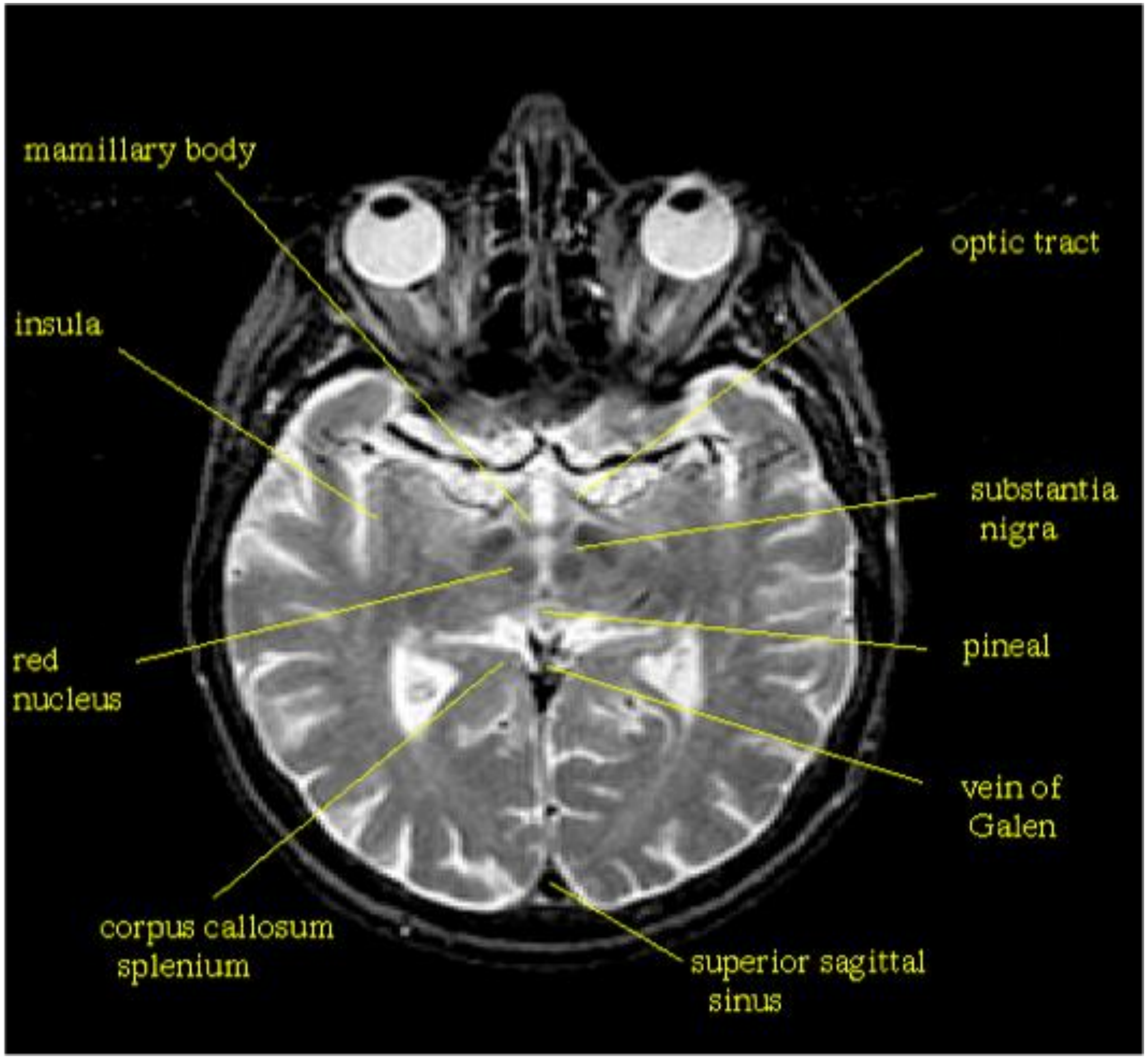
Example of an electroenchephalogram (EEG) signal



Example of a stock market signal

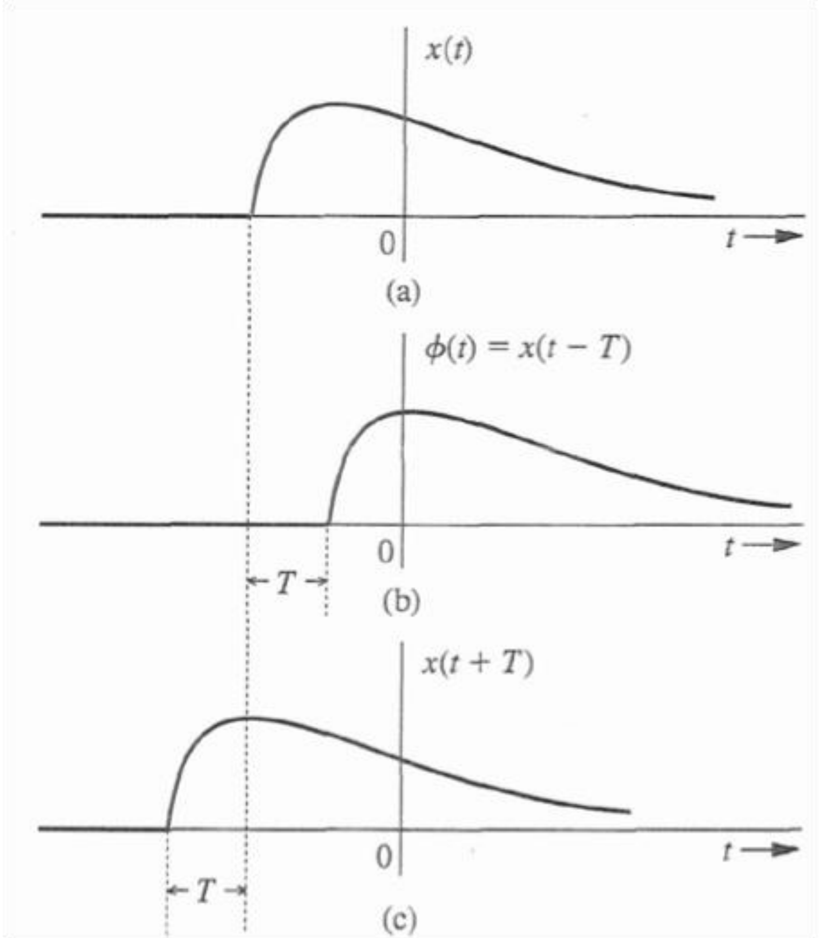


Magnetic resonance tomography image.
This is a signal in 2 dimensions.



Useful signal operations: time shifting

- Consider a signal $x(t)$ in continuous time shown in the figure right.
- The signal $x(t)$ may be delayed by T units of time; in that case the signal $\phi(t) = x(t - T)$ is obtained.
- The signal $x(t)$ may be advanced by T units of time; in that case the signal $\phi(t) = x(t + T)$ is obtained.

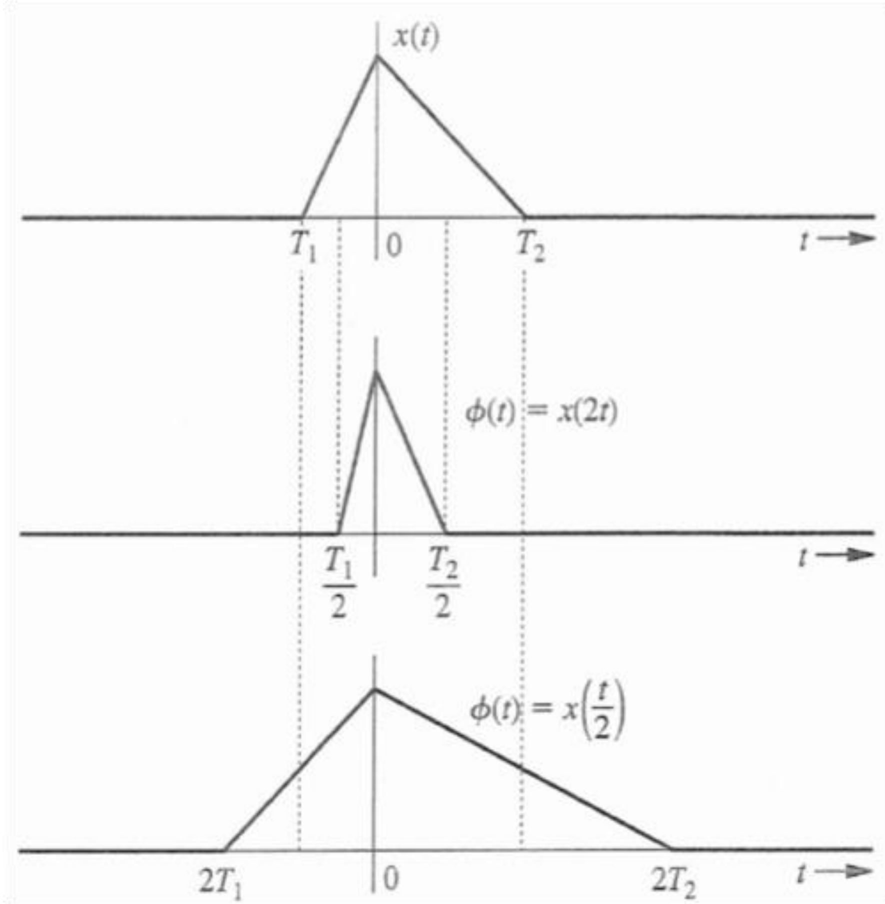


Useful signal operations cont.: time scaling

- Consider a signal $x(t)$ in continuous time shown in the figure right.

- The signal $x(t)$ may be compressed by a factor of 2; in that case the signal $\phi(t) = x(2t)$ is obtained.

- The signal $x(t)$ may be stretched by a factor of 2; in that case the signal $\phi(t) = x\left(\frac{t}{2}\right)$ is obtained.



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Classification of signals

- Signals may be classified into:
  - Energy and power signals
  - Discrete-time and continuous-time signals
  - Analogue and digital signals
  - Deterministic and probabilistic signals
  - Periodic and aperiodic signals
  - Even and odd signals

Energy of a signal

- How do we measure the “size” of a signal $x(t)$?
  - By the signal energy E_x

$$E_x = \int_{-\infty}^{\infty} x^2(t)dt$$

  - For a complex-valued signal the above relationship becomes:

$$E_x = \int_{-\infty}^{\infty} |x^2(t)|dt$$

  - The energy of a signal must be finite. This implies that:

$$\lim_{t\to\pm\infty} |x(t)| = 0$$

Power of a signal

- If $\lim_{t \to \pm\infty} |x(t)| \neq 0$ we must use the signal power instead

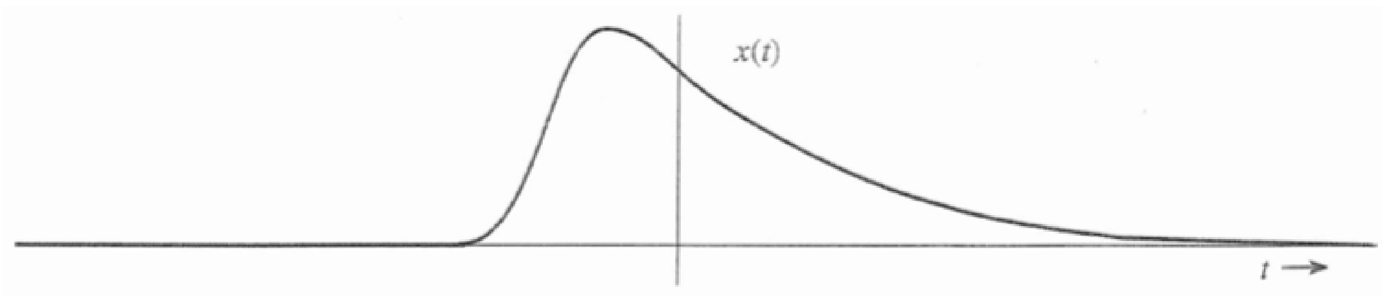
$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

- For a complex-valued signal the above relationship becomes:

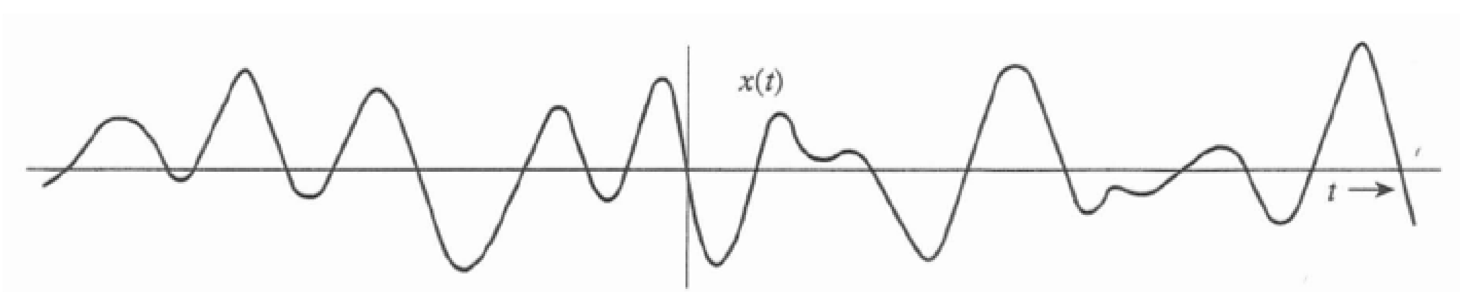
$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x^2(t)| dt$$

Energy and power cont.

- Example of a signal with finite energy, which implies that the power of the signal is zero.



- Example of a signal with finite power, which implies that the energy of the signal is infinite.



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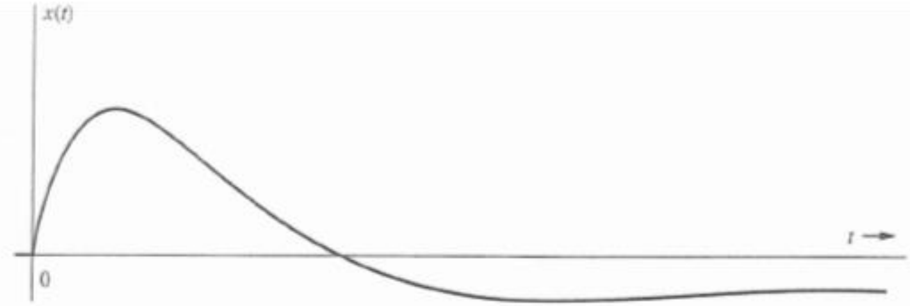
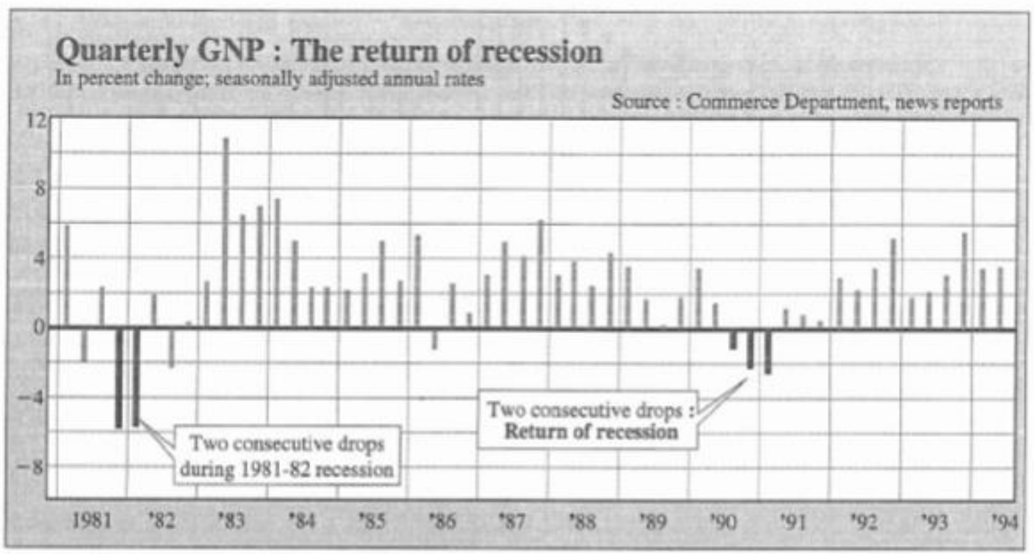
Continuous - and Discrete - Time Signals

This classification refers to time *t*

- Example of a continuous-time signal. This is defined for any value of time.
- Example of a discrete-time signal.
  - This is defined for certain instants of time only.
  - It arises from a continuous-time signal if only its values at these certain time instants are kept.
  - This process is called **sampling**.



(a)

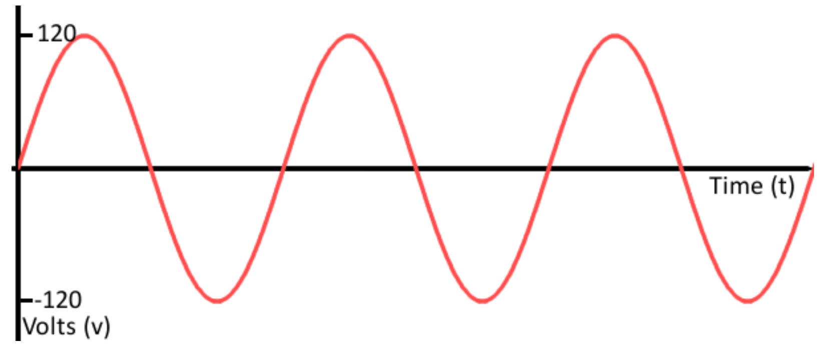


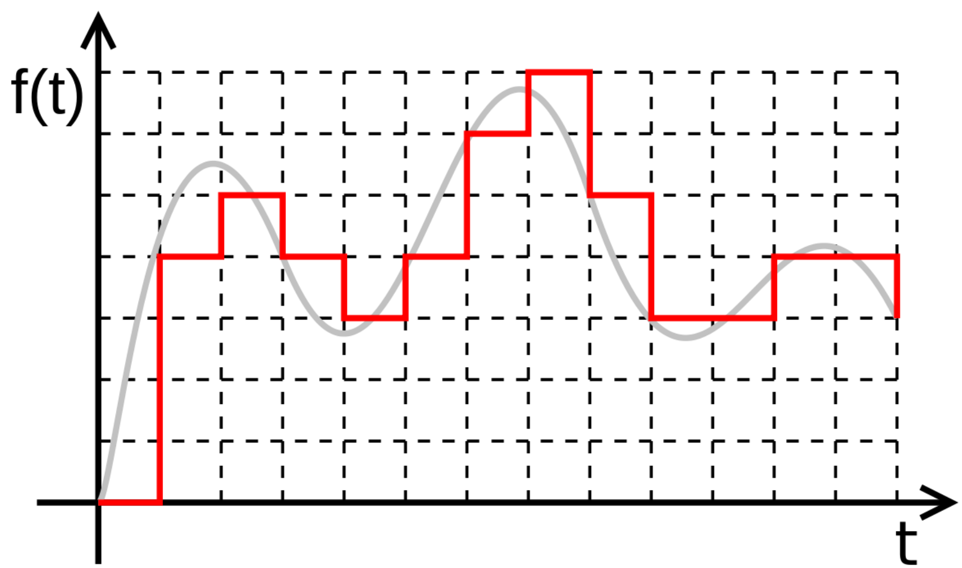
(b)

Analog and Digital Signals

This classification refers to the value of the signal $x(t)$

- An analog signal can take infinite number of values.
- A digital signal can take only a finite number of values.
  - It arises from an analog signal if all values are replaced with a finite number of values.
  - The total error arising from this process must be as small as possible.
  - This process is called **quantization**.





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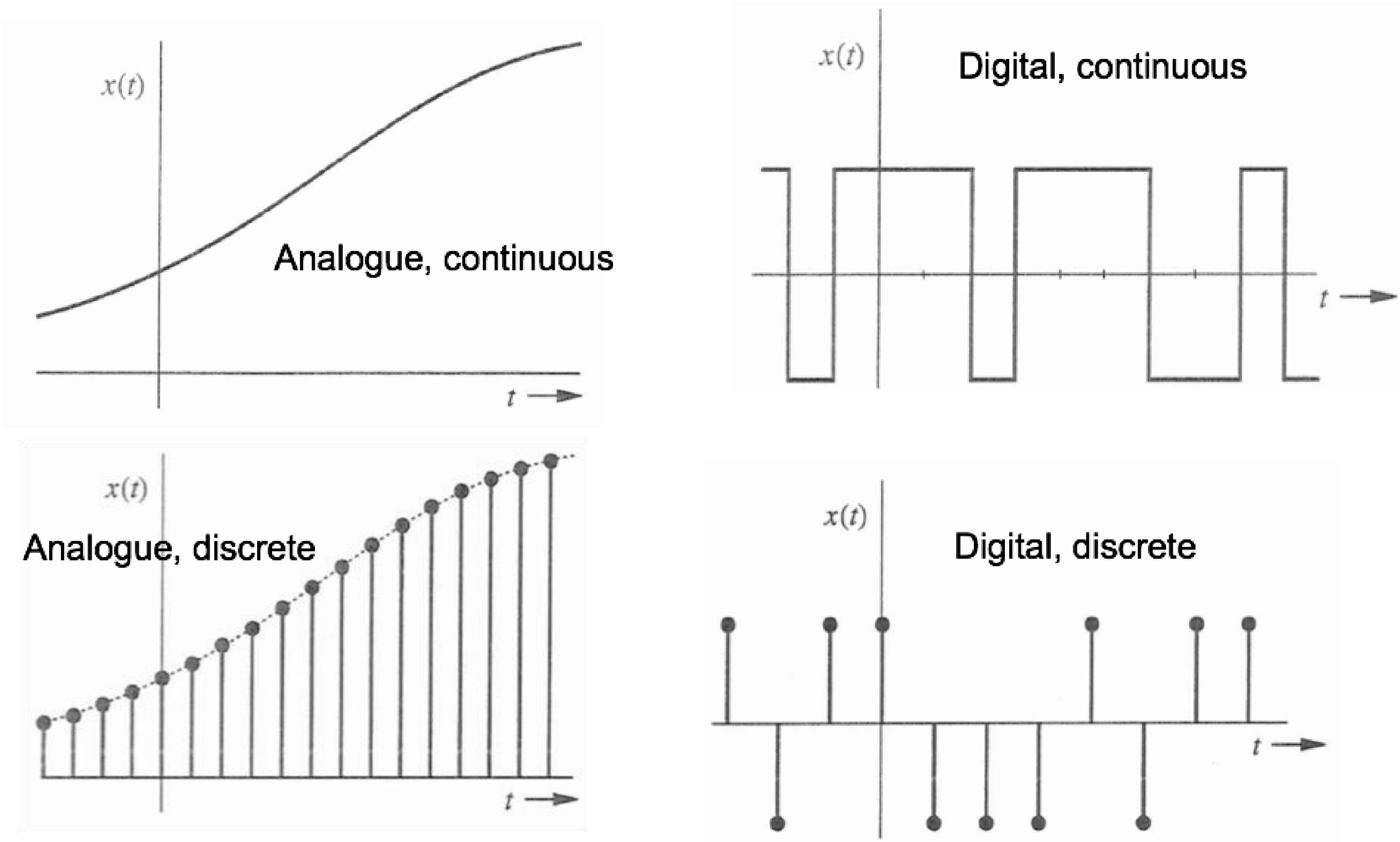
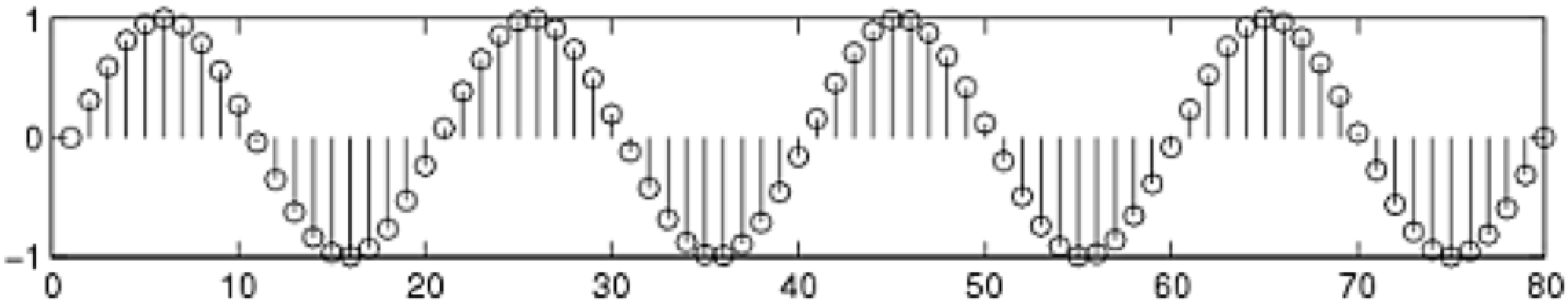
We now have 4 types of signals



Deterministic and Stochastic Signals

- The values of a deterministic signal can be obtained from a closed-form mathematical expressions. For example, $x(t) = \sin(t)$.



- The values of a stochastic signal can only be given as the outputs of a probabilistic model.

  Experiment. Drop a coin 1000 times and create the following 1000-sample digital signal: Every time you get head the value of your signal will be 1, every time you get tails the value of your signal will be -1.

  - Can you tell the value of the signal at time instant n?
  - Can you tell the probability of the value being +1 or -1?
  - What is approximately the mean (average value) of the signal you created?

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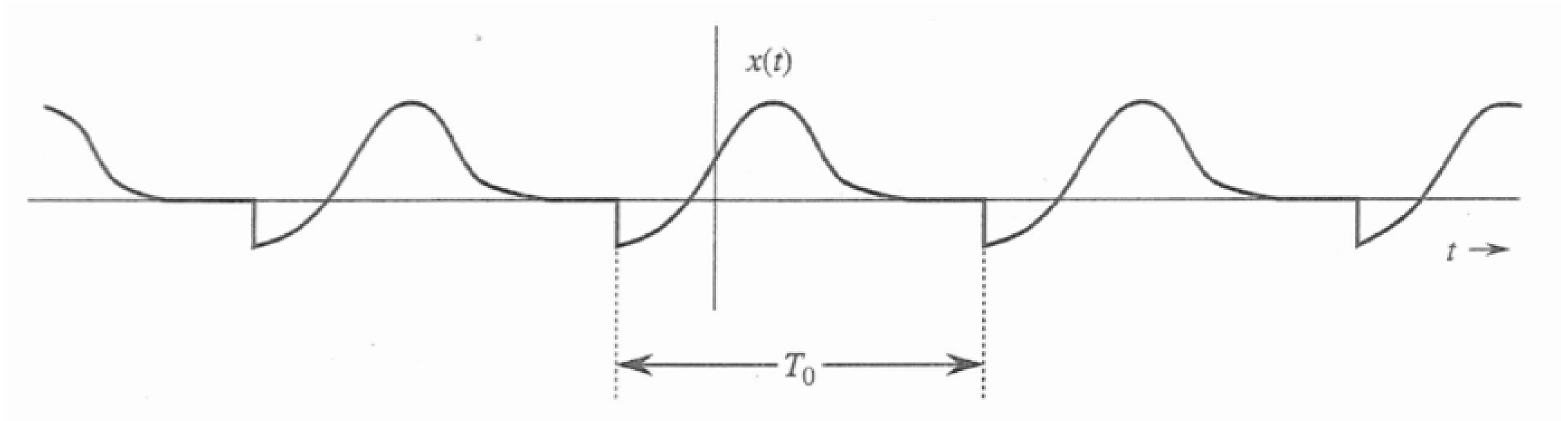
Periodic and Aperiodic Signals

- A signal is said to be periodic if for some constant T_0 the following holds:

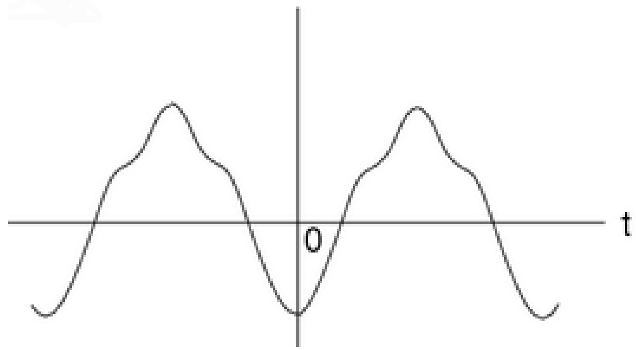
$$x(t + T_0) = x(t), \forall t$$

- The smallest T_0 that satisfies the above relationship is called the fundamental period of the signal.

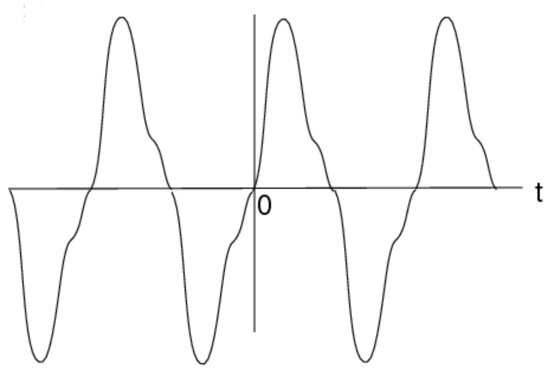


Even and Odd Signals

- An even signal remains the same if you rotate it along the vertical axis. In mathematical terms this property is defined as $x(-t) = x(t)$.



- An odd signal gets reflected along the horizontal axis if you rotate it along the vertical axis. In mathematical terms this property is defined as $x(-t) = -x(t)$.



Even and Odd Signals

Easy problems

- Verify that the signal $x(t) + x(-t)$ is always even.
- Verify that the signal $x(t) - x(-t)$ is always odd.

Based on the above, any signal can be written as a the sum of an even and an odd signal as follows:

$$x(t) = \frac{1}{2}[x(t) + x(-t)] + \frac{1}{2}[x(t) - x(-t)]$$

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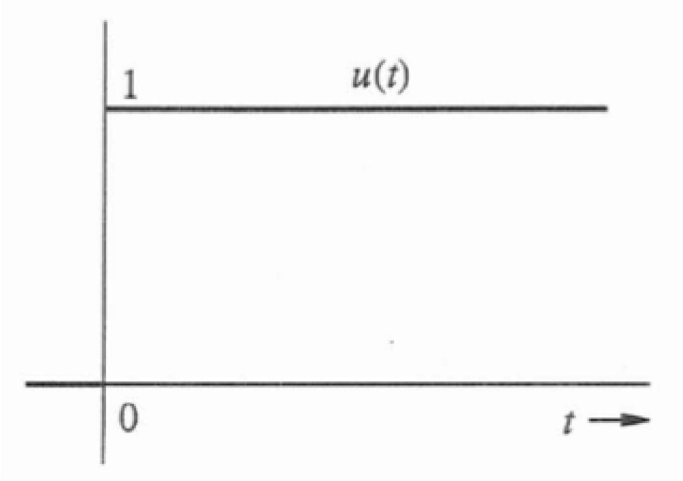
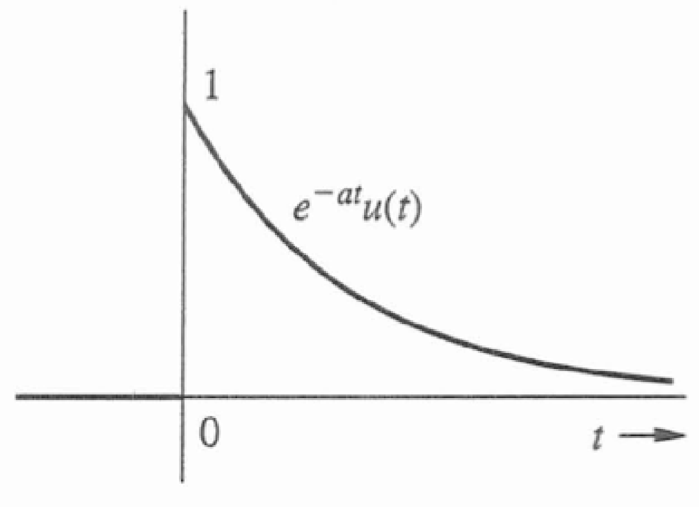
Unit step function

- The unit step function is defined as follows:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- The unit step function is often used to describe a signal that starts at $t = 0$.
- For example, consider the everlasting exponential signal $x(t) = e^{-t}$.
- The causal form of the above signal is $e^{-t}u(t)$.

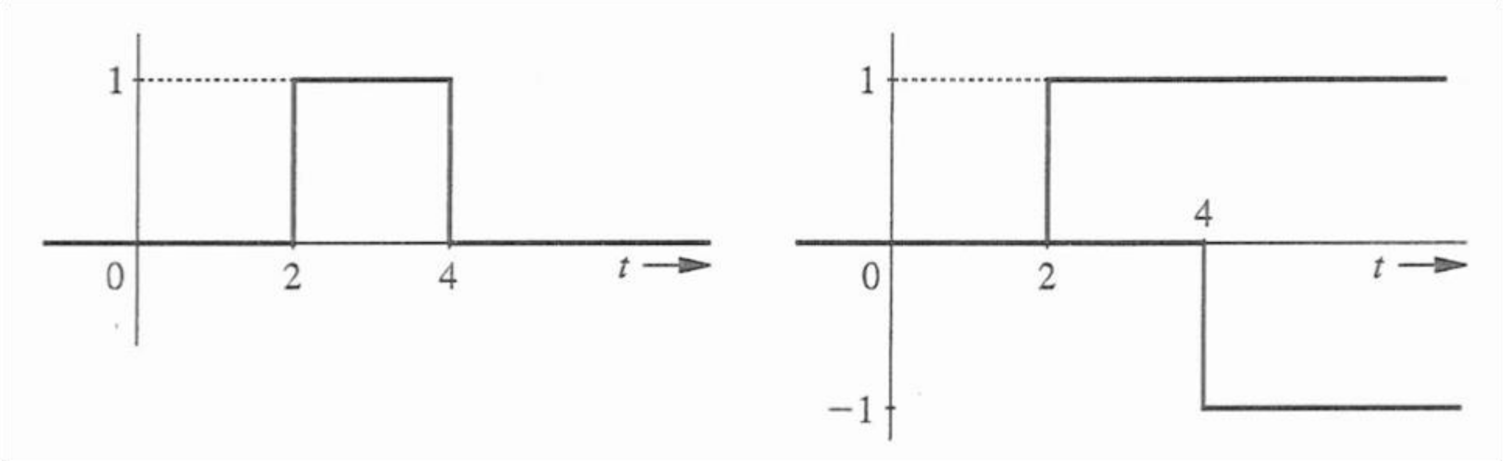




Pulse signals

- A pulse signal can be generated using two step functions, as for example:

$$x(t) = u(t-2) - u(t-4)$$



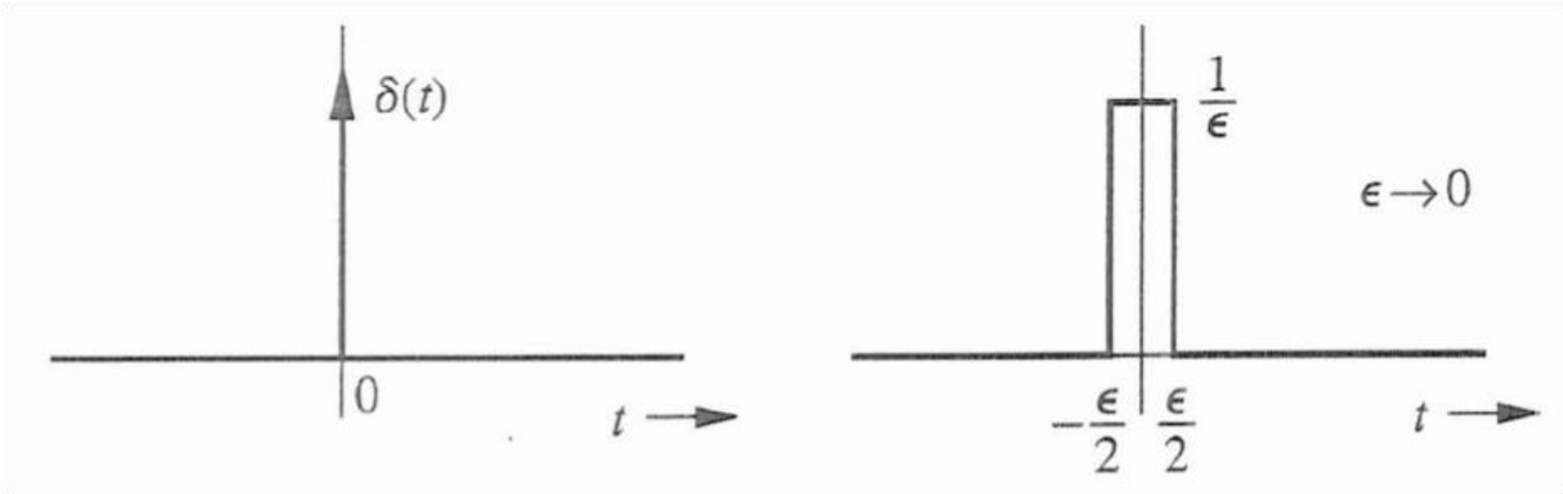
Unit impulse (Dirac) function

- The Dirac function is defined as follows:

$$\delta(t) = 0, t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- The Dirac function is the limit of a family of functions $\delta_\epsilon(t)$ when $\epsilon \to 0$. The simplest of these functions is the rectangular pulse shown in the figure below right. The width and height depend on ϵ but the entire area under the function is equal to 1.



Sampling property of the unit impulse function

- Since the unit impulse function is non-zero only at $t = 0$, for any function $\phi(t)$ we have

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

- From the above we get:

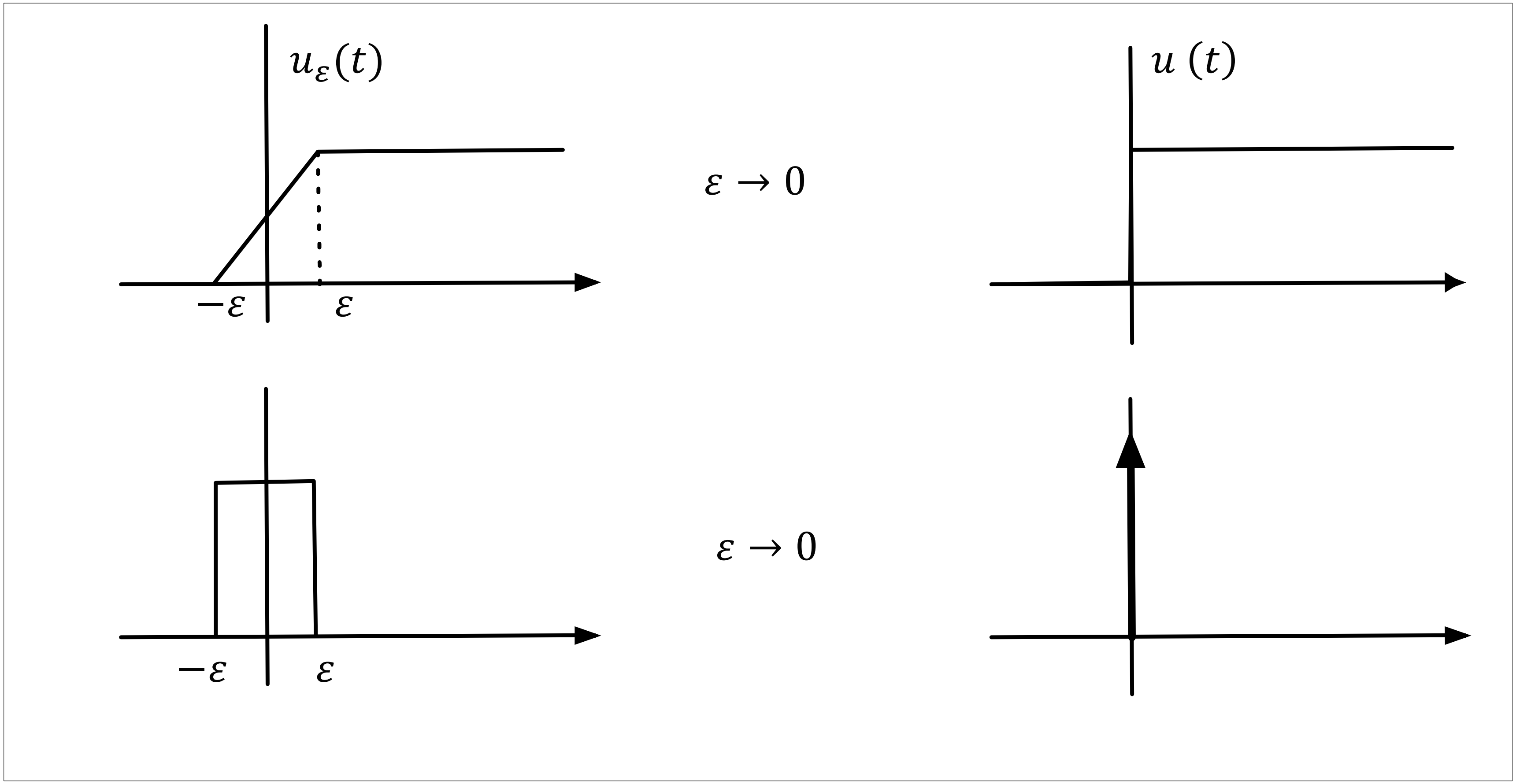
$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \int_{-\infty}^{\infty} \phi(0)\delta(t)dt = \phi(0) \int_{-\infty}^{\infty} \delta(t)dt = \phi(0)$$

- Furthermore, we have:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \int_{-\infty}^{\infty} \phi(\tau+T)\delta(\tau)d\tau = \phi(T)$$

Unit Step and Unit Impulse function

- It is proven that $\frac{du(t)}{dt} = \delta(t)$



The Exponential Function e^{st}

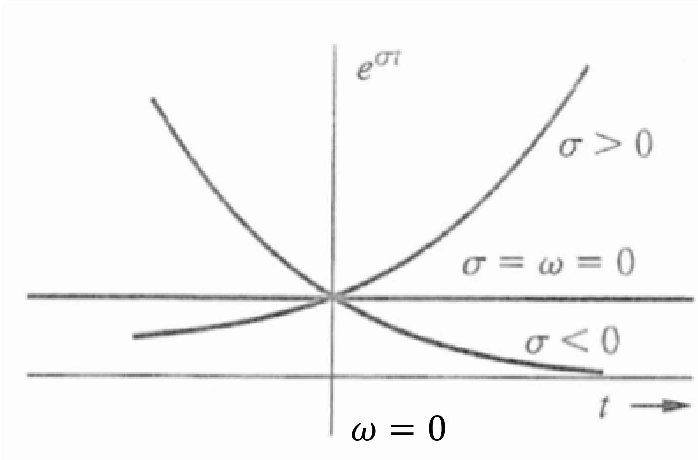
- The exponential function is very important in Signals and Systems.
- The parameter s is a complex variable given by $s = \sigma + j\omega$.
- Therefore, $e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}\, e^{j\omega t} = e^{\sigma t}(\cos\omega t + j\sin\omega t)$
- If s^* is the complex conjugate of s then $s = \sigma - j\omega$

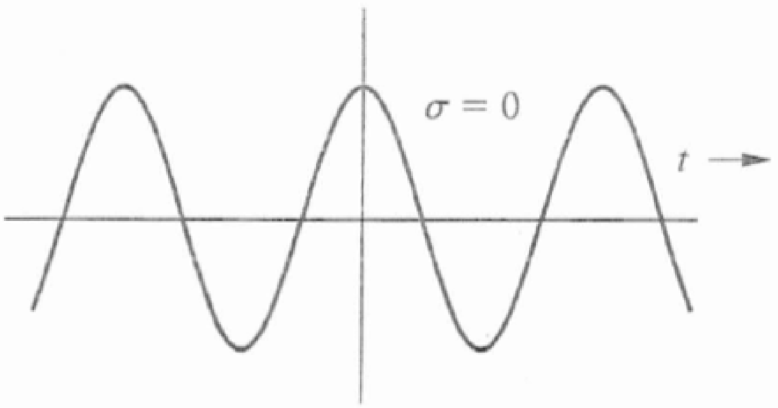
$$e^{s^*t} = e^{(\sigma - j\omega)t} = e^{\sigma t}\, e^{-j\omega t} = e^{\sigma t}(\cos\omega t - j\sin\omega t)$$

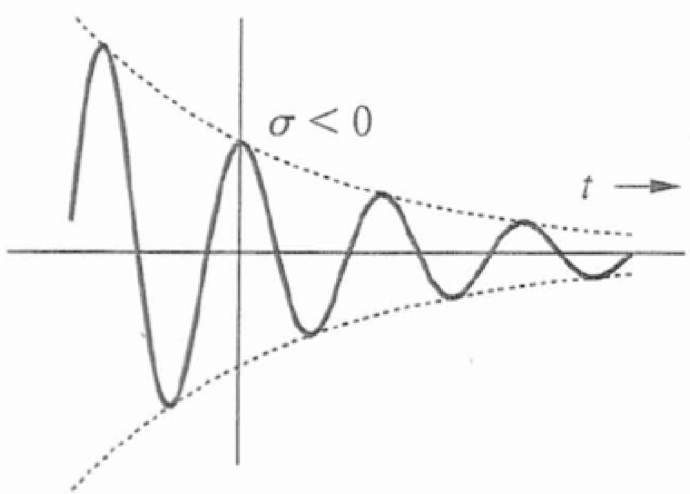
$$e^{\sigma t}\cos\omega t = \frac{1}{2}\left(e^{st} + e^{s^*t}\right)$$

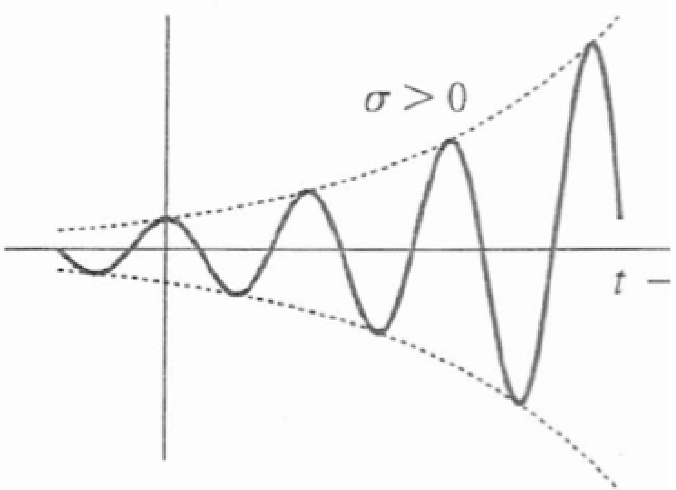
- The exponential function can be used to model a large class of signals.
  - A constant $k = ke^{0t}, s = 0$
  - A monotonic exponential $e^{\sigma t}, s = \sigma,\ \omega = 0$
  - A sinusoid $\cos\omega t$, $\sigma = 0, s = \pm j\omega$
  - An exponentially varying sinusoid $e^{\sigma t}\cos\omega t$, $s = \sigma \pm j\omega$

The Exponential Function e^{st} cont.









Discrete-Time Exponential γ^n

- A continuous-time exponential e^{st} can be expressed in alternate form as $e^{st} = \gamma^t$ with $\gamma = e^s$.
- Similarly for discrete time exponentials we have $e^{\lambda n} = \gamma^n$.
- When $\mathrm{Re}\{\lambda\} < 0$ then $|\gamma| < 1$ and the exponential decays.
- When $\mathrm{Re}\{\lambda\} > 0$ then $|\gamma| > 1$ and the exponential grows.
- When $\mathrm{Re}\{\lambda\} = 0$ then $|\gamma| = 1$. The exponential is of constant-amplitude and oscillates.

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Discrete-Time Exponential γ^n cont.

Mapping from λ to γ