Imperial College London

Signals and Systems

Lecture 10 Tuesday 21st November 2017

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Frequency response of a LTI system to an everlasting exponential

- We have seen that a LTI system's response to an everlasting exponential $x(t) = e^{st}$ is $H(s)e^{st}$. We represent such input-output pair as:

$$e^{st} \Rightarrow H(s)e^{st}$$

- Instead of using a complex frequency, we set $s = j\omega$. This yields:

$$e^{j\omega t} \Rightarrow H(j\omega)e^{j\omega t}$$

$$\cos(\omega t) = \mathrm{Re}\{e^{j\omega t}\} \Rightarrow \mathrm{Re}\{H(j\omega)e^{j\omega t}\}$$

- It is often better to express $H(j\omega)$ in polar form as:

$$H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)}$$

- Therefore,

$$\cos(\omega t) \Rightarrow |H(j\omega)|\cos[\,\omega t + \angle H(j\omega)]$$

  - $H(j\omega)$: Frequency response
  - $|H(j\omega)|$: Amplitude response
  - $\angle H(j\omega)$: Phase response

Frequency response of a LTI system to an everlasting exponential

- We can also show that a LTI system's response to an everlasting exponential $x(t) = e^{j\omega t+\theta}$ is $e^{j\omega t+\theta}H(j\omega)$.

  Proof:

  If $h(t)$ is the unit impulse response of a LTI system then:

$$y(t) = h(t) * e^{j\omega t+\theta} = \int_{-\infty}^{\infty} h(\tau)e^{j\omega(t-\tau)+\theta}d\tau = e^{j\omega t+\theta}\int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau$$

$$= e^{j\omega t+\theta}H(j\omega)$$

- Therefore,

$$\cos(\omega t + \theta) \Rightarrow |H(j\omega)|\cos[\,\omega t + \theta + \angle H(j\omega)]$$

  - $H(j\omega)$: Frequency response
  - $|H(j\omega)|$: Amplitude response
  - $\angle H(j\omega)$: Phase response

Frequency response example

- Find the frequency response (amplitude and phase response) of a system with transfer function:

$$H(s) = \frac{s + 0.1}{s + 5}$$

Then find the system's response $y(t)$ for inputs $x(t) = \cos 2t$ and $x(t) = \cos(10t - 50^{o})$.

- We substitute $s = j\omega$. Then, we obtain $H(j\omega) = \frac{j\omega+0.1}{j\omega+5}$.
  - Amplitude response: $|H(j\omega)| = \frac{\sqrt{\omega^2+0.01}}{\sqrt{\omega^2+25}}$.
  - Phase response: $\angle H(j\omega) = \Phi(\omega) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$.

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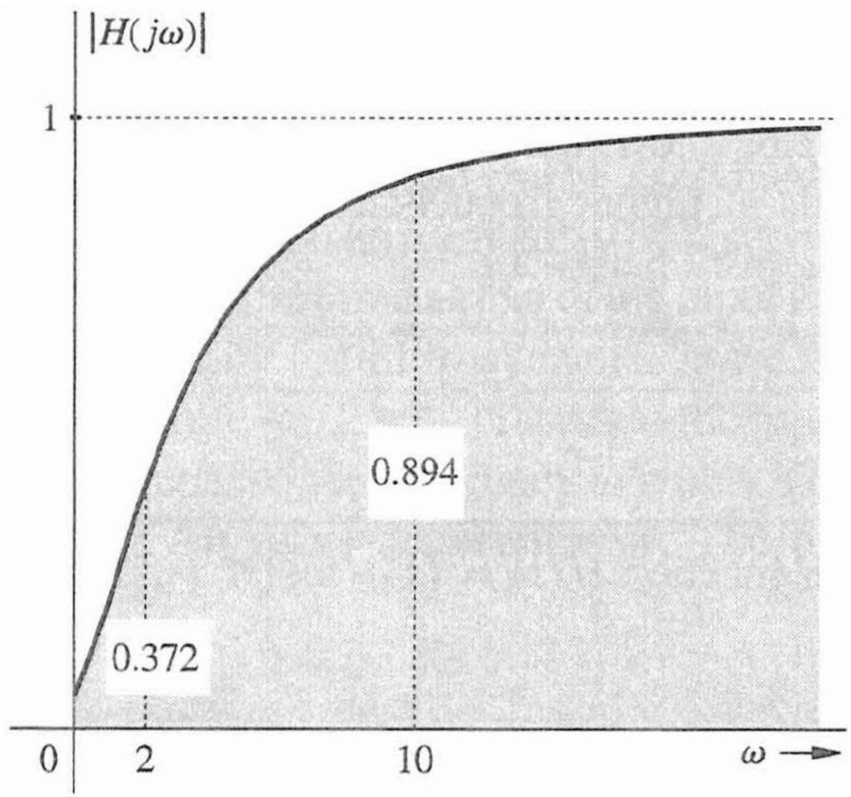
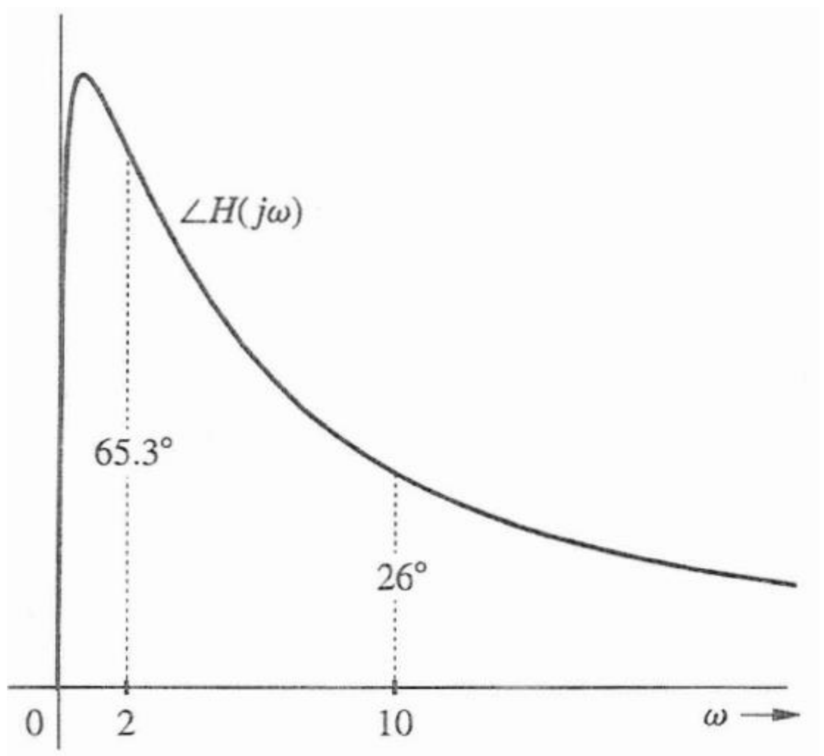
Frequency response example cont.

- Amplitude response: $|H(j\omega)| = \frac{\sqrt{\omega^2+0.01}}{\sqrt{\omega^2+25}}$.
- Phase response: $\angle H(j\omega) = \Phi(\omega) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$.

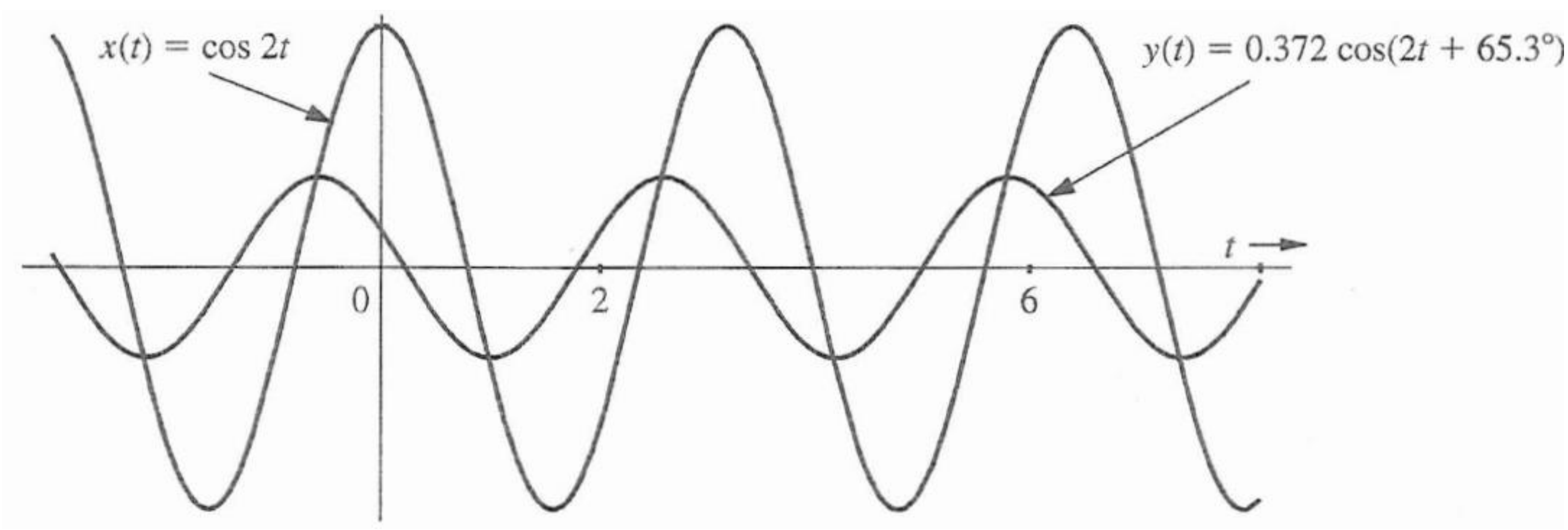




Frequency response example cont.

- For input $x(t) = \cos 2t$ we have:
  - Amplitude response: $|H(j2)| = \frac{\sqrt{2^2+0.01}}{\sqrt{2^2+25}} = 0.372.$
  - Phase response: $\angle H(j2) = \Phi(j2) = \tan^{-1}\left(\frac{2}{0.1}\right) - \tan^{-1}\left(\frac{2}{5}\right) = 65.3^{\text{o}}.$
- Therefore,

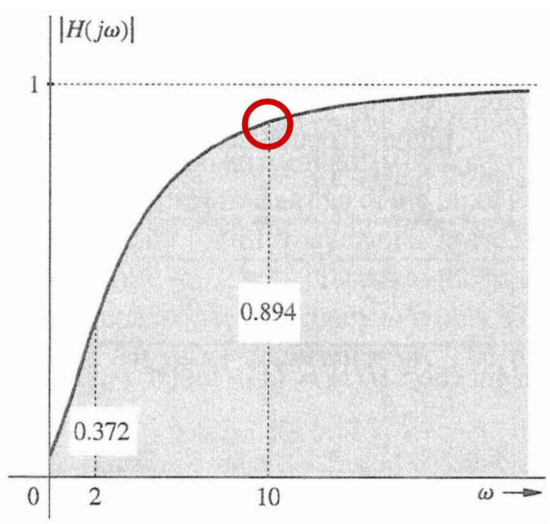
$$y(t) = 0.372\cos(2t + 65.3^{\text{o}})$$

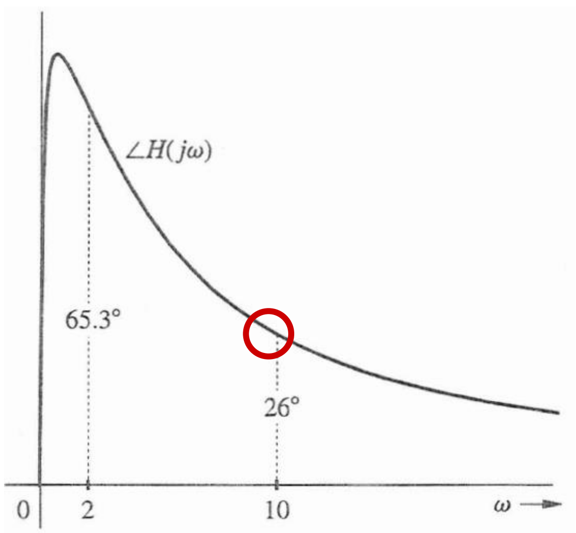


Frequency response example cont.

- For input $x(t) = \cos(10t - 50^o)$, instead of computing the values $|H(j\omega)|$ and $\angle H(j\omega)$ as previously, we shall read them directly from the frequency response plots corresponding to $\omega = 10$.
  - Amplitude response: $|H(j10)| = 0.894$.
  - Phase response: $\angle H(j10) = \Phi(j10) = 26^o$.
- Therefore,

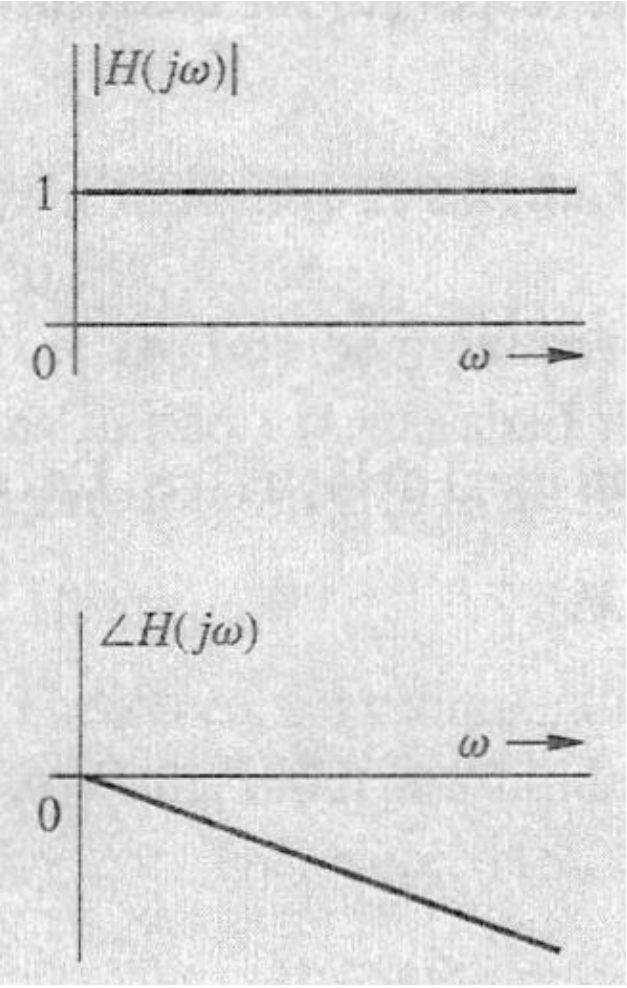
$$y(t) = 0.894\cos(10t - 50^o + 26^o)$$





Frequency response of a system that causes delay of T sec

- The transfer function of an ideal delay is $H(s) = e^{-sT}$ (proven previously).
- Therefore,
  - Amplitude response: $|H(j\omega)| = \left|e^{-j\omega T}\right| = 1.$
  - Phase response: $\angle H(j\omega) = \Phi(j\omega) = -\omega T.$
- Therefore:
  - Delaying a signal by T has no effect on its amplitude.
  - It introduces a linear phase shift with a gradient of $-T$.
  - The quantity $-\frac{d\Phi(\omega)}{d\omega} = \tau_g = T$ is known as <u>Group Delay</u>.



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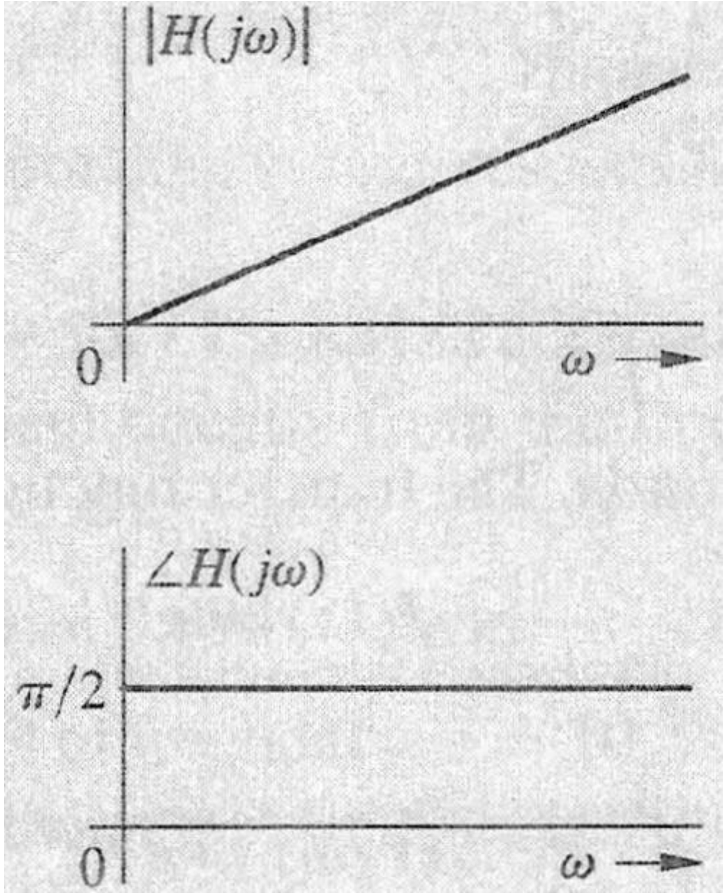
Frequency response of an ideal differentiator

- The transfer function of an ideal differentiator is $H(s) = s$.
- Therefore,
  - Frequency response: $H(j\omega) = j\omega$.
  - Amplitude response: $|H(j\omega)| = \omega$.
  - Phase response: $\angle H(j\omega) = \frac{\pi}{2}$.

  (Recall that $j = e^{j\frac{\pi}{2}}$)
- This agrees with:

$$\frac{d}{dt}(\cos\omega t) = -\omega\sin\omega t = \omega\cos(\omega t + \frac{\pi}{2})$$

- That is why differentiator is not a nice component to work with; it amplifies high frequency components (i.e., noise).

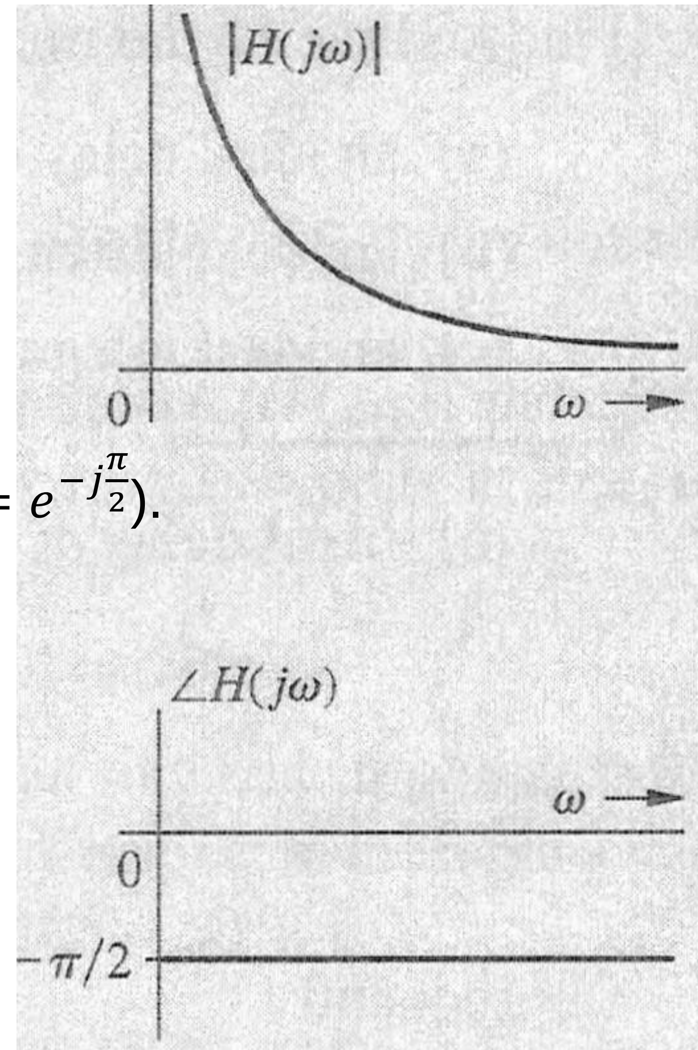


Frequency response of an ideal integrator

- The transfer function of an ideal integrator is $H(s) = \frac{1}{s}$.
- Therefore,
  - Frequency response: $H(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega}$.
  - Amplitude response: $|H(j\omega)| = \frac{1}{\omega}$.
  - Phase response: $\angle H(j\omega) = -\frac{\pi}{2}$ (since $-j = e^{-j\frac{\pi}{2}}$).
- This agrees with:

$$\int cos\omega t\, dt = \frac{1}{\omega} sin\omega t = \frac{1}{\omega}\cos(\omega t - \frac{\pi}{2})$$

- That is why an integrator is a nice component to work with; it supresses high frequency components (i.e., noise).



Bode Plots
Asymptotic behaviour of amplitude and phase response

- Consider a system with transfer function:

$$H(s) = \frac{K(s+a_1)(s+a_2)}{s(s+b_1)(s^2+b_2 s+b_3)} = \frac{K a_1 a_2}{b_1 b_3} \frac{(\frac{s}{a_1}+1)(\frac{s}{a_2}+1)}{s(\frac{s}{b_1}+1)(\frac{s^2}{b_3}+\frac{b_2}{b_3}s+1)}$$

- The **poles** are the roots of the denominator polynomial. In this case, the poles of the system are $s=0$, $s=-b_1$ and the solutions of the quadratic

$$s^2 + b_2 s + b_3 = 0$$

which we assume to form a complex conjugate pair.

- The **zeros** are the roots of the numerator polynomial. In this case, the zeros of the system are $s=-a_1$, $s=-a_2$.

Bode Plots
Asymptotic behaviour of amplitude and phase response

- Now let $s = j\omega$. The amplitude response $|H(j\omega)|$ can be rearranged as:

$$|H(j\omega)| = \frac{Ka_1a_2}{b_1b_3}\frac{\left|1+\frac{j\omega}{a_1}\right|\left|1+\frac{j\omega}{a_2}\right|}{|j\omega|\left|1+\frac{j\omega}{b_1}\right|\left|1+j\frac{b_2\omega}{b_3}+\frac{(j\omega)^2}{b_3}\right|}$$

- We express the above in decibel (i.e., $20\log(\cdot)$):

$$20\log|H(j\omega)| = 20\log\frac{Ka_1a_2}{b_1b_3} + 20\log\left|1+\frac{j\omega}{a_1}\right| + 20\log\left|1+\frac{j\omega}{a_2}\right|$$
$$-20\log|j\omega| - 20\log\left|1+\frac{j\omega}{b_1}\right| - 20\log\left|1+j\frac{b_2\omega}{b_3}+\frac{(j\omega)^2}{b_3}\right|$$

- By imposing a log operation the amplitude response (in dB) is broken into building block components that are added together.
- We have three types of building block terms: A term $j\omega$, a first order term $1+\frac{j\omega}{a}$ and a second order term with complex conjugate roots.

Advantages of logarithmic units

- They are desirable in several applications, where the variables considered have a very large range of values.
- The above is particularly true in frequency response amplitude plots since we require to plot values from 10^{-6} to 10^{6} or higher.
- A plot of such a large range on a linear scale will bury much of the useful information at lower frequencies.
- In humans the relationship between stimulus and perception is logarithmic.
  - This means that if a stimulus varies as a geometric progression (i.e., multiplied by a fixed factor), the corresponding perception is altered in an arithmetic progression (i.e., in additive constant amounts). For example, if a stimulus is tripled in strength (i.e., 3 x 1), the corresponding perception may be two times as strong as its original value (i.e., 1 + 1).
  - There is a theory behind the above observations developed by Weber and Frechner.

Bode plots – a pole at the origin: amplitude

- A pole at the origin gives rise to the amplitude term $-20\log|j\omega| = -20\log\omega$
  - This function can be plotted as function of ω.
  - We can effect further simplification by using the logarithmic function for the variable ω itself. Therefore, we define $u = \log\omega$.
- Therefore, $-20\log\omega = -20u$.
  - This is a straight line with a slope of -20.
  - A ratio of 10 in ω is called a <u>decade</u>. If $\omega_2 = 10\omega_1$ then $u_2 = \log\omega_2 = \log 10\omega_1 = \log 10 + \log\omega_1 = 1 + \log\omega_1 = 1 + u_1$.
  - A ratio of 2 in ω is called an <u>octave</u>. If $\omega_2 = 2\omega_1$ then $u_2 = \log\omega_2 = \log 2\omega_1 = \log 2 + \log\omega_1 = 0.301 + \log\omega_1 = 0.301 + u_1$
- Based on the above, equal increments in u are equivalent to equal ratios in ω.
- The amplitude plot has a slope of $-20d\text{B}/$ decade or $-20(0.301) = -6.02d\text{B}/$octave.
- The amplitude plot crosses the ω axis at $\omega = 1$, since $u = \log\omega = 0$ for $\omega = 1$.

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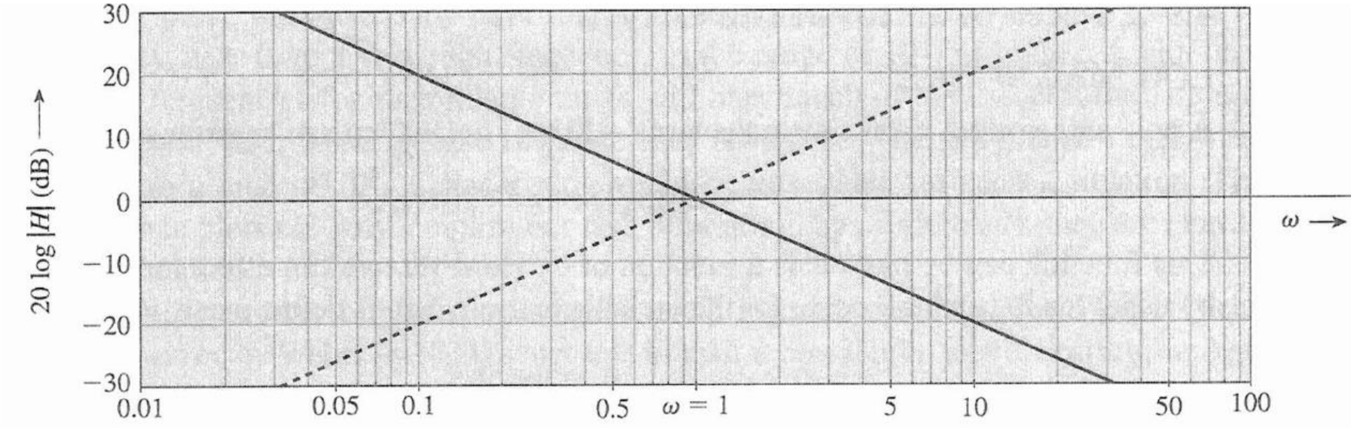
Bode plots – a zero at the origin: amplitude

- A zero at the origin gives rise to the term $20\log|j\omega| = 20\log\omega$.
- Therefore, $20\log\omega = 20u$.
- The amplitude plot has a slope of $20dB/$ decade or $20(0.301) = 6.02dB/$octave.
- The amplitude plot for a zero at the origin is a mirror image about the ω axis of the plot for a pole at the origin.



Amplitudes of a pole and a zero at the origin

Bode plots – first order pole: amplitude

- The log amplitude of a first order pole at $-a$ is $-20\log\left|1+\frac{j\omega}{a}\right|$.
  - $\omega \ll a \Rightarrow -20\log\left|1+\frac{j\omega}{a}\right| \approx -20\log 1 = 0$
  - $\omega \gg a \Rightarrow -20\log\left|1+\frac{j\omega}{a}\right| \approx -20\log(\frac{\omega}{a}) = -20\log\omega + 20\log a$

    This represents a straight line (when plotted as a function of u, the log of ω) with a slope of $-20dB$/decade or $-20(0.301) = -6.02dB$/octave.

    When $\omega = a$ the log amplitude is zero. Hence, this line crosses the ω axis at $\omega = a$. Note that the asymptotes meet at $\omega = a$.
- The exact log amplitude for this pole is:

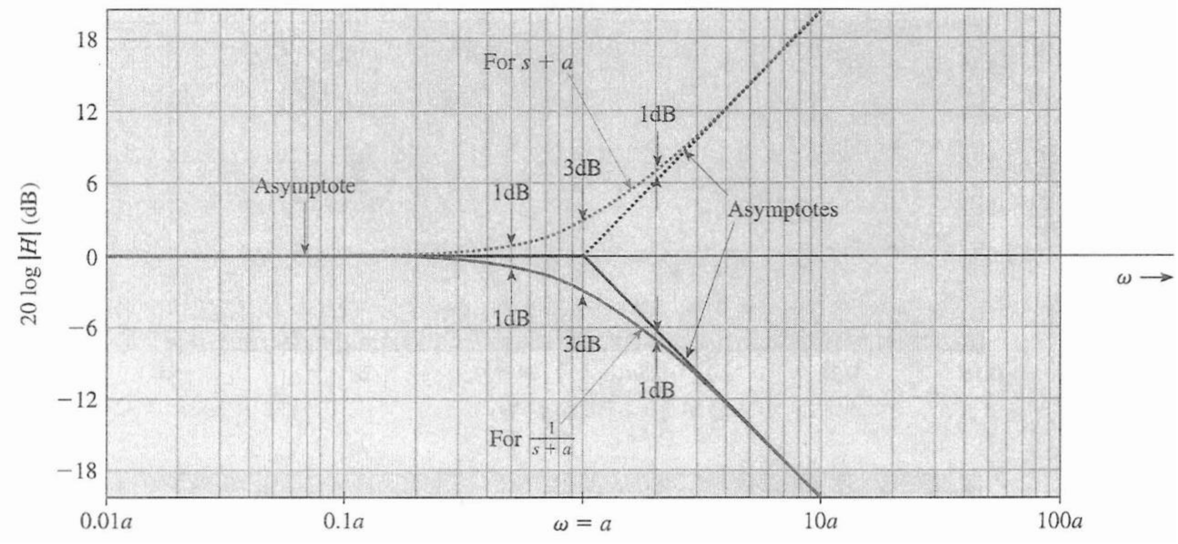
$$-20\log\left|1+\frac{j\omega}{a}\right| = -20\log\left(1+\frac{\omega^2}{a^2}\right)^{\frac{1}{2}} = -10\log\left(1+\frac{\omega^2}{a^2}\right)$$

- The maximum error between the actual and asymptotic plots occurs at $\omega = a$ and is $-3d$B. The frequency $\omega = a$ is called <u>corner frequency</u> or <u>break frequency</u>.

Bode plots – first order zero: amplitude

- A first order zero at $-a$ gives rise to the term $20\log\left|1+\frac{j\omega}{a}\right|$.
  - $\omega \ll a \Rightarrow 20\log\left|1+\frac{j\omega}{a}\right| \approx 20\log 1 = 0$.
  - $\omega \gg a \Rightarrow 20\log\left|1+\frac{j\omega}{a}\right| \approx 20\log(\frac{\omega}{a}) = 20\log\omega + 20\log a$.

    This represents a straight line with a slope of $20dB$/decade.
- The amplitude plot for a zero at $-a$ is a mirror image about the ω axis of the plot for a pole at $-a$.



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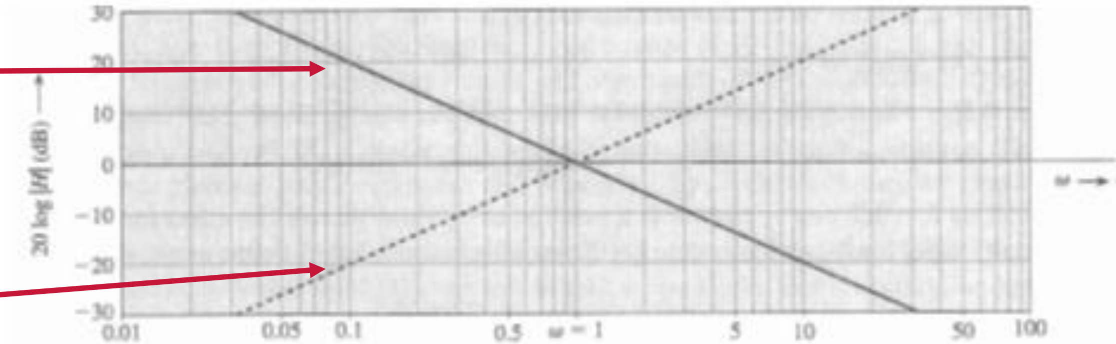
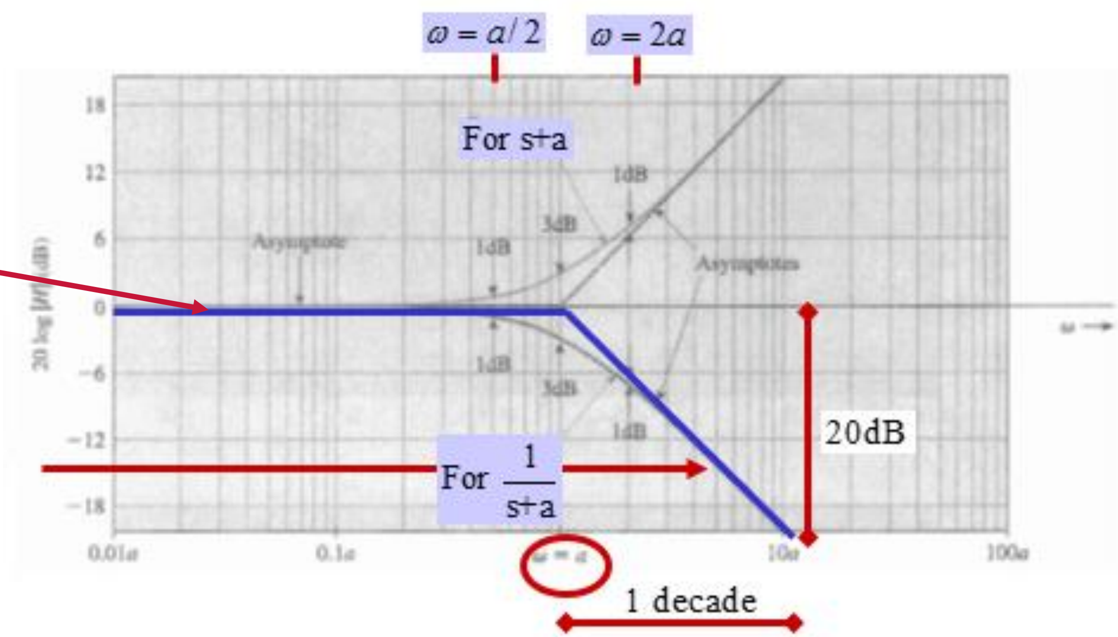
Summary of first order building blocks for Bode plots: amplitude

- Pole term: $-20\log|j\omega|$
  $= -20\log\omega$
- Zero term: $20\log|j\omega|$
  $= 20\log\omega$



- Pole term: $-20\log\left|1+\frac{j\omega}{a}\right|$
  - $\omega \ll a \Rightarrow -20\log\left|1+\frac{j\omega}{a}\right|$
    $\approx -20\log 1 = 0$
  - $\omega \gg a \Rightarrow -20\log\left|1+\frac{j\omega}{a}\right|$
    $\approx -20\log(\frac{\omega}{a})$
    $= -20\log\omega + 20\log a$
  - $\omega = a$
    $-20\log|1+j| \approx -20\log\sqrt{2} \approx -3db$



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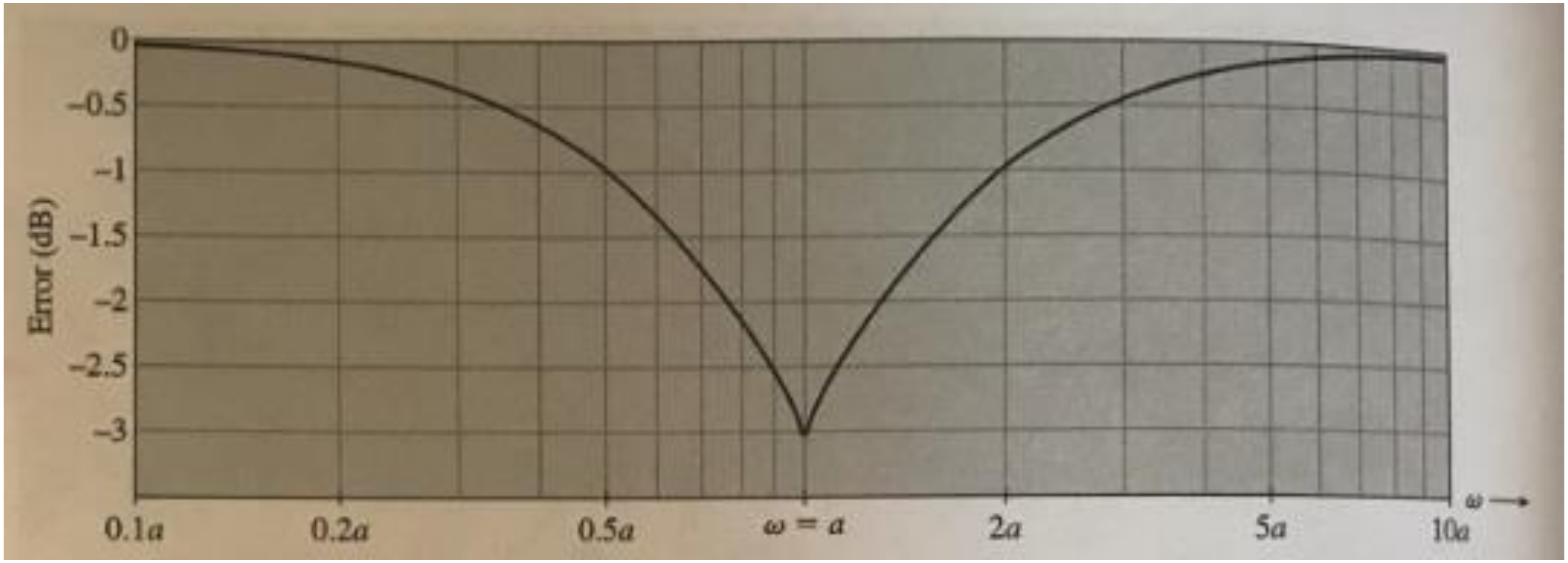
Error in the asymptotic approximation of amplitude due to a first order pole

- The error of the approximation as a function of ω is shown in the figure below.
- The actual plot can be obtained if we add the error to the asymptotic plot.
- **Problem:** Find the error when the frequency is equal to the corner frequency and 2, 5 and 10 times larger or smaller.

  (**Answers**: $-3dB, -1dB, -0.17dB,$ negligible)



Bode plots – second order pole : amplitude

- Now consider the quadratic term: $s^2 + b_2 s + b_3$.
- It is quite common to express the above term as: $s^2 + 2\zeta\omega_n s + \omega_n^2$.
  - The scalar ζ is called damping factor.
  - The scalar ω_n is called natural frequency.
- The log amplitude response is:

log amplitude $= -20\log\left|1 + 2j\zeta(\frac{\omega}{\omega_n}) + (\frac{j\omega}{\omega_n})^2\right|$

  - $\omega \ll \omega_n$, log amplitude $\approx -20\log 1 = 0$
  - $\omega \gg \omega_n$, log amplitude $\approx -20\log\left|-(\frac{\omega}{\omega_n})^2\right| = -40\log(\frac{\omega}{\omega_n})$

$= -40\log\omega - 40\log\omega_n = -40u - 40\log\omega_n$

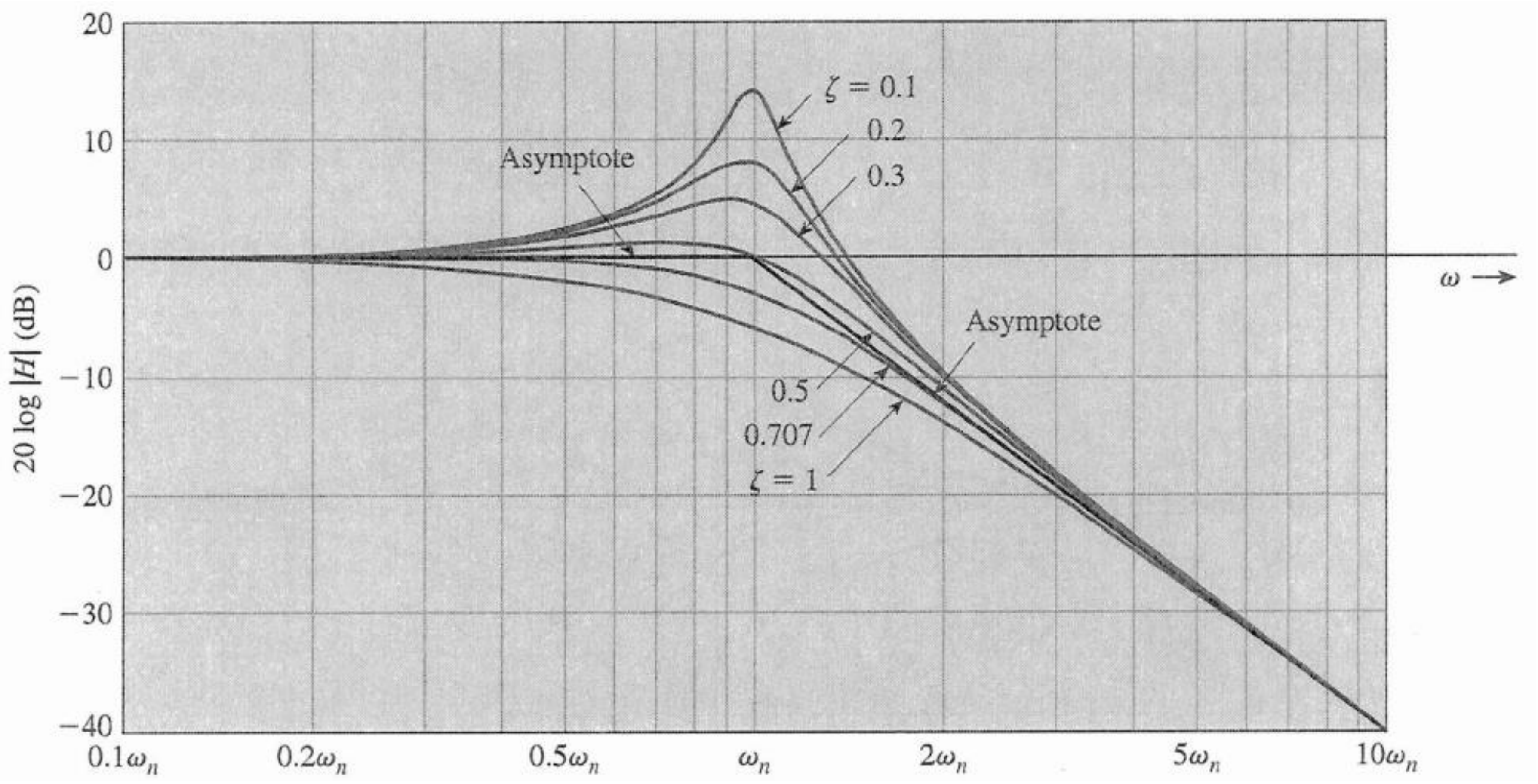
The exact log amplitude is $-20\log\left\{[1 - (\frac{\omega}{\omega_n})^2]^2 + 4\zeta^2(\frac{\omega}{\omega_n})^2\right\}^{1/2}$

Bode plots – second order pole : amplitude

- The log amplitude involves a parameter ζ, resulting in a different plot for each value of ζ.
- It can be proven that for complex-conjugate poles $\zeta < 1$.
- For $\zeta \geq 1$, the two poles in the second order factor are not longer complex but real, and each of these two real poles can be dealt with as a separate first order factor.
- The amplitude plot for a pair of complex conjugate zeros is a mirror image about the ω axis of the plot for a pair of complex conjugate poles.

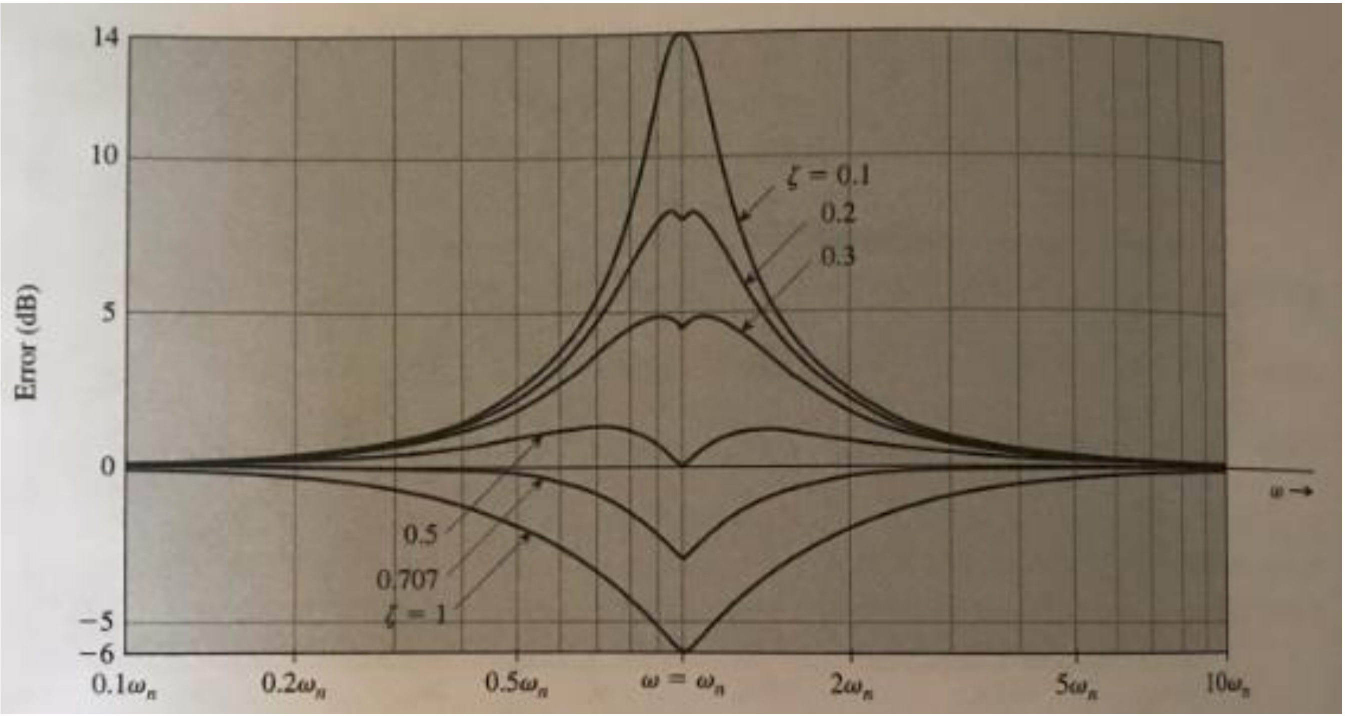
Bode plots – second order pole : amplitude

- The exact log amplitude is $= -20\log\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}^{1/2}$



Error in the asymptotic approximation of amplitude due to a pair of complex conjugate poles

- The error of the approximation as a function of ω is shown in the figure below for various values of ζs.
- The actual plot can be obtained if we add the error to the asymptotic plot.



Bode plots example: amplitude

- Consider a system with transfer function:

$$H(s) = \frac{20s(s+100)}{(s+2)(s+10)}$$

$$H(s) = \frac{20\times100}{2\times10}\,\frac{s(1+\frac{s}{100})}{(1+\frac{s}{2})(1+\frac{s}{10})} = 100\,\frac{s(1+\frac{s}{100})}{(1+\frac{s}{2})(1+\frac{s}{10})}$$

- Step 1: Establish where $x-$ axis crosses the $y-$ axis.
  - Since the constant term is $100 = 40dB$, x $-$axis cuts the vertical axis at 40 (i.e., relabel the horizontal axis as the $40dB$ line).
- Step 2: For each pole and zero term draw an asymptotic plot.
  - We need to draw straight lines for zero terms at origin and $\omega = -100$.
  - We need to draw straight lines for pole terms at $\omega = -2$ and $\omega = -10$.
- Step 3: Add all the asymptotes.
- Step 4: Apply corrections if possible.

Bode plots example: amplitude. Corrections.

- **Correction at $\boldsymbol{\omega} = 1$**
  - Due to corner frequency at $\omega = 2$ is $-1dB$.
  - Due to corner frequency at $\omega = 10$ is negligible.
  - Due to corner frequency at $\omega = 100$ is negligible.

  Total correction at $\omega = 1$ is $-1dB$.
- **Correction at $\boldsymbol{\omega} = 2$**
  - Due to corner frequency at $\omega = 2$ is $-3dB$.
  - Due to corner frequency at $\omega = 10$ is $-0.17dB$.
  - Due to corner frequency at $\omega = 100$ is negligible.

  Total correction at $\omega = 2$ is $-3.17dB$.
- **Correction at $\boldsymbol{\omega} = 10$**
  - Due to corner frequency at $\omega = 10$ is $-3dB$.
  - Due to corner frequency at $\omega = 2$ is $-0.17dB$.
  - Due to corner frequency at $\omega = 100$ is negligible.

  Total correction at $\omega = 10$ is $-3.17dB$.

Bode plots example: amplitude. Corrections cont.

- **Correction at $\boldsymbol{\omega = 100}$**
  - Due to corner frequency at $\omega = 100$ is $3dB$.
  - Due to corner frequency at $\omega = 2$ is negligible.
  - Due to corner frequency at $\omega = 10$ is negligible.

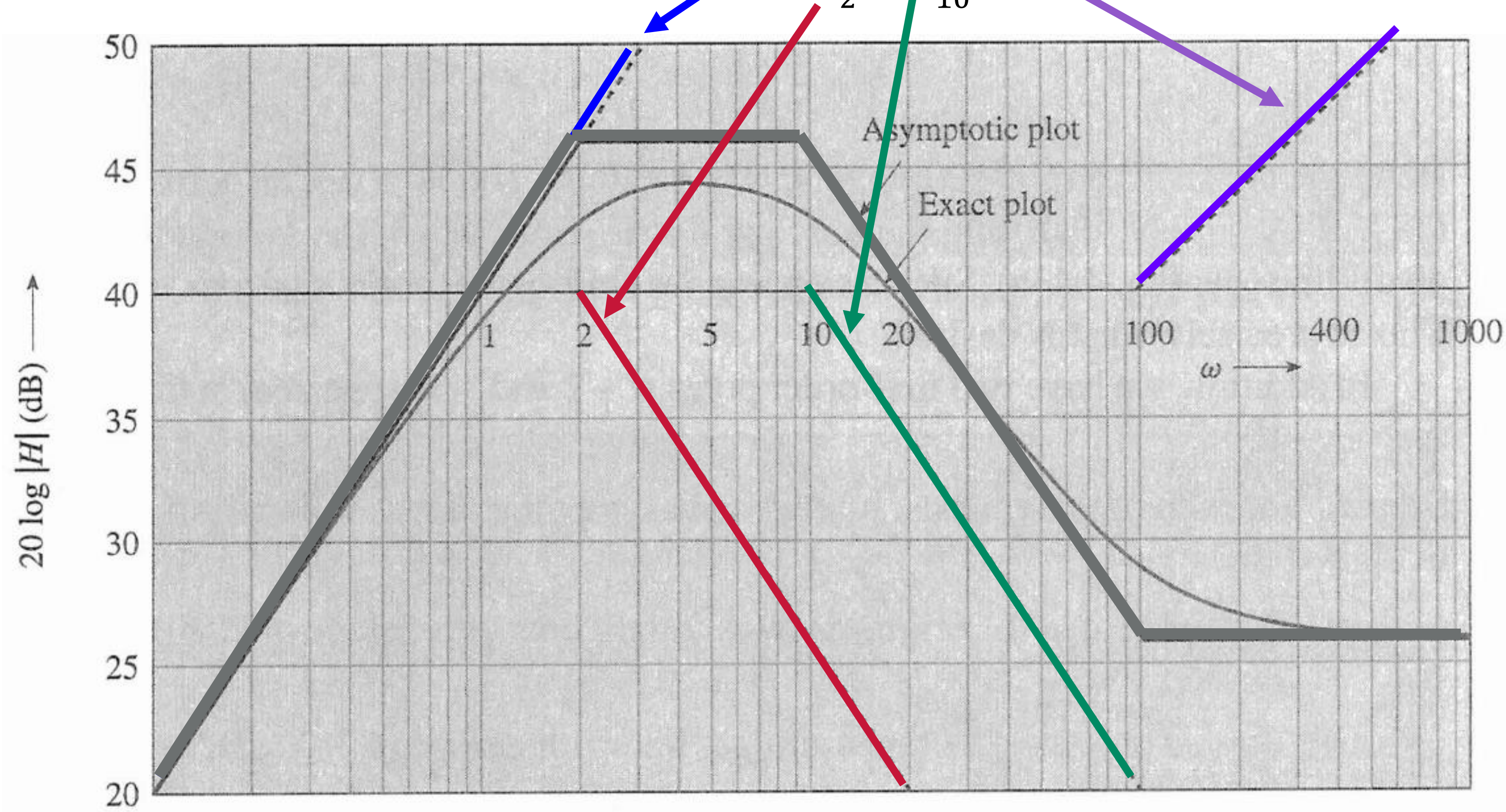
  Total correction at $\omega = 100$ is $3dB$.

- Correction at intermediate points other than corner frequencies may be considered for more accurate plots.

Bode plots example: total amplitude

- Observe now the final plot for the previous system with transfer function:

$$H(s) = 100\frac{s(1+\frac{s}{100})}{(1+\frac{s}{2})(1+\frac{s}{10})}$$



Bode plots: phase

- Now consider the phase response for the earlier transfer function:

$$H(j\omega) = \frac{Ka_1a_2}{b_1b_3}\frac{(1+\frac{j\omega}{a_1})(1+\frac{j\omega}{a_2})}{j\omega(1+\frac{j\omega}{b_1})(1+j\frac{b_2\omega}{b_3}+\frac{(j\omega)^2}{b_3})}$$

- The phase response is:

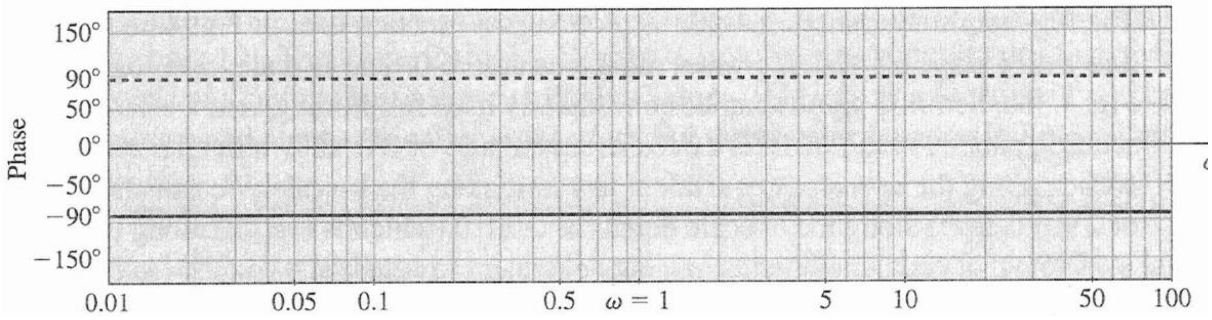
$$\angle H(j\omega) = \angle\left(1+\frac{j\omega}{a_1}\right) + \angle\left(1+\frac{j\omega}{a_2}\right) - \angle j\omega$$
$$-\angle(1+\frac{j\omega}{b_1}) - \angle(1+j\frac{b_2\omega}{b_3}+\frac{(j\omega)^2}{b_3})$$

- Again, we have three types of terms.

Bode plots – a pole or zero at the origin: phase

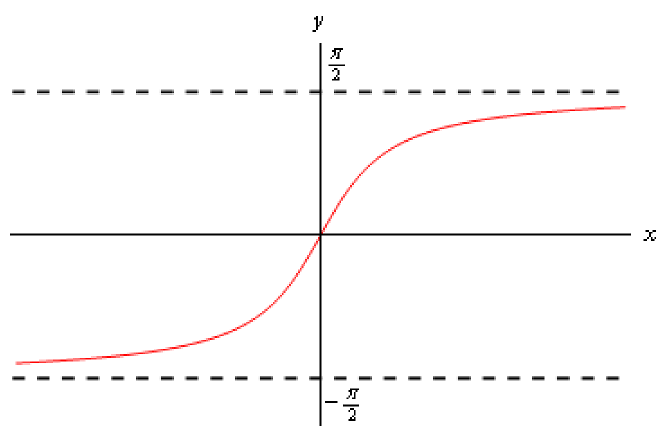
- A pole at the origin gives rise to the term $-j\omega$.
  - $\angle H(j\omega) = -\angle j\omega = -90^{o}$. The phase is therefore, constant for all values of ω.

- A zero at the origin gives rise to the term $j\omega$.
  - $\angle H(j\omega) = \angle j\omega = 90^{o}$. The phase plot for a zero at the origin is a mirror image about the ω axis of the phase plot for a pole at the origin.



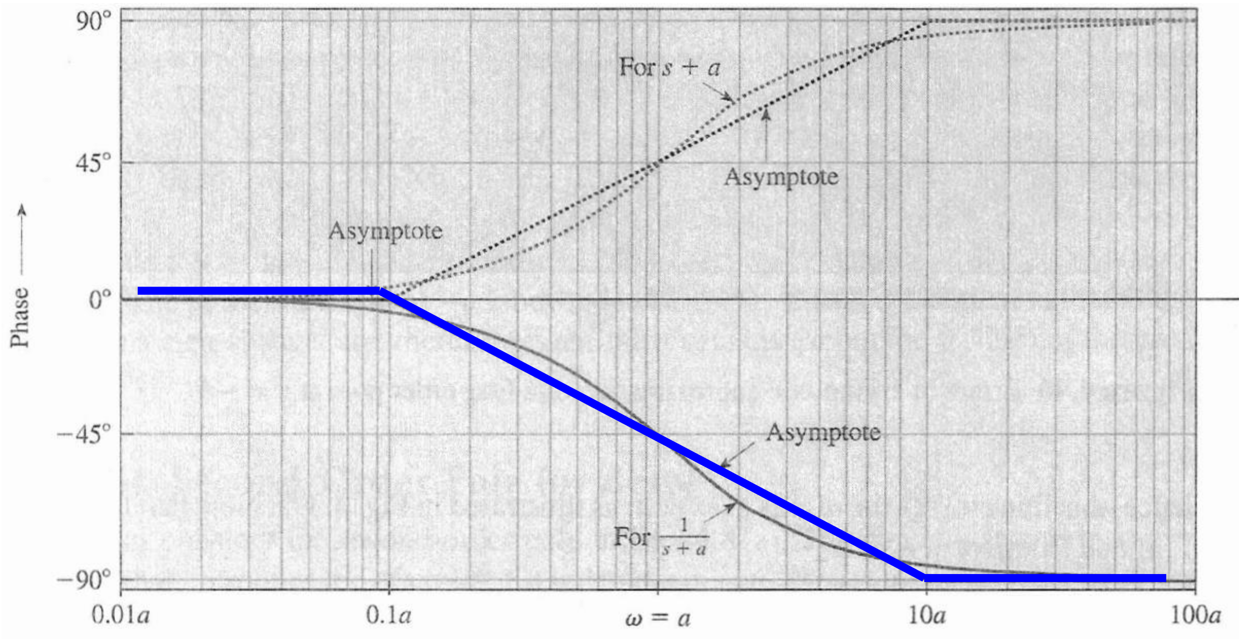
Bode plots – a first order pole or zero: phase

- A pole at $-a$ gives rise to the term $1+\frac{j\omega}{a}$.
  - $\angle H(j\omega) = -\angle\left(1+\frac{j\omega}{a}\right) = -\tan^{-1}\left(\frac{\omega}{a}\right)$.
  - $\omega \ll a \Rightarrow -\tan^{-1}\left(\frac{\omega}{a}\right) \approx 0$
  - $\omega \gg a \Rightarrow -\tan^{-1}\left(\frac{\omega}{a}\right) \approx -90^{o}$
- The phase plot for a zero at $-a$ is a mirror image about the ω axis of the phase plot for a pole at the origin.



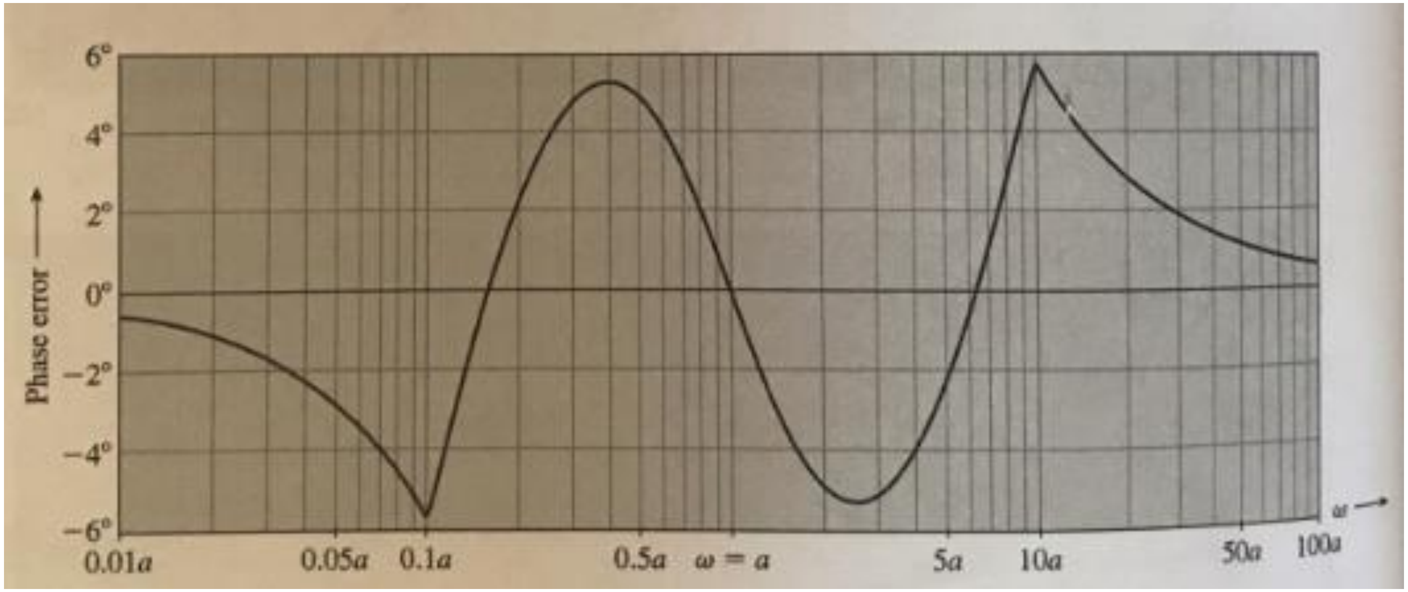
Bode plots – a first order pole or zero: phase

- We use a three-line segment asymptotic plot for greater accuracy. The asymptotes are:
  - $\omega \leq a/10 \Rightarrow 0^o$
  - $\omega \geq 10a \Rightarrow -90^o$
  - A straight line with slope -45^o /decade connects the above two asymptotes (from $\omega = a/10$ to $\omega = 10a$) crossing the ω axis at $\omega = a/10$.



Bode plots – a first order pole or zero: phase error

- The asymptotes are very close to the real curve and the maximum error is 5.7°.
- The actual phase can be obtained if we add the error to the asymptotic plot.



Bode plots – second order complex conjugate poles : phase

- Now consider the term:

$$1 + 2j\zeta\left(\frac{\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2 = 1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\ 2\zeta(\frac{\omega}{\omega_n})$$

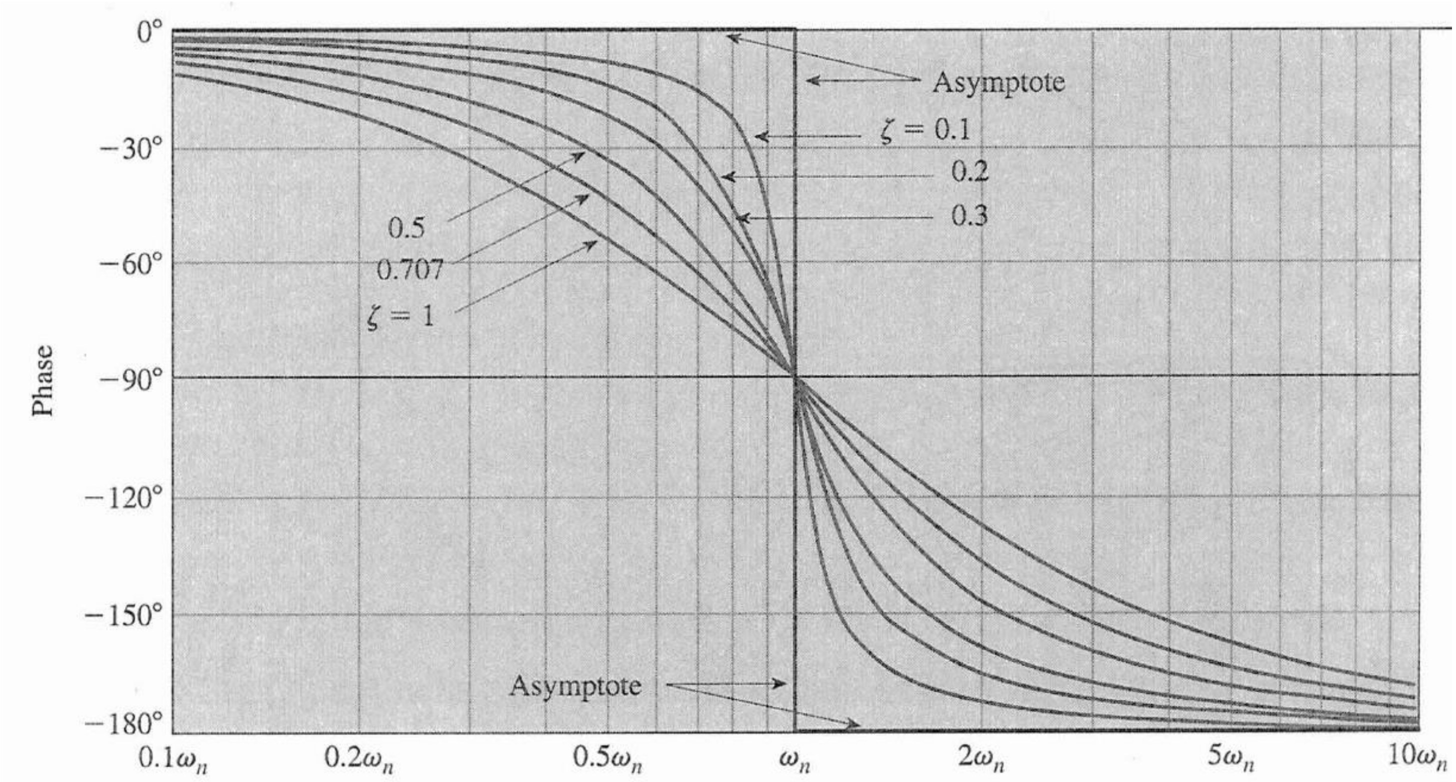
$$\angle H(j\omega) = -\tan^{-1}\left[\frac{2\zeta(\frac{\omega}{\omega_n})}{1-(\frac{\omega}{\omega_n})^2}\right]$$

  - $\omega \ll \omega_n,\ \angle H(j\omega) \approx -\tan^{-1}0 \approx 0$
  - $\omega \gg \omega_n,\ \angle H(j\omega) \approx -\tan^{-1}0 \approx -180^{\mathrm{o}}$

- The phase involves a parameter ζ, resulting in a different plot for each value of ζ.

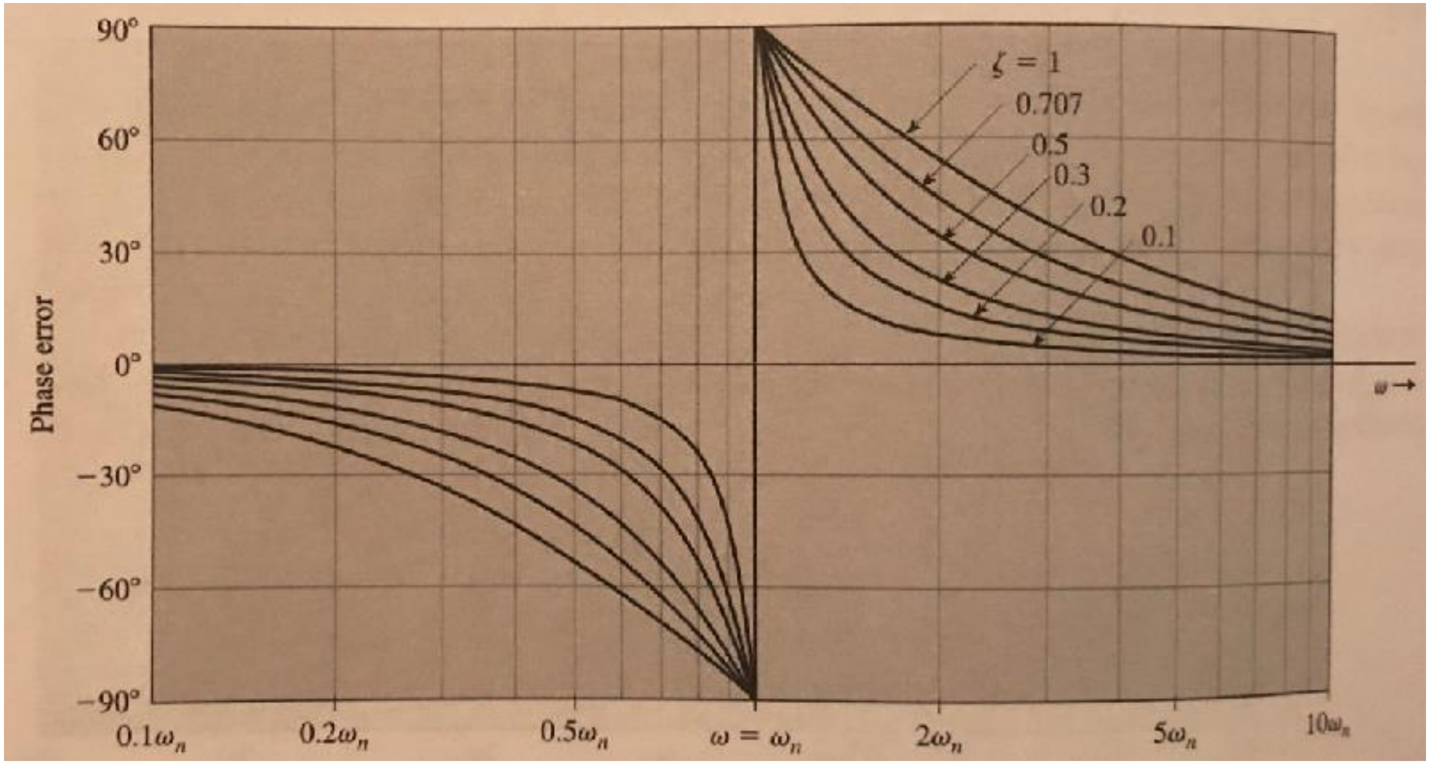
Bode plots – second order complex conjugate poles : phase

- A convenient asymptote for the phase of complex conjugate poles is a step function that is 0^o for $\omega < \omega_n$ and -180^o for $\omega > \omega_n$.
- For complex conjugate zeros, the amplitude and phase plots are mirror images of those for complex conjugate plots.



Bode plots – second order complex conjugate poles : phase error

- An error plot is shown in the figure below for various values of ζ.
- The actual phase can be obtained if we add the error to the asymptotic plot.



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Bode plots example: phase

- Consider the previous system with transfer function:

$$H(s) = \frac{20s(s+100)}{(s+2)(s+10)} = 100\,\frac{s(1+\frac{s}{100})}{(1+\frac{s}{2})(1+\frac{s}{10})}$$

- For the pole at $s = -2$ ($a = -2$) the phase plot is:
  - $\omega \leq \frac{2}{10} = 0.2 \Rightarrow 0^{o}$
  - $\omega \geq 10 \cdot 2 = 20 \Rightarrow -90^{o}$
  - A straight line with slope -45^{o} /decade connects the above two asymptotes (from $\omega = 0.2$ to $\omega = 20$) crossing the ω axis at $\omega = 0.2$.
- For the pole at $s = -10$ ($a = -10$) the phase plot is:
  - $\omega \leq \frac{10}{10} = 1 \Rightarrow 0^{o}$
  - $\omega \geq 10 \cdot 10 = 100 \Rightarrow -90^{o}$
  - A straight line with slope -45^{o} /decade connects the above two asymptotes (from $\omega = 1$ to $\omega = 100$) crossing the ω axis at $\omega = 1$.

Bode plots example: phase cont.

- Consider the previous system with transfer function:

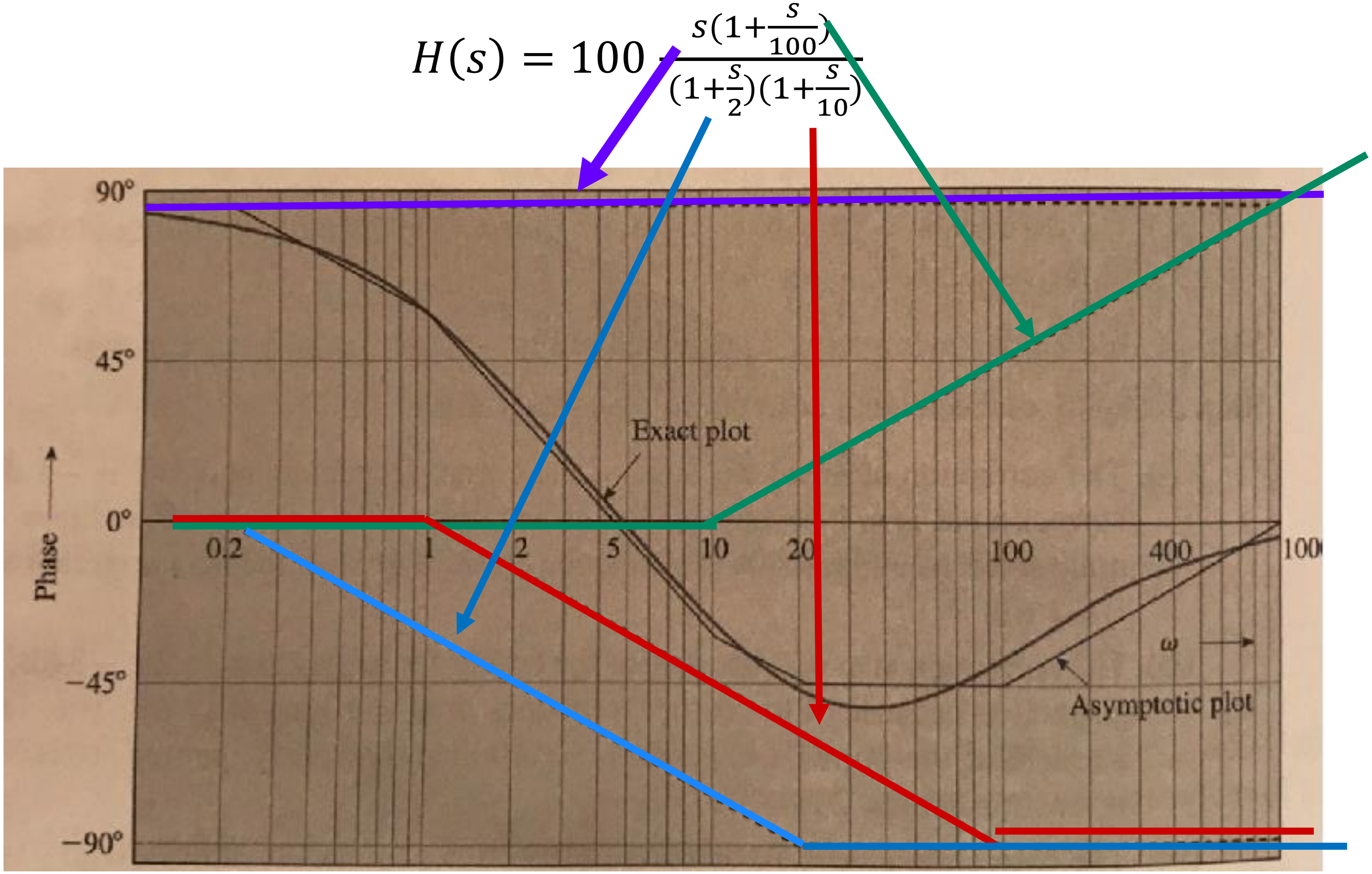
$$H(s) = \frac{20s(s+100)}{(s+2)(s+10)} = 100\,\frac{s(1+\frac{s}{100})}{(1+\frac{s}{2})(1+\frac{s}{10})}$$

- The zero at the origin causes a 90^{o} phase shift.
- For the zero at $s = -100$ ($a = -100$) the phase plot is:
  - $\omega \leq \frac{100}{10} = 10 \Rightarrow 0^{o}$
  - $\omega \geq 10 \cdot 100 = 1000 \Rightarrow 90^{o}$
  - A straight line with slope 45^{o}/decade connects the above two asymptotes (from $\omega = 10$ to $\omega = 1000$) crossing the ω axis at $\omega = 10$.

Bode plots example: total phase cont.

- Consider the previous system with transfer function:

$$H(s) = 100\,\frac{s\left(1+\frac{s}{100}\right)}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$



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Relating this lecture to other courses

- You will be applying frequency response in various areas such as filters and 2nd year control. You have also used frequency response in the 2nd year analogue electronics course. Here we explore this as a special case of the general concept of complex frequency, where the real part is zero.

- You have come across Bode plots from 2nd year analogue electronics course. Here we go deeper into where all these rules come from.

- We will apply much of what we have done so far in the frequency domain to analyse and design some filters in the next lecture.