

# Laplace transform Pairs (1)

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$

# Laplace transform Pairs (2)

No.	$x(t)$	$X(s)$
5	$e^{\lambda t} u(t)$	$\frac{1}{s - \lambda}$
6	$t e^{\lambda t} u(t)$	$\frac{1}{(s - \lambda)^2}$
7	$t^n e^{\lambda t} u(t)$	$\frac{n!}{(s - \lambda)^{n+1}}$
8a	$\cos bt u(t)$	$\frac{s}{s^2 + b^2}$
8b	$\sin bt u(t)$	$\frac{b}{s^2 + b^2}$
9a	$e^{-at} \cos bt u(t)$	$\frac{s + a}{(s + a)^2 + b^2}$
9b	$e^{-at} \sin bt u(t)$	$\frac{b}{(s + a)^2 + b^2}$

# Laplace transform Pairs (3)

No.	$x(t)$	$X(s)$
10a	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{(r \cos \theta)s + (ar \cos \theta - br \sin \theta)}{s^2 + 2as + (a^2 + b^2)}$
10b	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{0.5re^{j\theta}}{s + a - jb} + \frac{0.5re^{-j\theta}}{s + a + jb}$
10c	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{As + B}{s^2 + 2as + c}$
	$r = \sqrt{\frac{A^2c + B^2 - 2ABa}{c - a^2}}$ $\theta = \tan^{-1} \left( \frac{Aa - B}{A\sqrt{c - a^2}} \right)$ $b = \sqrt{c - a^2}$	
10d	$e^{-at} \left[ A \cos bt + \frac{B - Aa}{b} \sin bt \right] u(t)$	$\frac{As + B}{s^2 + 2as + c}$
	$b = \sqrt{c - a^2}$	

# Summary of Laplace Transform Properties (1)

Operation	$x(t)$	$X(s)$
Addition	$x_1(t) + x_2(t)$	$X_1(s) + X_2(s)$
Scalar multiplication	$kx(t)$	$kX(s)$
Time differentiation	$\frac{dx}{dt}$	$sX(s) - x(0^-)$
	$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
	$\frac{d^3x}{dt^3}$	$s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$
	$\frac{d^n x}{dt^n}$	$s^n X(s) - \sum_{k=1}^n s^{n-k} x^{(k-1)}(0^-)$
Time integration	$\int_{0^-}^t x(\tau) d\tau$	$\frac{1}{s} X(s)$
	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{s} X(s) + \frac{1}{s} \int_{-\infty}^{0^-} x(t) dt$

# Summary of Laplace Transform Properties (2)

Operation	$x(t)$	$X(s)$
Time shifting	$x(t - t_0)u(t - t_0)$	$X(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shifting	$x(t)e^{s_0 t}$	$X(s - s_0)$
Frequency differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$
Frequency integration	$\frac{x(t)}{t}$	$\int_s^\infty X(z) dz$
Scaling	$x(at), a \geq 0$	$\frac{1}{a} X\left(\frac{s}{a}\right)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi j} X_1(s) * X_2(s)$
Initial value	$x(0^+)$	$\lim_{s \rightarrow \infty} sX(s) \quad (n > m)$
Final value	$x(\infty)$	$\lim_{s \rightarrow 0} sX(s) \quad [\text{poles of } sX(s) \text{ in LHP}]$