

▷ **2: Three Different
Fourier Transforms**

Fourier Transforms

**Convergence of
DTFT**

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

MATLAB routines

2: Three Different Fourier Transforms

Fourier Transforms

2: Three Different Fourier Transforms

▷ Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

MATLAB routines

Three different Fourier Transforms:

- CTFT (Continuous-Time Fourier Transform): $x(t) \rightarrow X(j\Omega)$
- DTFT (Discrete-Time Fourier Transform): $x[n] \rightarrow X(e^{j\omega})$
- DFT a.k.a. FFT (Discrete Fourier Transform): $x[n] \rightarrow X[k]$

Forward Transform

Inverse Transform

CTFT	$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} dt$	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t} d\Omega$
DTFT	$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$	$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$
DFT	$X[k] = \sum_0^{N-1} x[n]e^{-j2\pi \frac{kn}{N}}$	$x[n] = \frac{1}{N} \sum_0^{N-1} X[k]e^{j2\pi \frac{kn}{N}}$

We use Ω for “real” and $\omega = \Omega T$ for “normalized” angular frequency.

Nyquist frequency is at $\Omega_{\text{Nyq}} = 2\pi \frac{f_s}{2} = \frac{\pi}{T}$ and $\omega_{\text{Nyq}} = \pi$.

For “power signals” (energy \propto duration), CTFT & DTFT are unbounded.

Fix this by normalizing:

$$X(j\Omega) = \lim_{A \rightarrow \infty} \frac{1}{2A} \int_{-A}^A x(t)e^{-j\Omega t} dt$$

$$X(e^{j\omega}) = \lim_{A \rightarrow \infty} \frac{1}{2A+1} \sum_{-A}^A x[n]e^{-j\omega n}$$

Convergence of DTFT

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of ▷ DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

MATLAB routines

DTFT: $X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$ does not converge for all $x[n]$.

Consider the finite sum: $X_K(e^{j\omega}) = \sum_{-K}^K x[n]e^{-j\omega n}$

Strong Convergence:

$x[n]$ absolutely summable $\Rightarrow X(e^{j\omega})$ converges **uniformly**

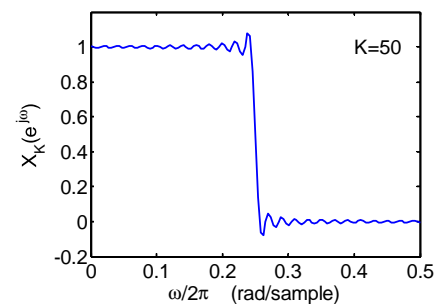
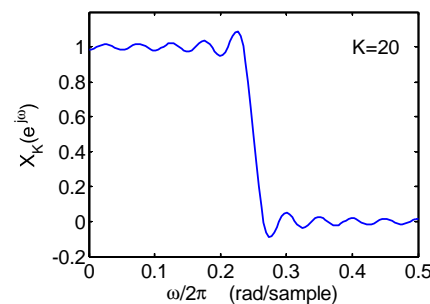
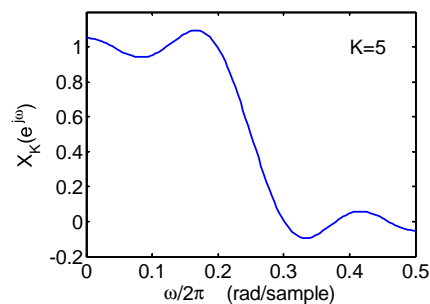
$$\sum_{-\infty}^{\infty} |x[n]| < \infty \Rightarrow \sup_{\omega} |X(e^{j\omega}) - X_K(e^{j\omega})| \xrightarrow{K \rightarrow \infty} 0$$

Weaker convergence:

$x[n]$ finite energy $\Rightarrow X(e^{j\omega})$ converges in **mean square**

$$\sum_{-\infty}^{\infty} |x[n]|^2 < \infty \Rightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega}) - X_K(e^{j\omega})|^2 d\omega \xrightarrow{K \rightarrow \infty} 0$$

Example: $x[n] = \frac{\sin 0.5\pi n}{\pi n}$



Gibbs phenomenon:

Converges at each ω as $K \rightarrow \infty$ but peak error does not get smaller.

[DTFT Convergence Proofs]

(1) Strong Convergence:

[these proofs are not examinable]

We are given that $\sum_{-\infty}^{\infty} |x[n]| < \infty \Rightarrow \forall \epsilon > 0, \exists N$ such that $\sum_{|n|>N} |x[n]| < \epsilon$

$$\begin{aligned} \text{For } K \geq N, \sup_{\omega} |X(e^{j\omega}) - X_K(e^{j\omega})| &= \sup_{\omega} \left| \sum_{|n|>K} x[n]e^{-j\omega n} \right| \\ &\leq \sup_{\omega} \left(\sum_{|n|>K} |x[n]e^{-j\omega n}| \right) = \sum_{|n|>K} |x[n]| < \epsilon \end{aligned}$$

(2) Weak Convergence:

We are given that $\sum_{-\infty}^{\infty} |x[n]|^2 < \infty \Rightarrow \forall \epsilon > 0, \exists N$ such that $\sum_{|n|>N} |x[n]|^2 < \epsilon$

Define $y^{[K]}[n] = \begin{cases} 0 & |n| \leq K \\ x[n] & |n| > K \end{cases}$ so that its DTFT is, $Y^{[K]}(e^{j\omega}) = \sum_{-\infty}^{\infty} y^{[K]}[n]e^{-j\omega n}$

$$\begin{aligned} \text{We see that } X(e^{j\omega}) - X_K(e^{j\omega}) &= \sum_{-\infty}^{\infty} x[n]e^{-j\omega n} - \sum_{-K}^K x[n]e^{-j\omega n} \\ &= \sum_{|n|>K} x[n]e^{-j\omega n} = \sum_{-\infty}^{\infty} y^{[K]}[n]e^{-j\omega n} = Y^{[K]}(e^{j\omega}) \end{aligned}$$

$$\begin{aligned} \text{From Parseval's theorem, } \sum_{-\infty}^{\infty} |y^{[K]}[n]|^2 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |Y^{[K]}(e^{j\omega})|^2 d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega}) - X_K(e^{j\omega})|^2 d\omega \end{aligned}$$

$$\text{Hence for } K \geq N, \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega}) - X_K(e^{j\omega})|^2 d\omega = \sum_{-\infty}^{\infty} |y^{[K]}[n]|^2 = \sum_{|n|>N} |x[n]|^2 < \epsilon$$

DTFT Properties

- 2: Three Different Fourier Transforms
- Fourier Transforms
- Convergence of DTFT
- ▷ DTFT Properties
- DFT Properties
- Symmetries
- Parseval's Theorem
- Convolution
- Sampling Process
- Zero-Padding
- Phase Unwrapping
- Uncertainty principle
- Summary
- MATLAB routines

DTFT: $X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$

- **DTFT** is **periodic** in ω : $X(e^{j(\omega+2m\pi)}) = X(e^{j\omega})$ for integer m .

- **DTFT** is the **z -Transform** evaluated at the point $e^{j\omega}$:

$$X(z) = \sum_{-\infty}^{\infty} x[n]z^{-n}$$

DTFT converges iff the ROC includes $|z| = 1$.

- **DTFT** is the same as the CTFT of a signal comprising **impulses at the sample times** (Dirac δ functions) of appropriate heights:

$$x_{\delta}(t) = \sum x[n]\delta(t - nT) = x(t) \times \sum_{-\infty}^{\infty} \delta(t - nT)$$

Equivalent to multiplying a continuous $x(t)$ by an impulse train.

Proof: $X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$

$$\sum_{n=-\infty}^{\infty} x[n] \int_{-\infty}^{\infty} \delta(t - nT) e^{-j\omega \frac{t}{T}} dt$$

$$\stackrel{(i)}{=} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT) e^{-j\omega \frac{t}{T}} dt$$

$$\stackrel{(ii)}{=} \int_{-\infty}^{\infty} x_{\delta}(t) e^{-j\Omega t} dt$$

(i) OK if $\sum_{-\infty}^{\infty} |x[n]| < \infty$. (ii) use $\omega = \Omega T$.

DFT Properties

- 2: Three Different Fourier Transforms
- Fourier Transforms
- Convergence of DTFT
- DTFT Properties
- ▷ DFT Properties
- Symmetries
- Parseval's Theorem
- Convolution
- Sampling Process
- Zero-Padding
- Phase Unwrapping
- Uncertainty principle
- Summary
- MATLAB routines

$$\text{DFT: } X[k] = \sum_0^{N-1} x[n]e^{-j2\pi \frac{kn}{N}}$$

$$\text{DTFT: } X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

Case 1: $x[n] = 0$ for $n \notin [0, N - 1]$

DFT is the same as DTFT at $\omega_k = \frac{2\pi}{N}k$.

The $\{\omega_k\}$ are uniformly spaced from $\omega = 0$ to $\omega = 2\pi \frac{N-1}{N}$.

DFT is the z -Transform evaluated at N equally spaced points around the unit circle beginning at $z = 1$.

Case 2: $x[n]$ is periodic with period N

DFT equals the normalized DTFT

$$X[k] = \lim_{K \rightarrow \infty} \frac{N}{2K+1} \times X_K(e^{j\omega_k})$$

$$\text{where } X_K(e^{j\omega}) = \sum_{-K}^K x[n]e^{-j\omega n}$$

[Proof of Case 2]

We want to show that if $x[n] = x[n + N]$ (i.e. $x[n]$ is periodic with period N) then

$$\lim_{K \rightarrow \infty} \frac{N}{2K+1} \times X_K(e^{j\omega_k}) \triangleq \lim_{K \rightarrow \infty} \frac{N}{2K+1} \times \sum_{-K}^K x[n]e^{-j\omega_k n} = X[k]$$

where $\omega_k = \frac{2\pi}{N}k$. We assume that $x[n]$ is bounded with $|x[n]| < B$.

We first note that the summand is periodic:

$$x[n + N]e^{-j\omega_k(n+N)} = x[n]e^{-j\omega_k n}e^{-jk\frac{2\pi}{N}N} = x[n]e^{-j\omega_k n}e^{-j2\pi k} = x[n]e^{-j\omega_k n}.$$

We now define M and R so that $2K + 1 = MN + R$ where $0 \leq R < N$ (i.e. MN is the largest multiple of N that is $\leq 2K + 1$). We can now write

$$\frac{N}{2K+1} \times \sum_{-K}^K x[n]e^{-j\omega_k n} = \frac{N}{MN+R} \times \sum_{-K}^{K-R} x[n]e^{-j\omega_k n} + \frac{N}{MN+R} \times \sum_{K-R+1}^K x[n]e^{-j\omega_k n}$$

The first sum contains MN consecutive terms of a periodic summand and so equals M times the sum over one period. The second sum contains R bounded terms and so its magnitude is $< RB < NB$.

$$\text{So } \frac{N}{2K+1} \times \sum_{-K}^K x[n]e^{-j\omega_k n} = \frac{MN}{MN+R} \times \sum_0^{N-1} x[n]e^{-j\omega_k n} + P = \frac{1}{1+\frac{R}{MN}} \times X[k] + P$$

$$\text{where } |P| < \frac{N}{MN+R} \times NB \leq \frac{N}{MN+0} \times NB = \frac{NB}{M}.$$

As $M \rightarrow \infty$, $|P| \rightarrow 0$ and $\frac{1}{1+\frac{R}{MN}} \rightarrow 1$ so the whole expression tends to $X[k]$.

Symmetries

- 2: Three Different Fourier Transforms
- Fourier Transforms
- Convergence of DTFT
- DTFT Properties
- DFT Properties
- ▷ Symmetries
- Parseval's Theorem
- Convolution
- Sampling Process
- Zero-Padding
- Phase Unwrapping
- Uncertainty principle
- Summary
- MATLAB routines

If $x[n]$ has a special property then $X(e^{j\omega})$ and $X[k]$ will have corresponding properties as shown in the table (and vice versa):

One domain	Other domain
Discrete	Periodic
Symmetric	Symmetric
Antisymmetric	Antisymmetric
Real	Conjugate Symmetric
Imaginary	Conjugate Antisymmetric
Real + Symmetric	Real + Symmetric
Real + Antisymmetric	Imaginary + Antisymmetric

Symmetric: $x[n] = x[-n]$
 $X(e^{j\omega}) = X(e^{-j\omega})$
 $X[k] = X[(-k)_{\text{mod } N}] = X[N - k] \text{ for } k > 0$

Conjugate Symmetric: $x[n] = x^*[-n]$

Conjugate Antisymmetric: $x[n] = -x^*[-n]$

Parseval's Theorem

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

MATLAB routines

Fourier transforms preserve “energy”

$$\text{CTFT} \quad \int |x(t)|^2 dt = \frac{1}{2\pi} \int |X(j\Omega)|^2 d\Omega$$

$$\text{DTFT} \quad \sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$\text{DFT} \quad \sum_0^{N-1} |x[n]|^2 = \frac{1}{N} \sum_0^{N-1} |X[k]|^2$$

More generally, they actually preserve **complex inner products**:

$$\sum_0^{N-1} x[n]y^*[n] = \frac{1}{N} \sum_0^{N-1} X[k]Y^*[k]$$

Unitary matrix viewpoint for DFT:

If we regard \mathbf{x} and \mathbf{X} as vectors, then $\mathbf{X} = \mathbf{F}\mathbf{x}$ where \mathbf{F} is a symmetric matrix defined by $f_{k+1,n+1} = e^{-j2\pi \frac{kn}{N}}$.

The inverse DFT matrix is $\mathbf{F}^{-1} = \frac{1}{N}\mathbf{F}^H$
equivalently, $\mathbf{G} = \frac{1}{\sqrt{N}}\mathbf{F}$ is a **unitary matrix** with $\mathbf{G}^H\mathbf{G} = \mathbf{I}$.

Convolution

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

▷ Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

MATLAB routines

DTFT: Convolution \rightarrow Product

$$x[n] = g[n] * h[n] = \sum_{k=-\infty}^{\infty} g[k]h[n-k]$$

$$\Rightarrow X(e^{j\omega}) = G(e^{j\omega})H(e^{j\omega})$$

DFT: Circular convolution \rightarrow Product

$$x[n] = g[n] \circledast_N h[n] = \sum_{k=0}^{N-1} g[k]h[(n-k)_{\text{mod}N}]$$

$$\Rightarrow X[k] = G[k]H[k]$$

DTFT: Product \rightarrow Circular Convolution $\div 2\pi$

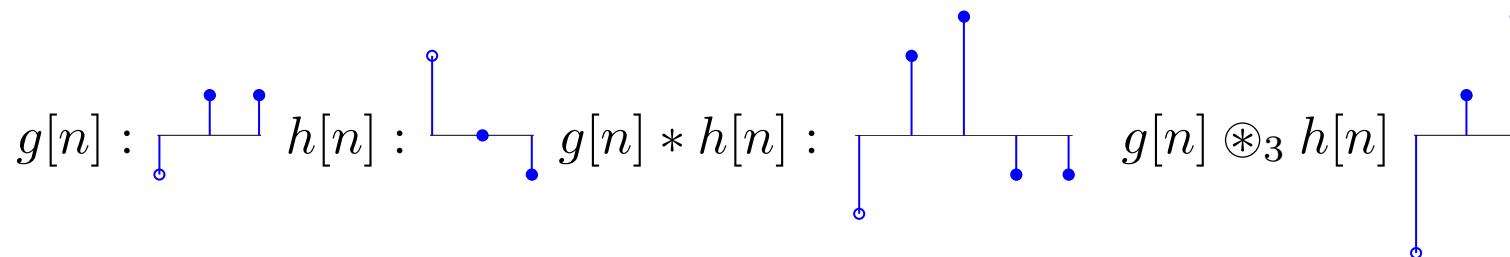
$$y[n] = g[n]h[n]$$

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{2\pi} G(e^{j\omega}) \circledast_{\pi} H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\theta})H(e^{j(\omega-\theta)})d\theta$$

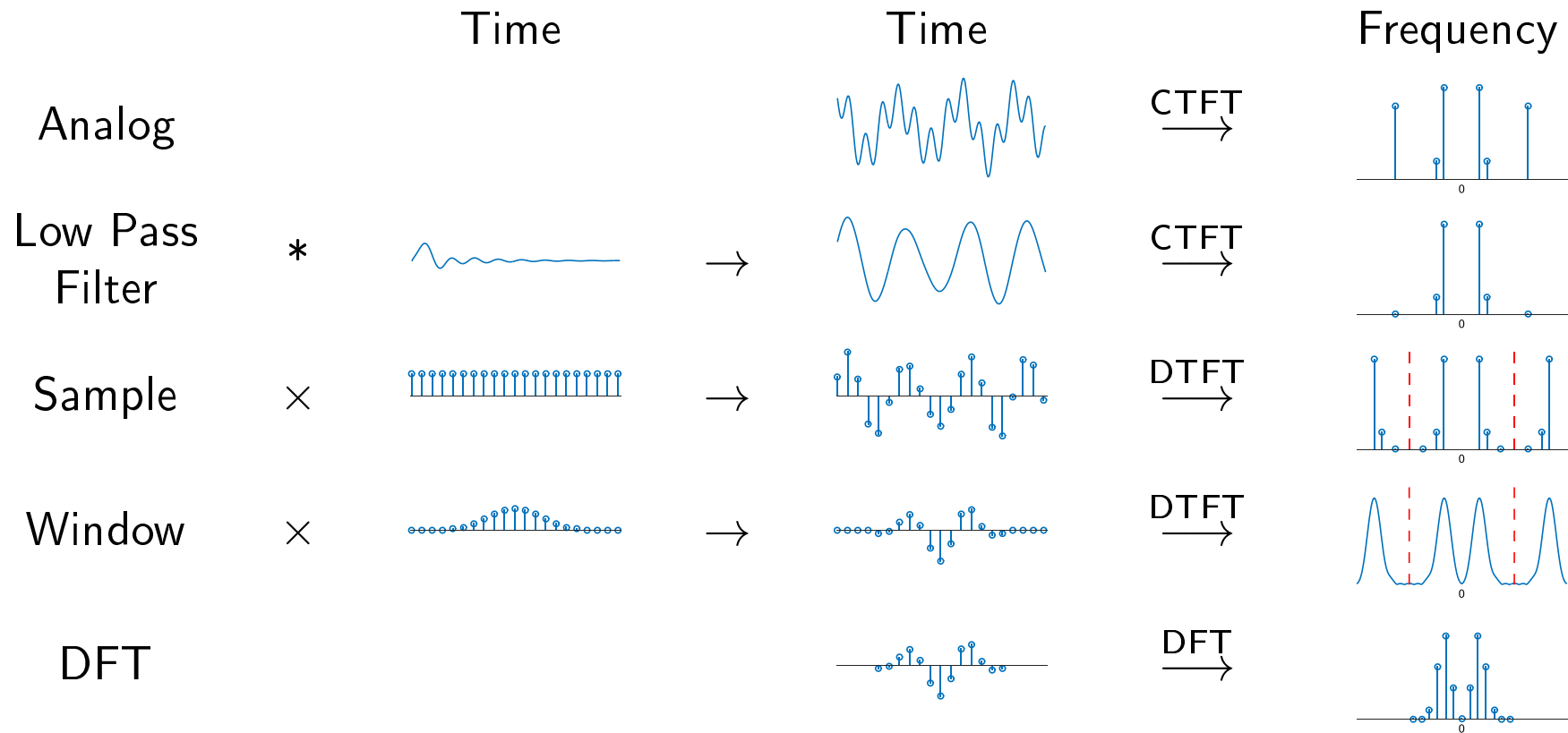
DFT: Product \rightarrow Circular Convolution $\div N$

$$y[n] = g[n]h[n]$$

$$\Rightarrow Y[k] = \frac{1}{N} G[k] \circledast_N H[k]$$



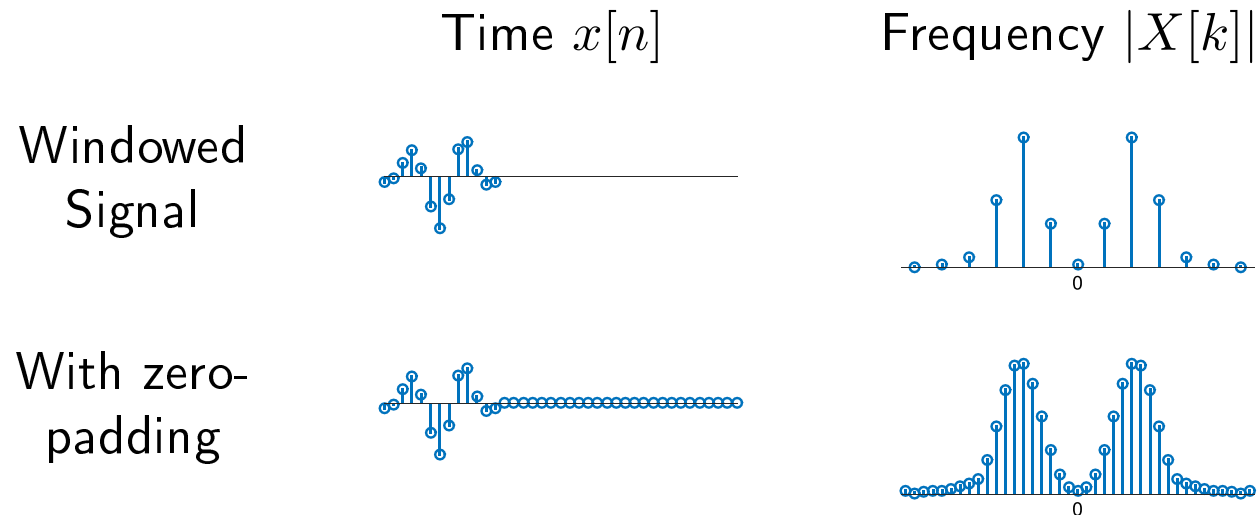
Sampling Process



Zero-Padding

- 2: Three Different Fourier Transforms
- Fourier Transforms
- Convergence of DTFT
- DTFT Properties
- DFT Properties
- Symmetries
- Parseval's Theorem
- Convolution
- Sampling Process
- ▷ Zero-Padding
- Phase Unwrapping
- Uncertainty principle
- Summary
- MATLAB routines

Zero padding means added extra zeros onto the end of $x[n]$ before performing the DFT.



- Zero-padding causes the DFT to evaluate the DTFT at more values of ω_k . Denser frequency samples.
- Width of the peaks remains constant: determined by the length and shape of the window.
- Smoother graph but increased frequency resolution is an illusion.

Phase Unwrapping

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

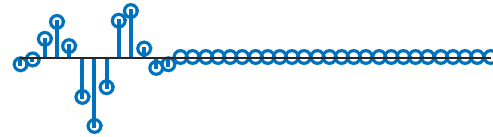
▷ Phase Unwrapping

Uncertainty principle

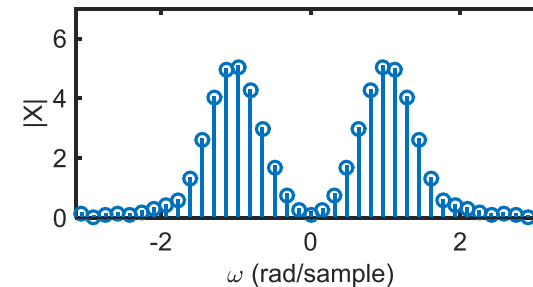
Summary

MATLAB routines

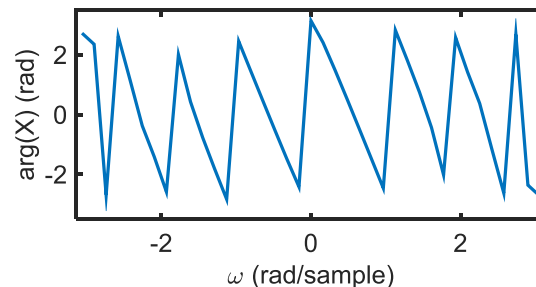
Phase of a DTFT is only defined to within an integer multiple of 2π .



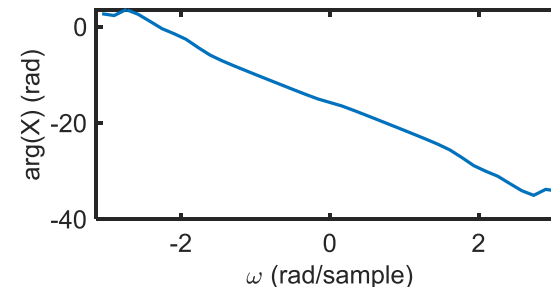
$x[n]$



$|X[k]|$



$\angle X[k]$



$\angle X[k]$ unwrapped

Phase unwrapping adds multiples of 2π onto each $\angle X[k]$ to make the phase as continuous as possible.

Uncertainty principle

2: Three Different
Fourier Transforms

Fourier Transforms

Convergence of
DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty

▷ principle

Summary

MATLAB routines

$$\text{CTFT uncertainty principle: } \left(\frac{\int t^2 |x(t)|^2 dt}{\int |x(t)|^2 dt} \right)^{\frac{1}{2}} \left(\frac{\int \omega^2 |X(j\omega)|^2 d\omega}{\int |X(j\omega)|^2 d\omega} \right)^{\frac{1}{2}} \geq \frac{1}{2}$$

The first term measures the “width” of $x(t)$ around $t = 0$.

It is like σ if $|x(t)|^2$ was a zero-mean probability distribution.

The second term is similarly the “width” of $X(j\omega)$ in frequency.

A signal **cannot be concentrated in both time and frequency.**

Proof Outline:

$$\text{Assume } \int |x(t)|^2 dt = 1 \Rightarrow \int |X(j\omega)|^2 d\omega = 2\pi \quad [\text{Parseval}]$$

$$\text{Set } v(t) = \frac{dx}{dt} \Rightarrow V(j\omega) = j\omega X(j\omega) \quad [\text{by parts}]$$

$$\text{Now } \int tx \frac{dx}{dt} dt = \frac{1}{2} tx^2(t) \Big|_{t=-\infty}^{\infty} - \int \frac{1}{2} x^2 dt = 0 - \frac{1}{2} \quad [\text{by parts}]$$

$$\text{So } \frac{1}{4} = \left| \int tx \frac{dx}{dt} dt \right|^2 \leq \left(\int t^2 x^2 dt \right) \left(\int \left| \frac{dx}{dt} \right|^2 dt \right) \quad [\text{Schwartz}]$$

$$= \left(\int t^2 x^2 dt \right) \left(\int |v(t)|^2 dt \right) = \left(\int t^2 x^2 dt \right) \left(\frac{1}{2\pi} \int |V(j\omega)|^2 d\omega \right)$$

$$= \left(\int t^2 x^2 dt \right) \left(\frac{1}{2\pi} \int \omega^2 |X(j\omega)|^2 d\omega \right)$$

No exact equivalent for DTFT/DFT but a similar effect is true

[Uncertainty Principle Proof Steps]

(1) Suppose $v(t) = \frac{dx}{dt}$. Then integrating the CTFT definition by parts w.r.t. t gives

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} dt = \left[\frac{-1}{j\Omega} x(t)e^{-j\Omega t} \right]_{-\infty}^{\infty} + \frac{1}{j\Omega} \int_{-\infty}^{\infty} \frac{dx(t)}{dt} e^{-j\Omega t} dt = 0 + \frac{1}{j\Omega} V(j\Omega)$$

(2) Since $\frac{d}{dt} \left(\frac{1}{2} x^2 \right) = x \frac{dx}{dt}$, we can apply integration by parts to get

$$\int_{-\infty}^{\infty} tx \frac{dx}{dt} dt = \left[t \times \frac{1}{2} x^2 \right]_{t=-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{dt}{dt} \times \frac{1}{2} x^2 dt = -\frac{1}{2} \int_{-\infty}^{\infty} x^2 dt = -\frac{1}{2} \times 1 = -\frac{1}{2}$$

It follows that $\left| \int_{-\infty}^{\infty} tx \frac{dx}{dt} dt \right|^2 = \left(-\frac{1}{2} \right)^2 = \frac{1}{4}$ which we will use below.

(3) The Cauchy-Schwarz inequality is that in a complex inner product space

$|\mathbf{u} \cdot \mathbf{v}|^2 \leq (\mathbf{u} \cdot \mathbf{u})(\mathbf{v} \cdot \mathbf{v})$. For the inner-product space of real-valued square-integrable functions, this becomes $\left| \int_{-\infty}^{\infty} u(t)v(t) dt \right|^2 \leq \int_{-\infty}^{\infty} u^2(t) dt \times \int_{-\infty}^{\infty} v^2(t) dt$. We apply this with $u(t) = tx(t)$ and $v(t) = \frac{dx(t)}{dt}$ to get

$$\frac{1}{4} = \left| \int_{-\infty}^{\infty} tx \frac{dx}{dt} dt \right|^2 \leq \left(\int_{-\infty}^{\infty} t^2 x^2 dt \right) \left(\int_{-\infty}^{\infty} \left(\frac{dx}{dt} \right)^2 dt \right) = \left(\int_{-\infty}^{\infty} t^2 x^2 dt \right) \left(\int_{-\infty}^{\infty} v^2(t) dt \right)$$

(4) From Parseval's theorem for the CTFT, $\int_{-\infty}^{\infty} v^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |V(j\Omega)|^2 d\Omega$. From step (1), we can substitute $V(j\Omega) = j\Omega X(j\Omega)$ to obtain $\int_{-\infty}^{\infty} v^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega^2 |X(j\Omega)|^2 d\Omega$. Making this substitution in (3) gives

$$\frac{1}{4} \leq \left(\int_{-\infty}^{\infty} t^2 x^2 dt \right) \left(\int_{-\infty}^{\infty} v^2(t) dt \right) = \left(\int_{-\infty}^{\infty} t^2 x^2 dt \right) \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 |X(j\Omega)|^2 d\Omega \right)$$

Summary

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

▷ Summary

MATLAB routines

- Three types: CTFT, DTFT, DFT
 - DTFT = CTFT of continuous signal \times impulse train
 - DFT = DTFT of periodic or finite support signal
 - ▷ DFT is a scaled unitary transform
- DTFT: Convolution \rightarrow Product; Product \rightarrow Circular Convolution
- DFT: Product \leftrightarrow Circular Convolution
- DFT: Zero Padding \rightarrow Denser freq sampling but same resolution
- Phase is only defined to within a multiple of 2π .
- Whenever you integrate over frequency you need a **scale factor**
 - $\frac{1}{2\pi}$ for CTFT and DTFT or $\frac{1}{N}$ for DFT
 - e.g. Inverse transform, Parseval, frequency domain convolution

For further details see Mitra: 3 & 5.

MATLAB routines

2: Three Different Fourier Transforms

Fourier Transforms

Convergence of DTFT

DTFT Properties

DFT Properties

Symmetries

Parseval's Theorem

Convolution

Sampling Process

Zero-Padding

Phase Unwrapping

Uncertainty principle

Summary

▷ MATLAB routines

fft, ifft	DFT with optional zero-padding
fftshift	swap the two halves of a vector
conv	convolution or polynomial multiplication (not circular)
$x[n] \circledast y[n]$	$\text{real}(\text{ifft}(\text{fft}(x) \cdot \text{fft}(y)))$
unwrap	remove 2π jumps from phase spectrum