Digital Image Procesing

Spatial Filters in Image Processing

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Spatial filters for image enhancement

- Spatial filters called spatial masks are used for specific operations on the image. Popular filters are the following:

  - Low pass filters
  - High boost filters
  - High pass filters

Image averaging in the case of many realizations

- We have M different noisy images:
$$g_i(x,y) = f(x,y) + n_i(x,y), i = 0, \dots, M-1$$
- Noise realizations are zero mean and white with the same variance, i.e.,
$$E\{n_i(x,y)\} = 0 \text{ and } R_{n_i}[k,l] = {\sigma_{n_i}}^2 \delta[k,l] = {\sigma_n}^2\, \delta[k,l]$$
- We define a new image which is the average
$$\bar{g}(x,y) = \frac{1}{M}\sum_{i=0}^{M-1} g_i(x,y) = f(x,y) + \frac{1}{M}\sum_{i=0}^{M-1} n_i(x,y)$$
- Notice that average is calculated across realizations.
- **Problem:** Find the mean and variance of the new image $\bar{g}(x,y)$.
- $E\{\bar{g}(x,y)\} = E\{\frac{1}{M}\sum_{i=0}^{M-1} g_i(x,y)\} = \frac{1}{M}\sum_{i=0}^{M-1} E\{g_i(x,y)\} = f(x,y)$
- ${\sigma_{\bar{g}(x,y)}}^2 = {\sigma_{f(x,y)}}^2 + \frac{1}{M}{\sigma_{n(x,y)}}^2 = \frac{1}{M}{\sigma_{n(x,y)}}^2$

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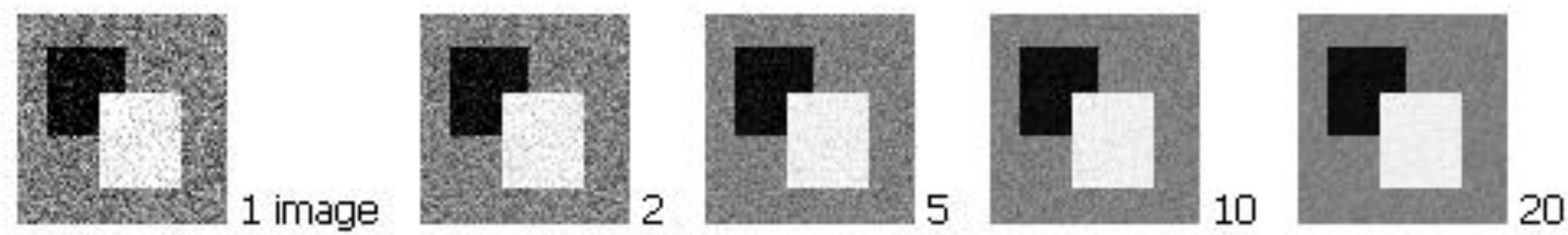
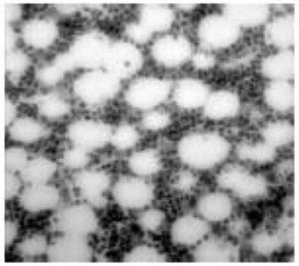
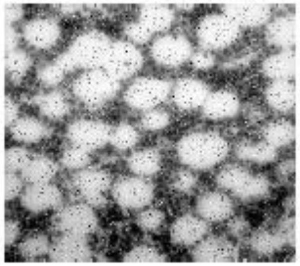
Example: Image Averaging (number refers to realizations)



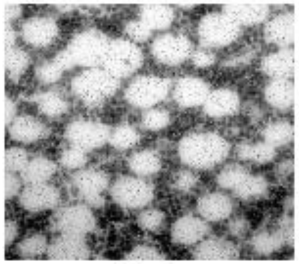
Example: Image Averaging



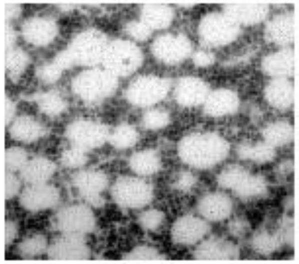
Noise-free
Image



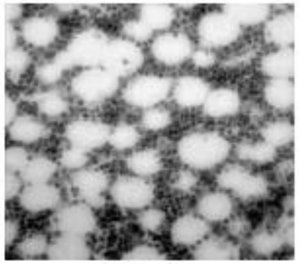
Noisy Image
Noise Variance = 0.05



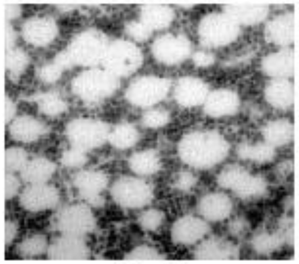
$M = 2$



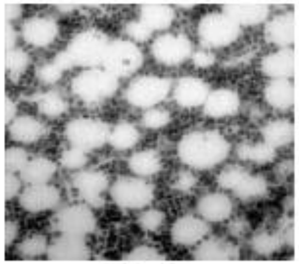
$M = 5$



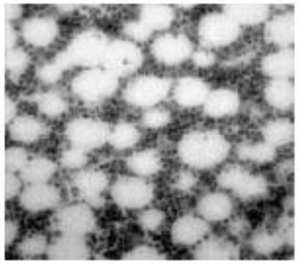
$M = 10$



$M = 25$

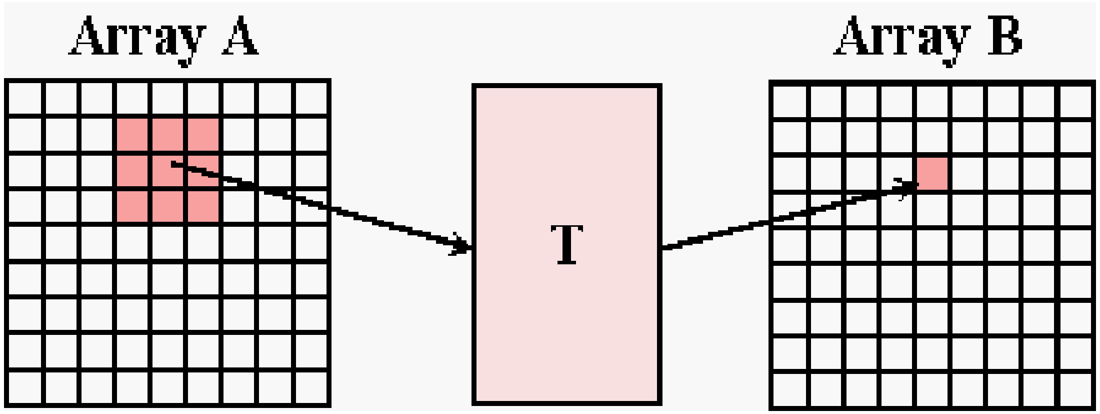


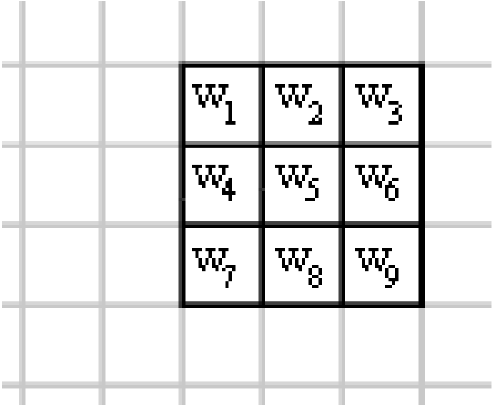
$M = 50$



$M = 100$

Spatial masks





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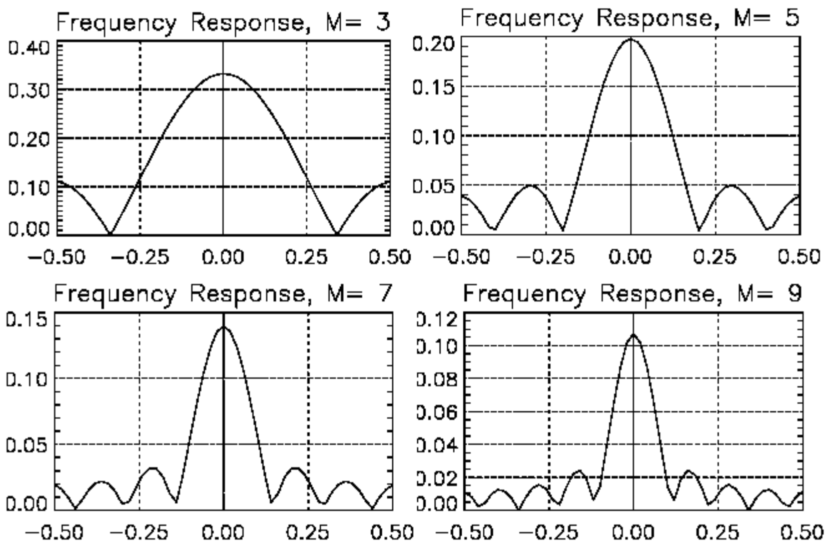
Local averaging spatial masks for image de-noising (smoothing)

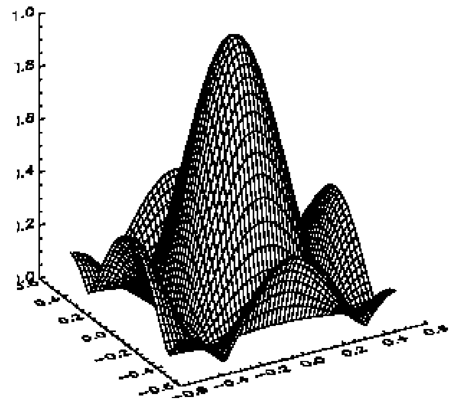
$$\frac{1}{9}\times\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$\frac{1}{25}\times\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$$

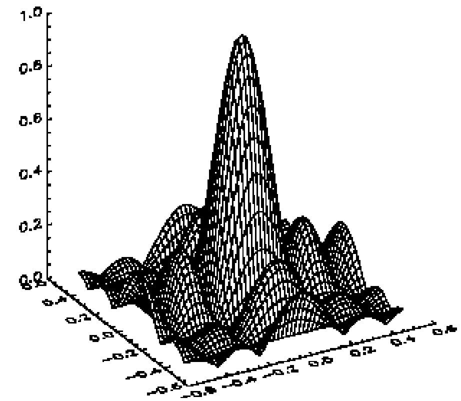
$$\frac{1}{49}\times\begin{bmatrix}1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\end{bmatrix}$$

Response of 1D and 2D local averaging spatial masks

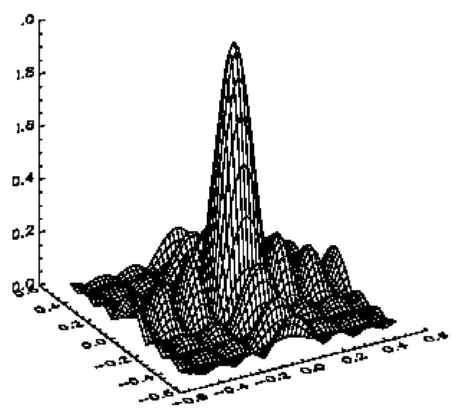




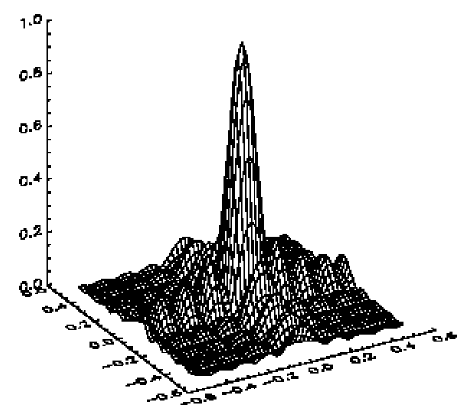
Frequency Response for M=3



Frequency Response for M=5



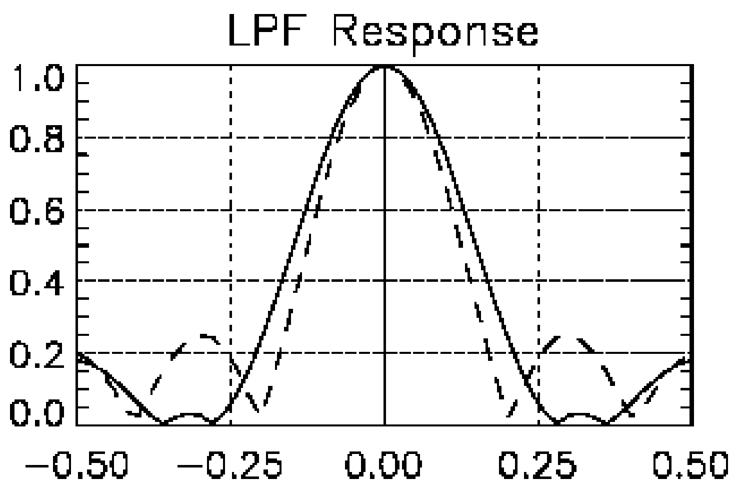
Frequency Response for M=7



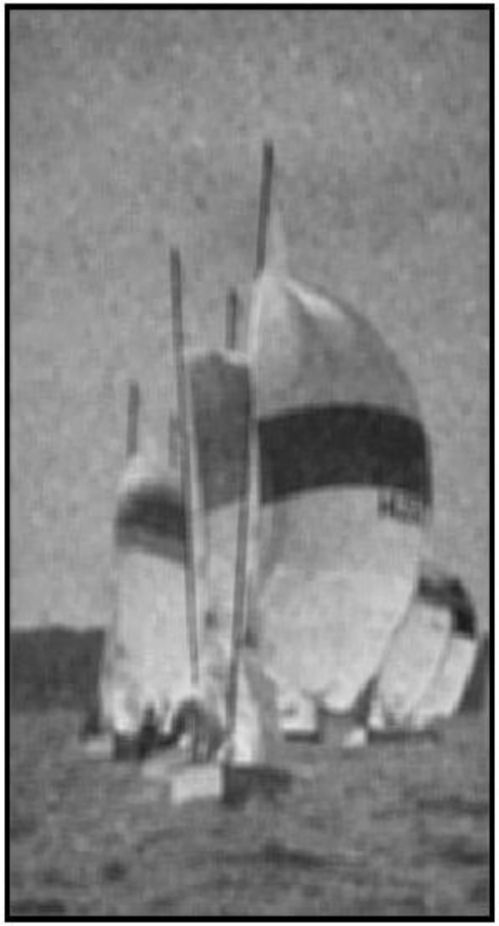
Frequency Response for M=9

Other (weighted) local averaging spatial masks

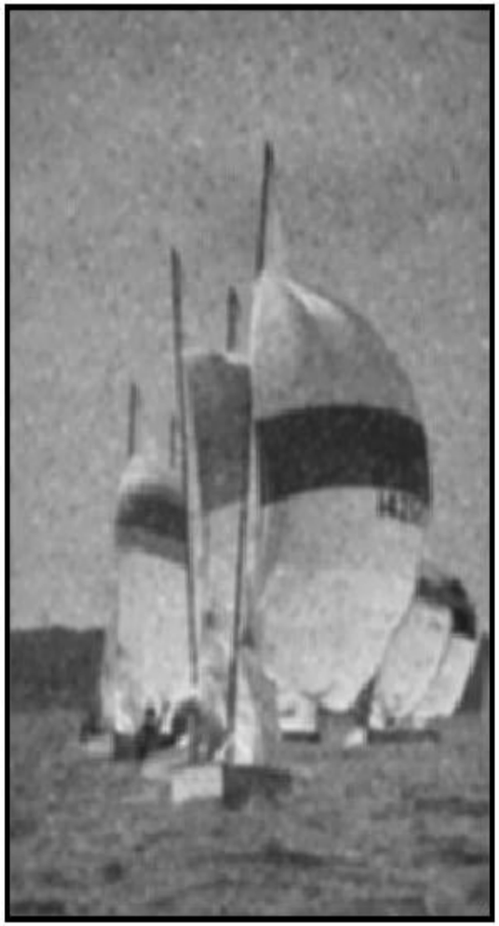
$$\frac{1}{25} \times \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 2 & 3 & 4 & 3 & 2 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$$



solid line shows weighted mask
dashed line shows simple averaging



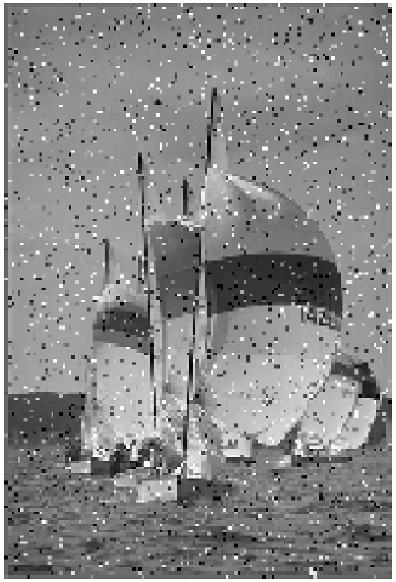
Averaging Filter



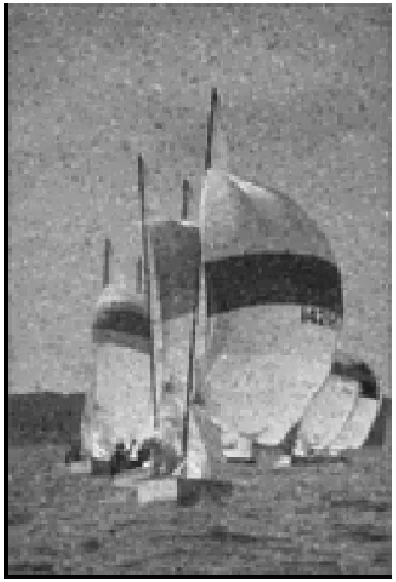
Lowpass Filter

Example of local image averaging

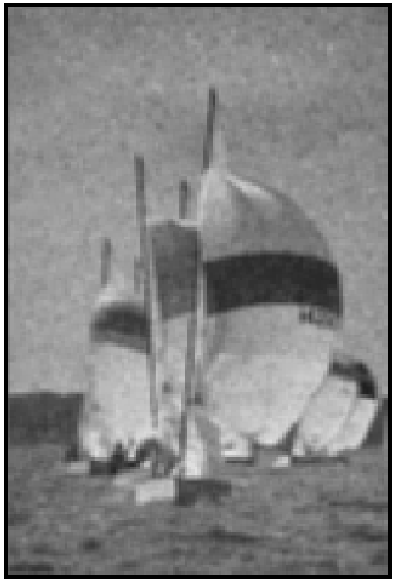
Original Image



Noisy Image

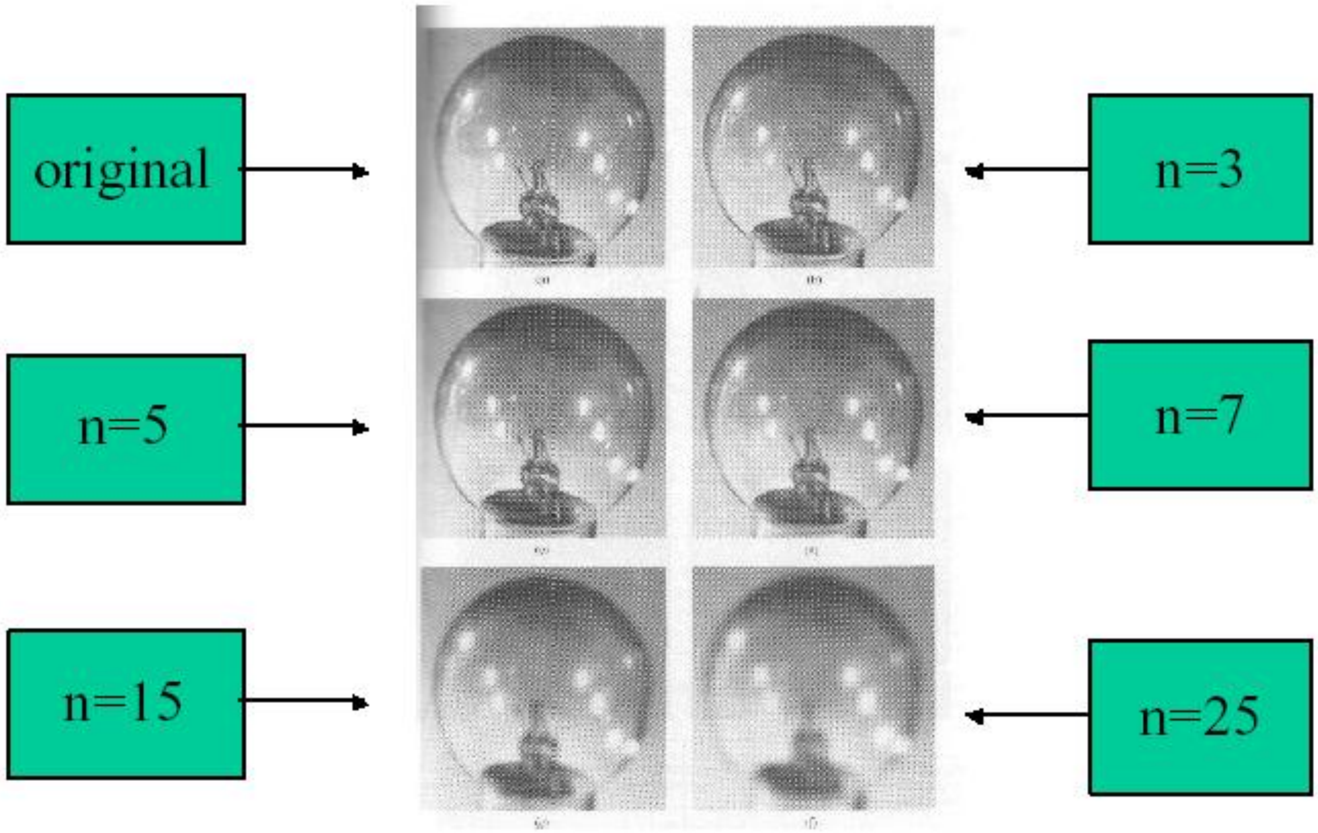


3×3 smoothing



5×5 smoothing

Example: Image Averaging



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A noiseless image and its noisy version

Original Lena image

Lena image with 10db noise

And some denoising!

3×3 averaging mask

5×5 averaging mask

Median filtering

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

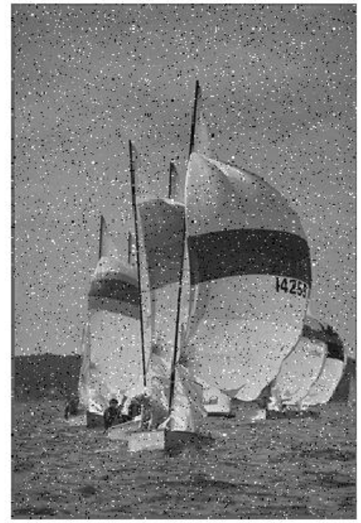
Neighbourhood values:

115, 119, 120, 123, 124,
125, 126, 127, 150

Median value: 124

Example of median filtering

Original

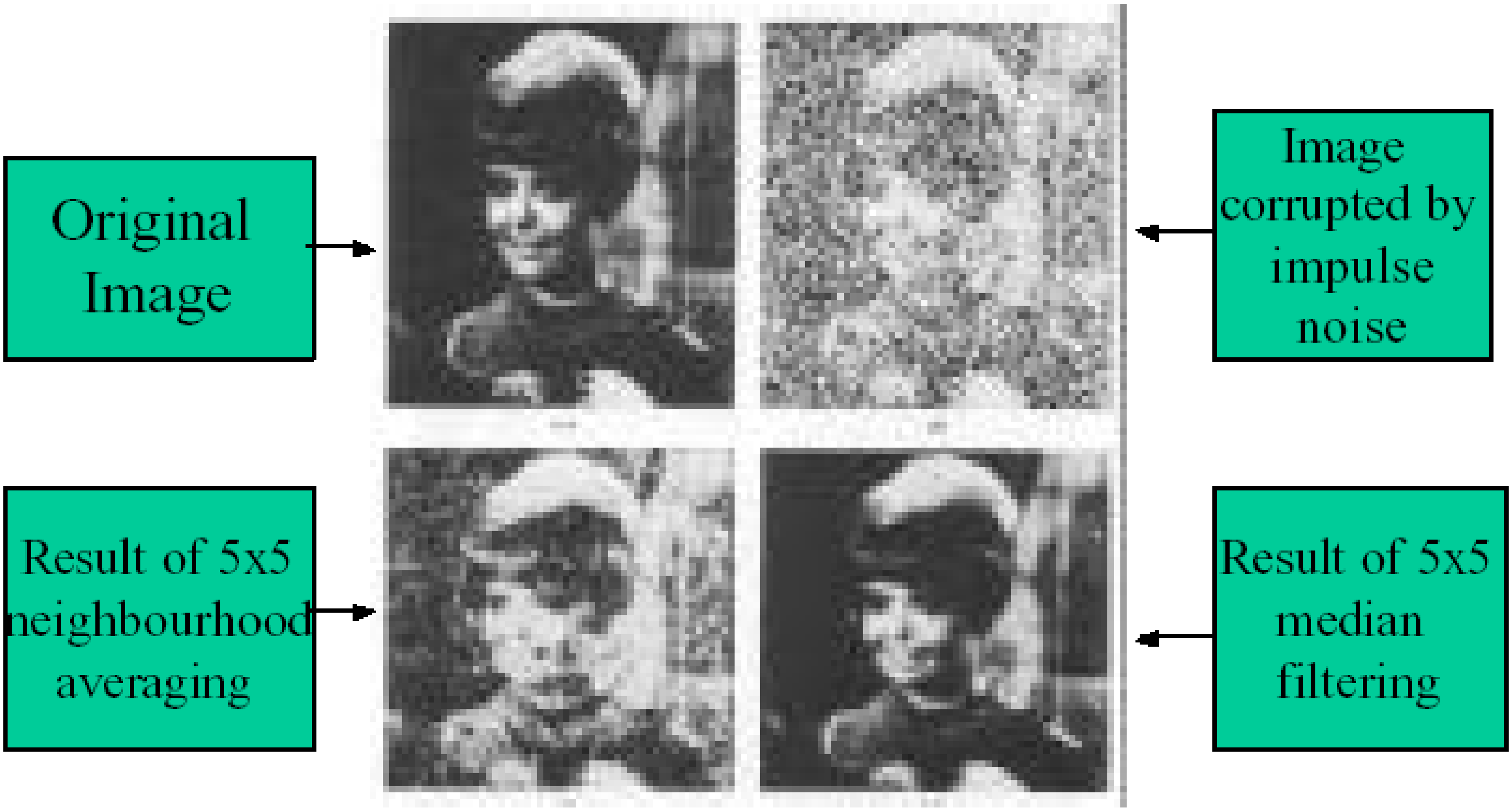


Original with Noise

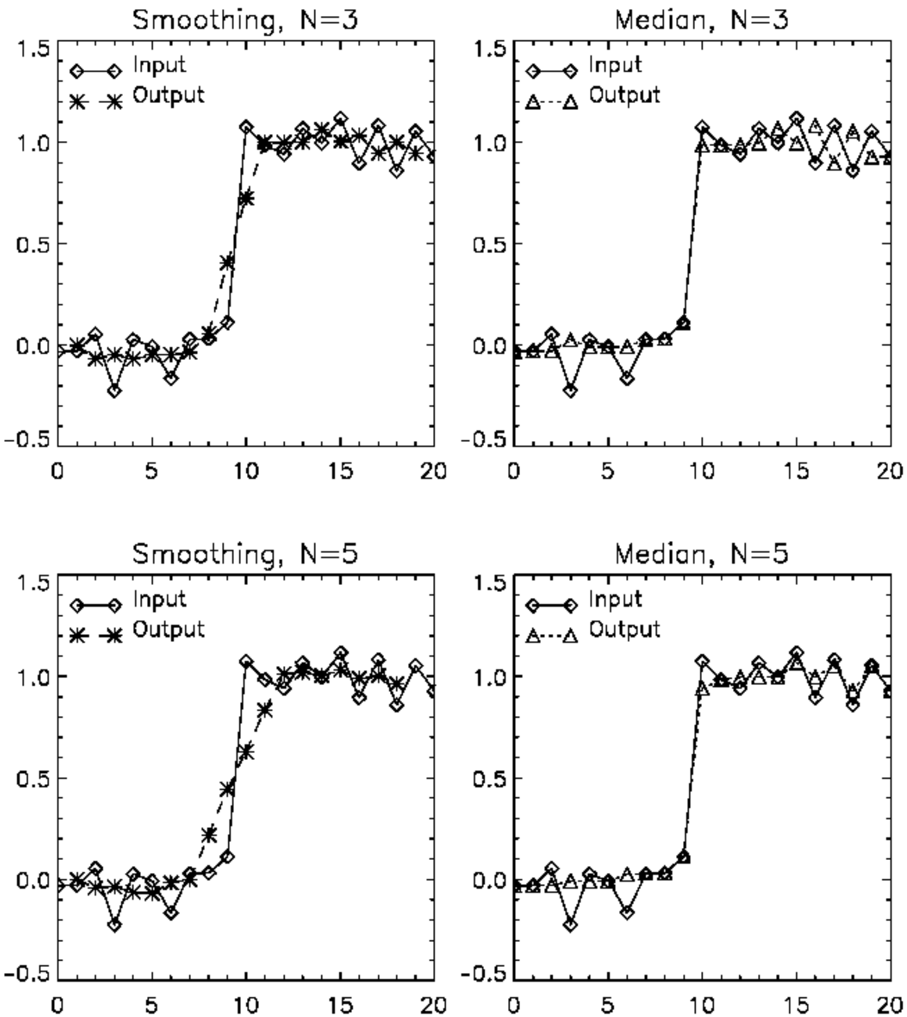
After N=3 Median Filter

After N=5 Median Filter

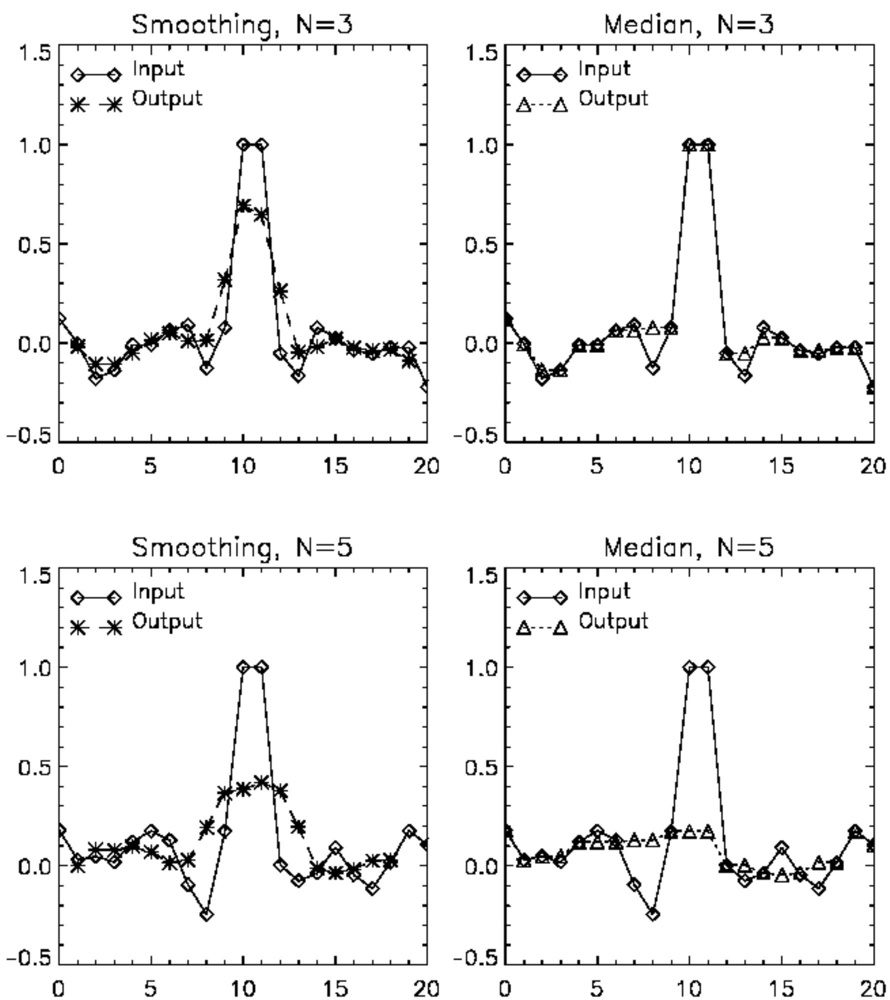
Comparison of local smoothing and median filtering



Median versus local averaging filter response around local step edge



Median versus local averaging filter response around local ridge edge

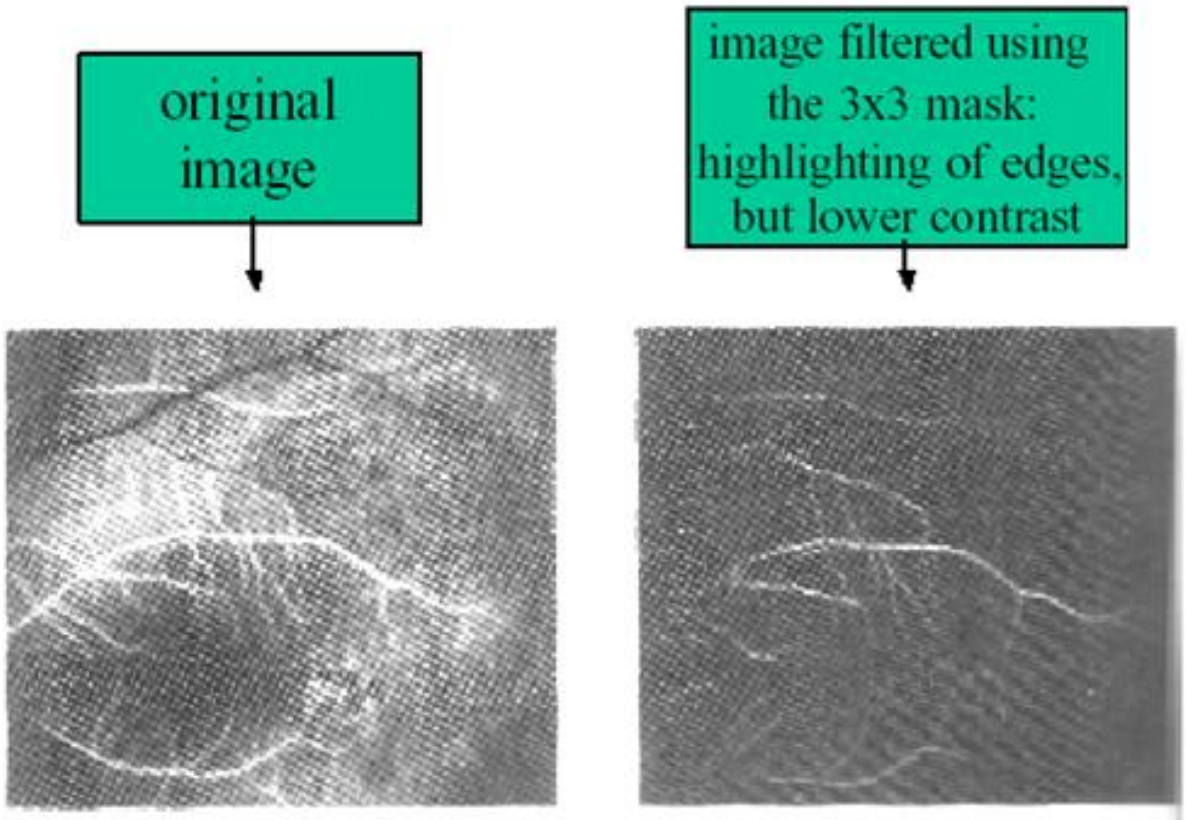


High pass filtering

- We use masks with positive coefficients around the centre of the mask and negative coefficients in the periphery.
- In the mask shown below the central coefficient is +8 and its 8 nearest neighbours are -1.
- The reversed signs are equally valid, i.e., -8 in the middle and +1 for the rest of the coefficients.
- The sum of the mask coefficients must be zero. This leads to the response of the mask within a constant (or slowly varying) intensity area being zero (or very small).

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Example of high pass filtering



Example of high pass filtering

High pass filtered image

High pass filtered image with 10db noise

High boost filtering

- The goal of high boost filtering is to enhance the high frequency information without completely eliminating the background of the image.
- We know that:

  (High-pass filtered image)=(Original image)-(Low-pass filtered image)
- We define:

  (High boost filtered image)=A $\times$(Original image)-(Low-pass filtered image)

  (High boost)=$(A-1)$ $\times$(Original)+(Original)-(Low-pass)

  (High boost)=$(A-1)$ $\times$ (Original)+(High-pass)
- As you can see, when $A > 1$, part of the original is added back to the high-pass filtered version of the image in order to partly restore the low frequency components that would have been eliminated with standard high-pass filtering.
- Typical values for A are values slightly higher than 1, as for example 1.15, 1.2, etc.

High boost filtering

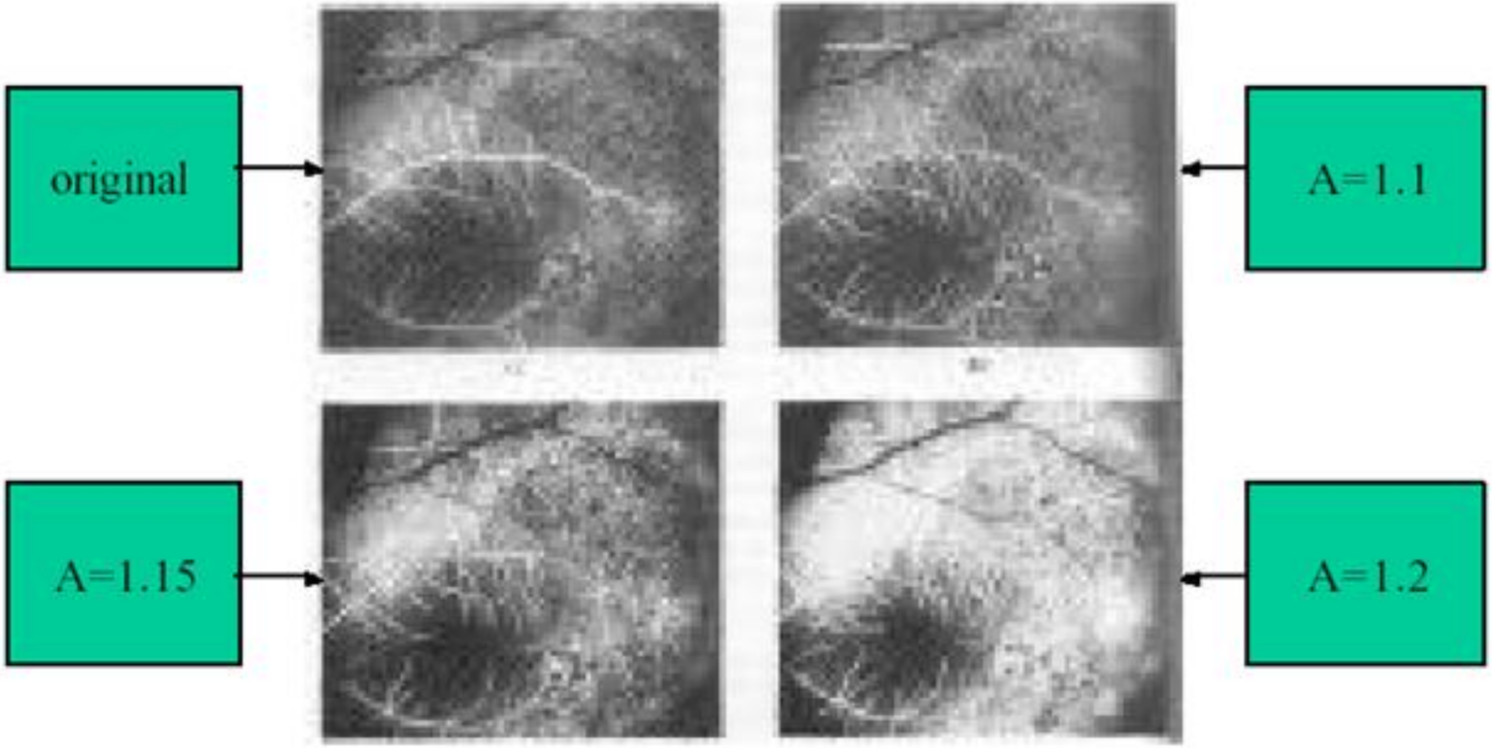
- The resulting image looks similar to the original image with some edge enhancement.
- The spatial mask that implements the high boost filtering algorithm is shown below.
- The resulting image depends on the choice of A.

$\frac{1}{9} \times$

-1	-1	-1
-1	$w = 9A - 1$	-1
-1	-1	-1

- High boost filtering is used in printing and publishing industry.

Example of high boost filtering



Example of high boost filtering

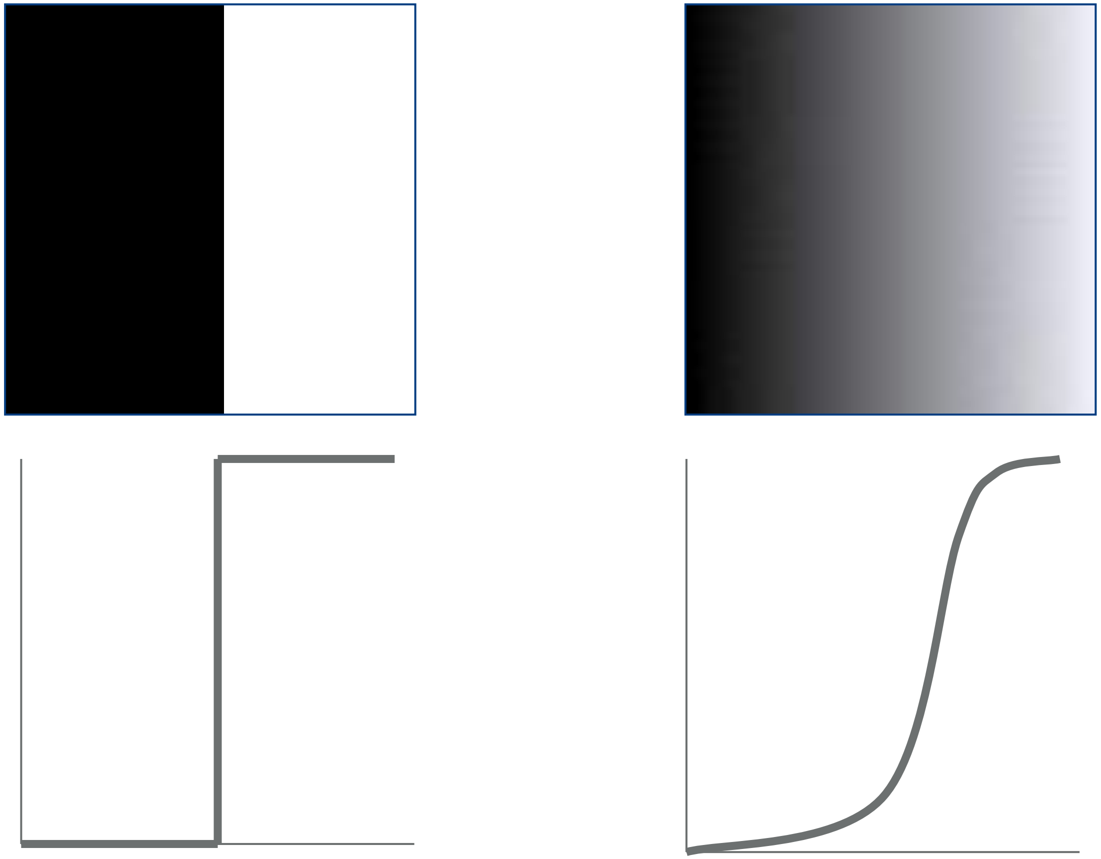
$A = 1.15$

$A = 1.2$

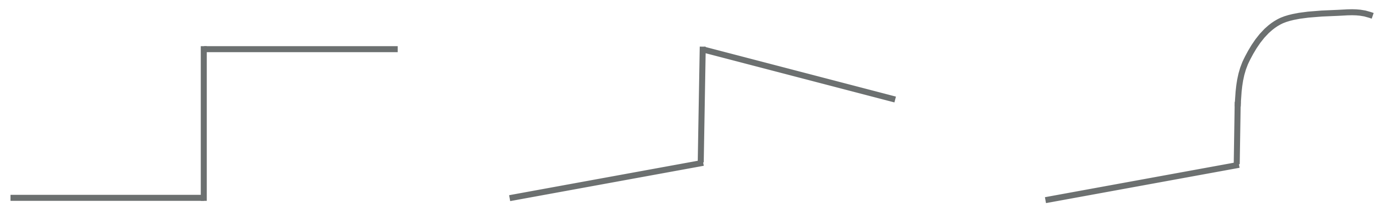
Edge detection

- Edges are abrupt local changes in the image value (for example grey level).
- Edges are of crucial important because they are related to the boundaries of the various objects present in the image.
- Object boundaries are key feature for object detection and identification.
- Edge information in an image is found by looking at the relationship a pixel has with its neighborhoods.
- If a pixel's grey-level value is similar to those around it, there is probably <u>not</u> an edge at that point.
- If a pixel's has neighbors with widely varying grey levels, it may present an edge point.

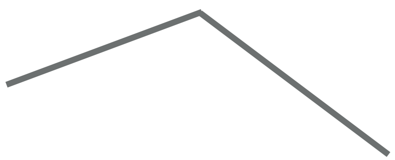
Abrupt versus gradual change in image intensity



Type of edges



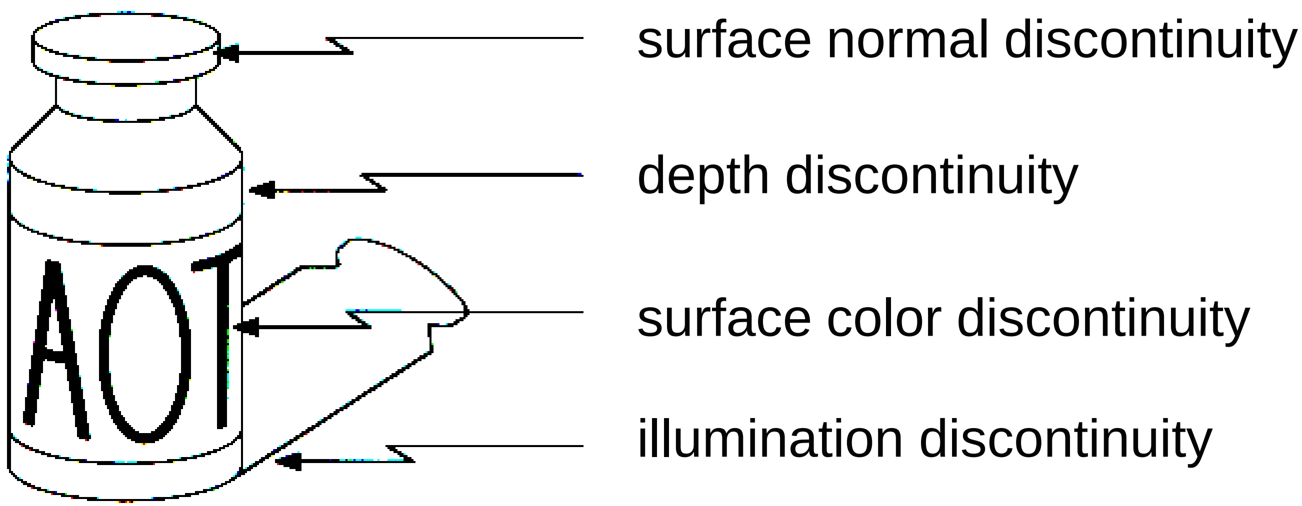
Step edges



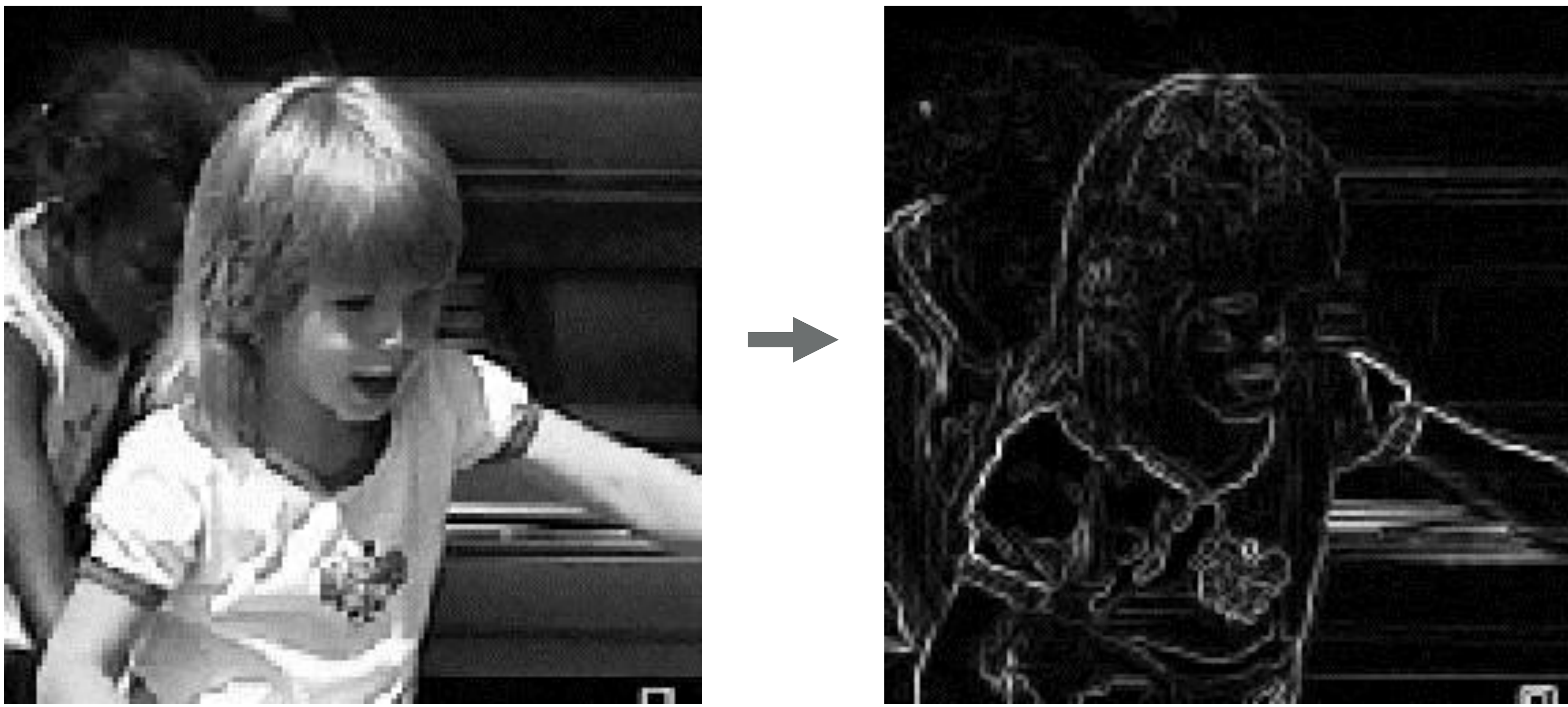
Roof edge

Line (ridge) edges

Edges are caused by a variety of factots



From a grey level image to an edge image (or edge map)



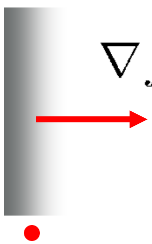
How do we detect the presence of a local edge?

- The gradient (first derivative) of the image around a pixel might give information about how detailed is the area around a pixel and whether there are abrupt intensity changes.
- The image is a 2-D signal and therefore, the gradient at a location (x, y) is a 2-D vector which contains the two partial derivatives of the image with respect to the coordinates x, y.

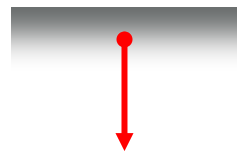
$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- Edge strength is given by $\|\nabla f\| = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$ or $\|\nabla f\| \cong \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right|$.
- Edge orientation is given by $\theta = tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$.

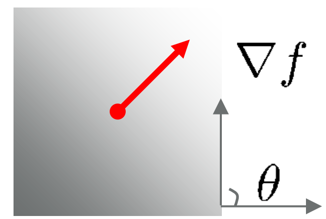
Examples of edge orientations



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- Edge strength is given by $\|\nabla f\| = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$ or $\|\nabla f\| \cong \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right|$.

- Edge orientation is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

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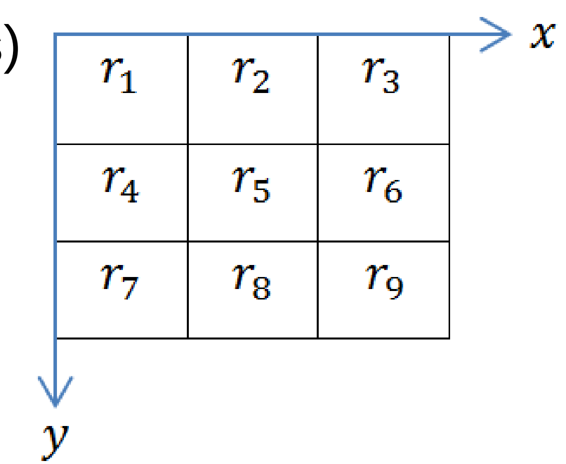
Spatial masks for edge detection

- Consider an image region of size 3×3 pixels. The coordinates x, y are shown.
- The quantity r denotes grey level values.
- The magnitude $||\nabla f|| = \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right|$ of the gradient at pixel with intensity r_5 can be approximated by:
  - ➢ differences along x and y
    $||\nabla f|| = |r_5 - r_6| + |r_5 - r_8|$
  - ➢ cross-differences (along the two main diagonals)
  - ➢ $||\nabla f|| = |r_6 - r_8| + |r_5 - r_9|$



r_1	r_2	r_3
r_4	r_5	r_6
r_7	r_8	r_9

Spatial masks for edge detection – Roberts operator

- The magnitude $\|\nabla f\| = \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right|$ of the gradient at pixel with intensity r_5 can be approximated by:
  - differences along x and y
    $\|\nabla f\| = |r_5 - r_6| + |r_5 - r_8|$
  - cross-differences (along the two main diagonals)
    $\|\nabla f\| = |r_6 - r_8| + |r_5 - r_9|$
- Each of the above approximation is the sum of the magnitudes of the responses of two masks. These sets of masks are the so called **Roberts operator**.

abs{

0	0	0
0	1	0
0	-1	0

}+abs{

0	0	0
0	1	-1
0	0	0

}

abs{

0	0	0
0	0	1
0	-1	0

}+abs{

0	0	0
0	1	0
0	0	-1

}

Spatial masks for edge detection – Prewitt and Sobel operators

- The magnitude $\|\nabla f\| = \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right|$ of the gradient at pixel with intensity r_5 can be also approximated by involving more pixels:
  - Roberts operator

    $\|\nabla f\| = |(r_3+r_6+r_9)-(r_1+r_4+r_7)| + |(r_7+r_8+r_9)-(r_1+r_2+r_3)|$
  - Sobel operator
  - $\|\nabla f\| = |(r_3+2r_6+r_9)-(r_1+2r_4+r_7)| + |(r_7+2r_8+r_9)-(r_1+2r_2+r_3)|$
- Each of the above approximation is the sum of the magnitudes of the responses of two masks.

abs{

-1	0	1
-1	0	1
-1	0	1

}+abs{

-1	-1	-1
0	0	0
1	1	1

}

abs{

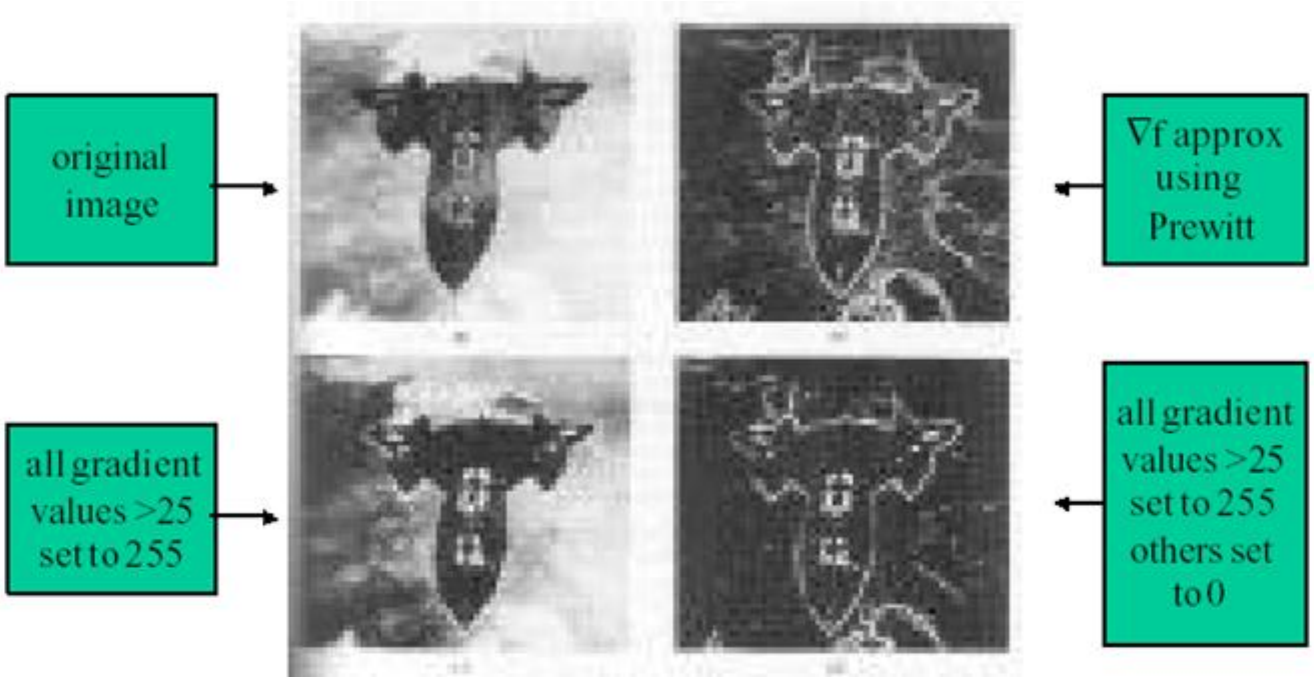
-1	-2	-1
0	0	0
1	2	1

}+abs{

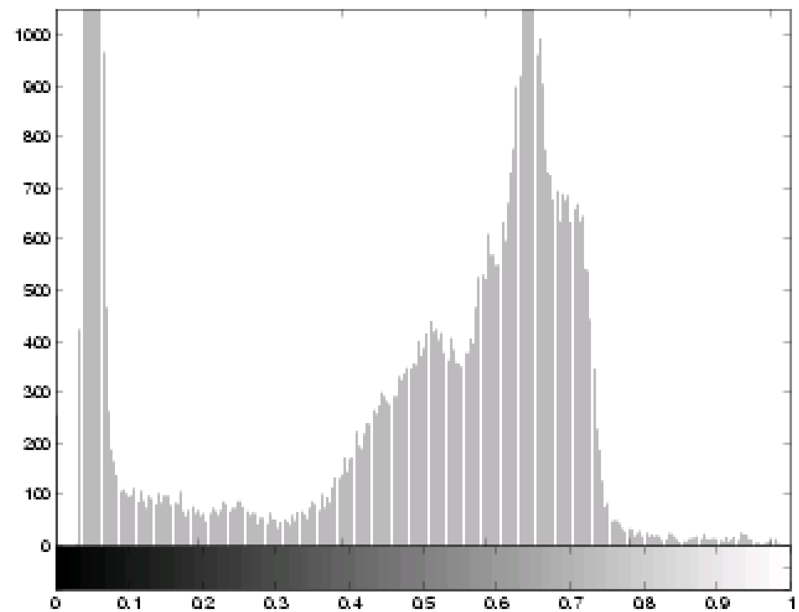
-1	0	1
-2	0	2
-1	0	1

}

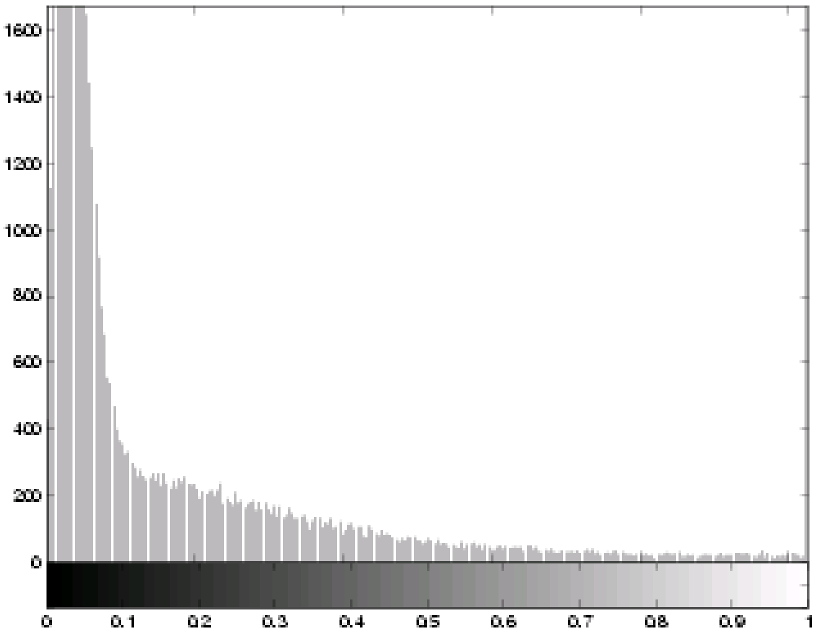
Example of edge detection using Prewitt



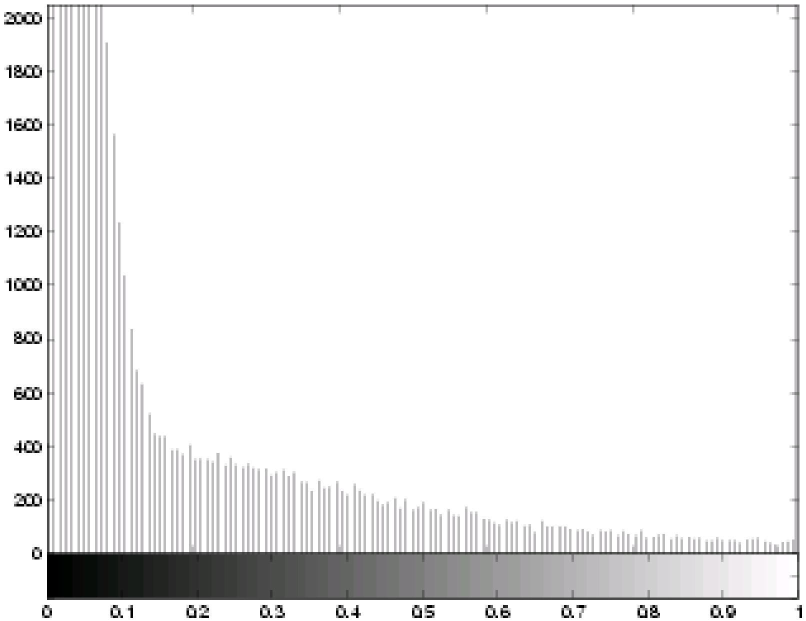
Original image and its histogram



Example of edge detection using Prewitt



Example of edge detection using Sobel



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Edge detection using second order gradient: Laplacian mask

- The second order gradient gives also information about how detailed is the area around a pixel.
- It is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- A mask that can approximate the above function is the Laplacian:

$$\nabla^2 f \cong 4r_5 - (r_2 + r_4 + r_6 + r_8)$$