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Digital Image Processing

Image Enhancement

Histogram Equalization

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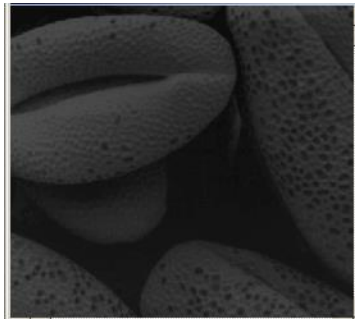
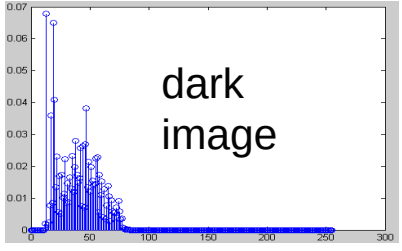
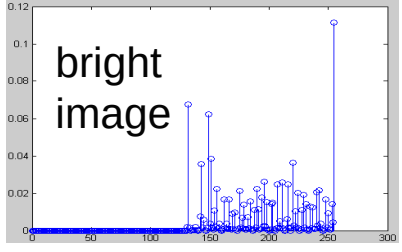
Histogram Processing: definition of image histogram

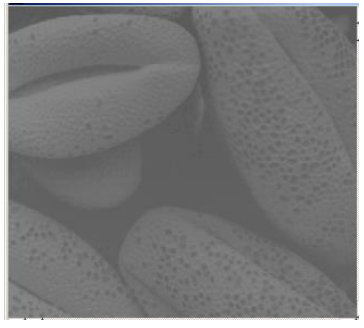
- Consider an image with intensity r_k, $k \in [0, L-1]$ and size $M \times N$.
- The number of pixels with intensity r_k is n_k.
- The histogram of the image is the function $h(r_k) = n_k$.
- The normalized histogram is the function

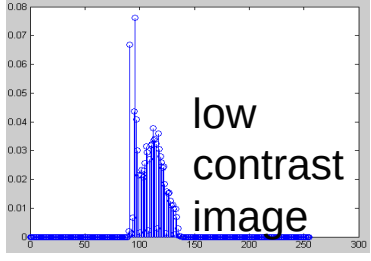
$$p(r_k) = \frac{n_k}{MN} \text{ for } k = 0, \ldots, L-1$$

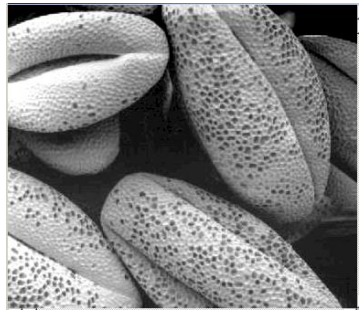


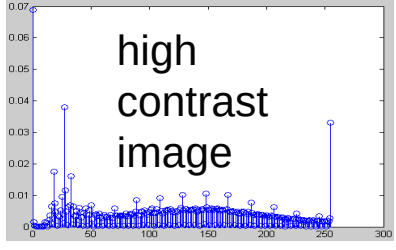






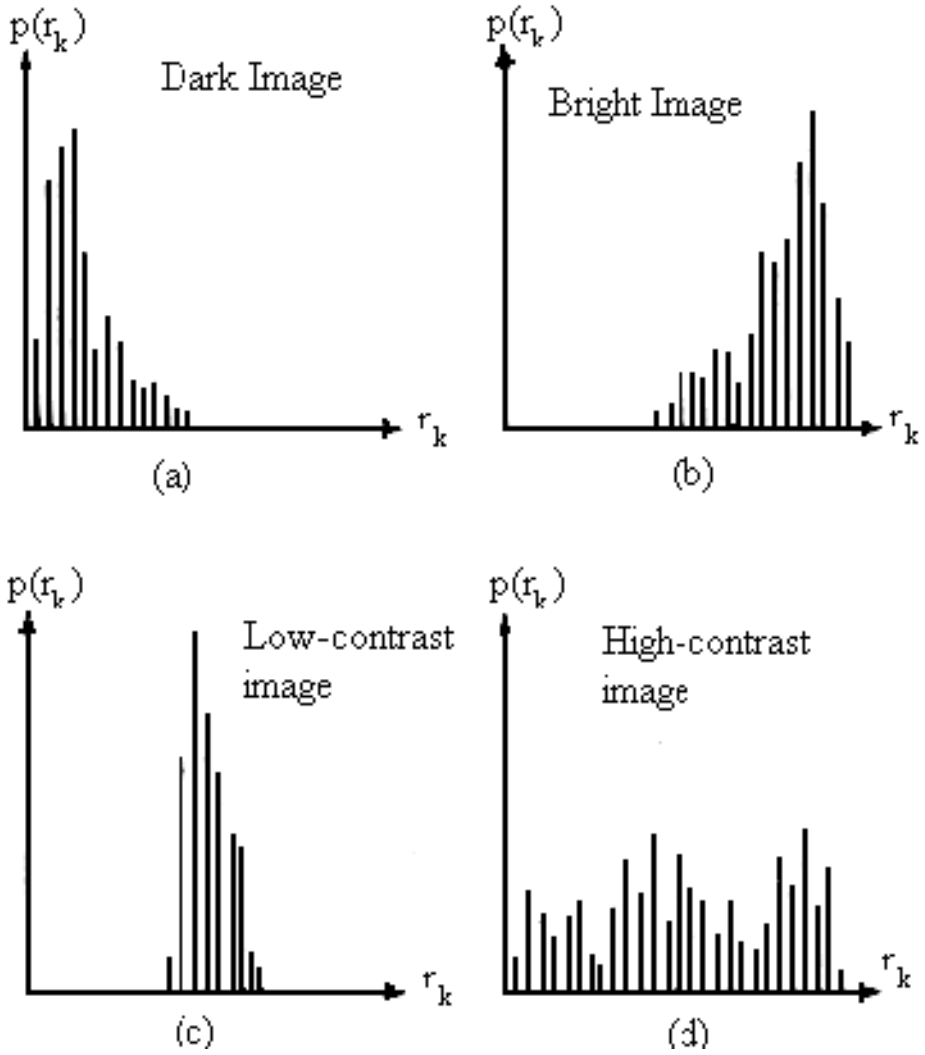






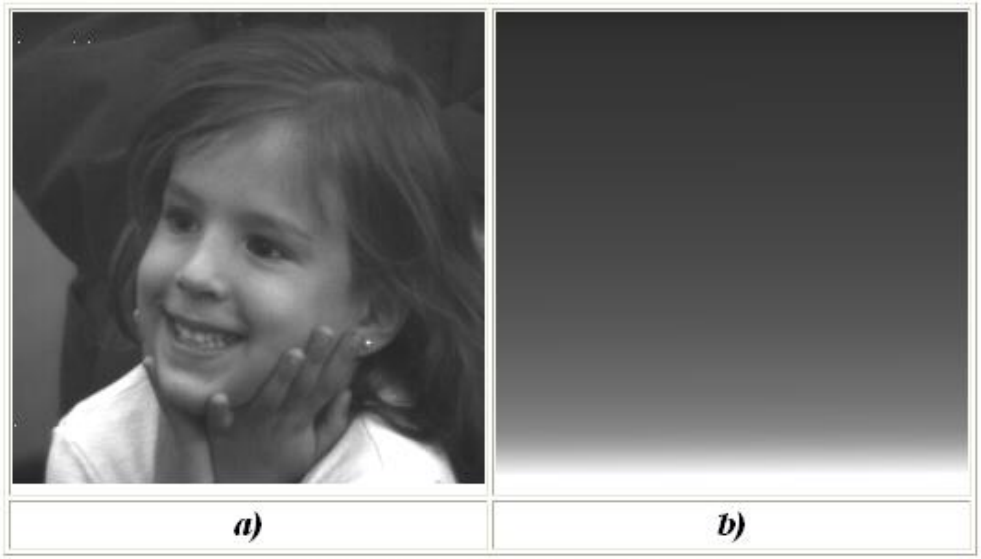
Generic figures of histograms

- The appearance of histogram reveals a lot of information about the contrast of the image and the mean gray level.
- An image of low contrast has a histogram that is concentrated around a small range of intensities.
- Images of high contrast are more interesting and pleasant for the human eye.

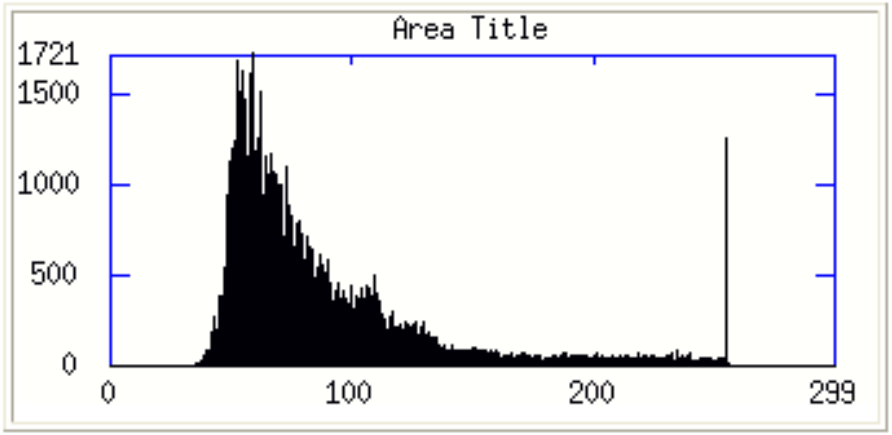


Two different images with the same histogram

- The appearance of histogram reveals a lot of information about the contrast of the image and the mean gray level **BUT :**
- It doesn't give any information regarding the location or the type of objects present in the image. This information is important because it is related to the image content.
- Two images can have identical histograms and still be completely different in terms of content.



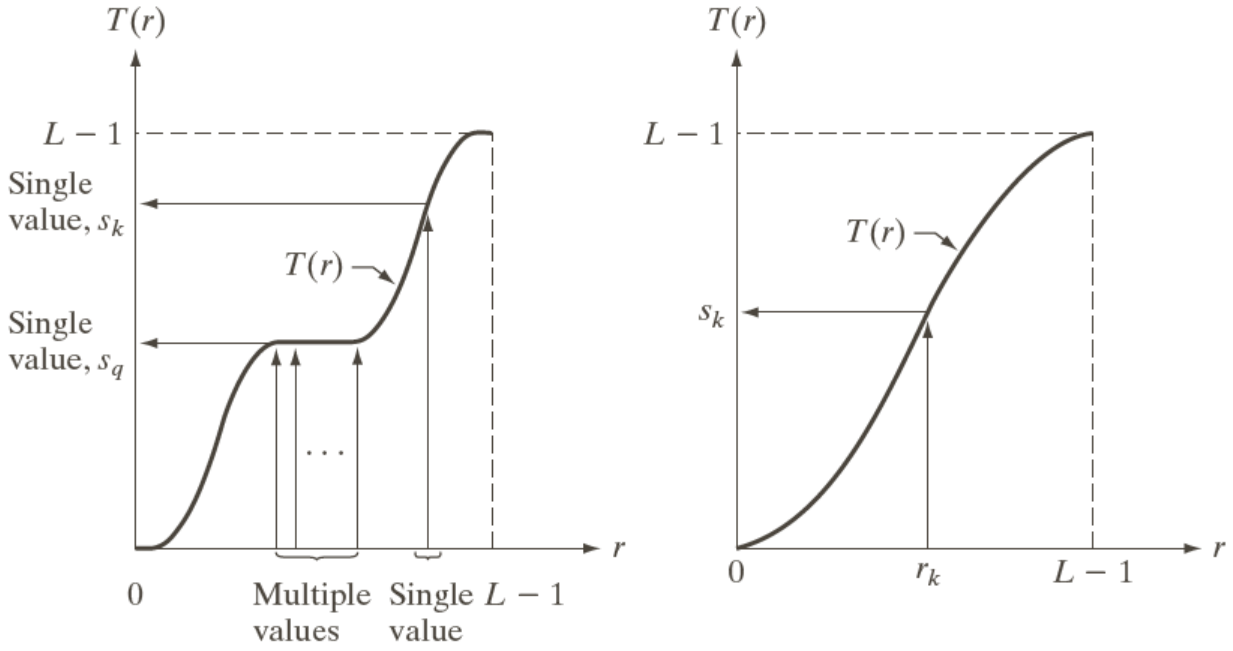
a)Original image; b)Sorted version



Histogram Processing: definition of intensity transformation

- Consider for the moment continuous intensity values $r \in [0, L-1]$.
- The value $r = 0$ represents black and the value $r = L-1$ represents white.
- We are looking for intensity transformations of the form:
$$s = T(r), 0 \leq r \leq L-1.$$
- The following conditions are imposed on $T(r)$:
  - $T(r)$ is monotonically increasing in $0 \leq r \leq L-1$ or strictly monotonically increasing in $0 \leq r \leq L-1$.
    - The above condition guarantees that ordering of the output intensity values will follow the ordering of the input intensity values (avoids reversal of intensities).
    - If $T(r)$ is strictly monotonically increasing then the mapping from s back to r will be 1-1.
  - $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$.
    This condition guarantees that the range of intensities of the output will be the same as the range of inthe input.

Monotonicity versus strict monotonicity



- In the above Figure on the left we cannot perform inverse mapping (from s to r).
- In the above Figure on the right inverse mapping is possible.

Modelling intensities as continuous variables

- Assume that an original intensity r is mapped to an intensity s through the transformation $s = T(r)$.
- We can view intensities r and s as random variables.
- Instead of histograms we use probability density functions (pdf) $p_r(r)$ and $p_s(s)$.
- Consider a minimal increment of the original intensity r to the intensity $r + dr$.
- The intensity $r + dr$ is mapped to an intensity $s + ds$ through the transformation $T(r)$.
- Since $T(r)$ is (monotonically) increasing we can easily say that $s + ds \geq s$.
- All values of the original intensity which are within the interval $[r, r + dr]$ will be mapped to new values within the interval $[s, s + ds]$.
- We can say that:

  Probability($r \leq$ original intensity $\leq r + dr$)=Probability($s \leq$ new intensity $\leq s + ds$) or in mathematical terms:

  $$\int_r^{r+dr} p_r(w)dw = \int_s^{s+ds} p_s(w)dw$$

Modelling intensities as continuous variables cont.

- Probability($r \leq$ original intensity $\leq r + dr$)=Probability($s \leq$ new intensity $\leq s + ds$) or in mathematical terms:

  $\int_r^{r+dr} p_r(w)dw = \int_s^{s+ds} p_s(w)dw$
- If dr is small enough we can assume that $p_r(r)$ remains almost constant within the interval $[r, r + dr]$ and equal to $p_r(r)$.
- We can choose dr to be as small as to be able to assume that ds is small enough and that $p_s(s)$ remains almost constant within the interval $[s, s + ds]$ and equal to $p_s(s)$.
- $\int_r^{r+dr} p_r(w)dw = \int_s^{s+ds} p_s(w)dw \Rightarrow p_r(r)dr = p_s(s)ds$
- The above analysis is depicted in the Figure of the next slide.

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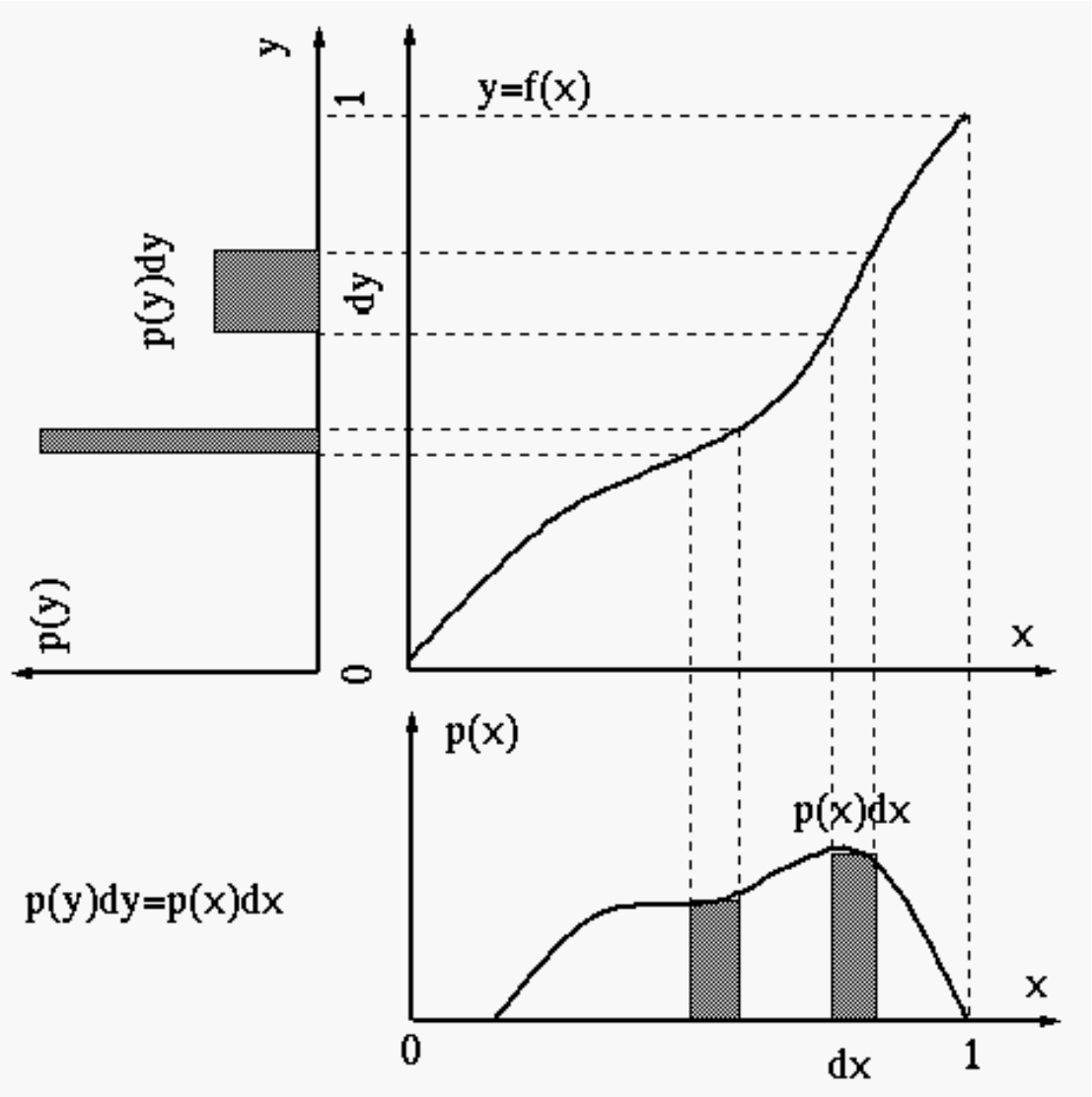
Modelling intensities as continuous variables



Histogram Equalization: continuous form

- A transformation of particular importance in Image Processing is the cumulative distribution function (CDF) of a random variable.

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

- It is an increasing function since for $r_2 \geq r_1$ we see that:

$$s_2 = T(r_2) = (L-1)\int_0^{r_2} p_r(w)dw$$

$$= (L-1)\int_0^{r_1} p_r(w)dw + (L-1)\int_{r_1}^{r_2} p_r(w)dw$$

$$= T(r_1) + (L-1)\int_{r_1}^{r_2} p_r(w)dw \geq T(r_1)$$

Histogram Equalization: continuous form

- We showed previously that $p_r(r)dr = p_s(s)ds$
- Therefore, $p_s(s) = \frac{p_r(r)}{\frac{ds}{dr}}$.
- From the definition

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

we have that:

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\int_0^r p_r(w)dw = (L-1)p_r(r)$$

Hence, $p_s(s) = \frac{p_r(r)}{\frac{ds}{dr}} = \frac{p_r(r)}{(L-1)p_r(r)} = \frac{1}{(L-1)}, s = 1, \ldots, L-1.$

Therefore, the new variable s is uniformly distributed.

Histogram Equalization: discrete case

- The formula for histogram equalisation in the discrete case is given by a straightforward modification of the formula that corresponds to the continuous-time case.
- Instead of probability density functions (pdf) $p_r(r)$ and $p_s(s)$ we now use histograms.
- The discrete input intensity r_k is mapped onto a new discrete intensity s_k through the following transformation:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j.$$

r_k: input intensity

s_k: new intensity

n_j: frequency of intensity j

MN: total number of image pixels

Histogram Equalization: discrete case cont.

- In the ideal continuous case histogram equalization produces a new variable s which is uniformly distributed.
- In the discrete case the histogram of the new discrete variable s_k is far from flat but:
  - The new histogram is still much more stretched (extended) than the original histogram.
  - The new intensity variable always reaches white since

$$s_{L-1} = T(r_{L-1}) = (L-1)\sum_{j=0}^{L-1} p_r(r_j) = L-1$$

  - **In other words, this process usually results in an enhanced image, with an increase in the dynamic range of pixel values.**

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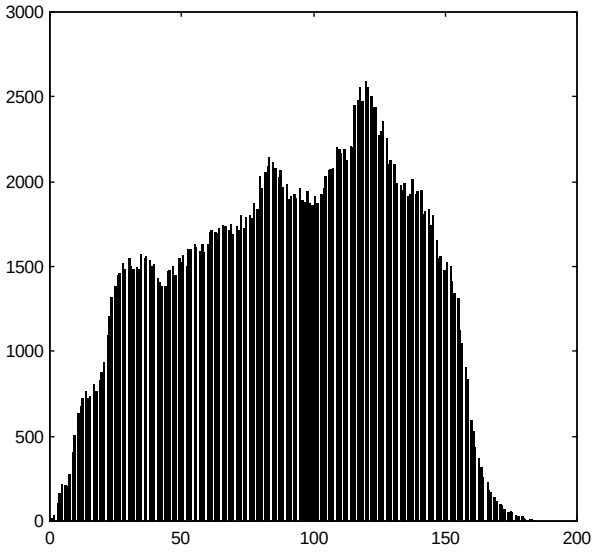
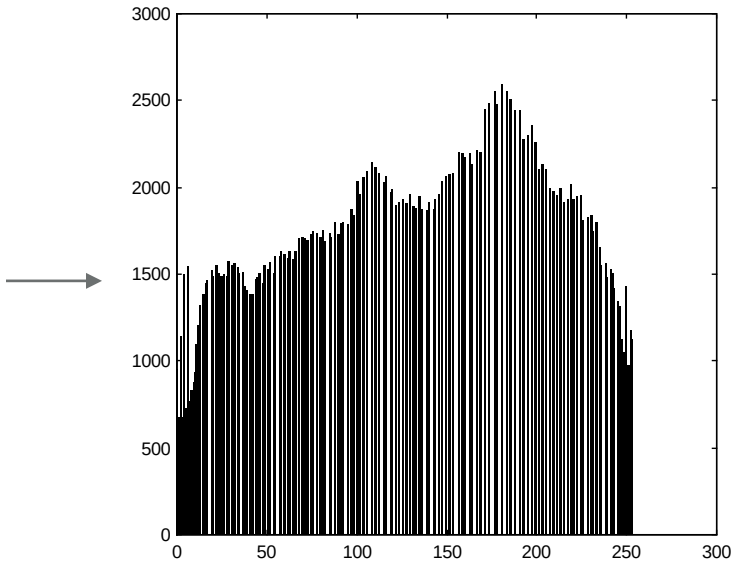
Histogram Equalization: discrete case cont.

- In the Figures below you can see how histogram could look like after equalizing a digital image.
- It is more “extended” and slightly “flatter” compared to the original histogram

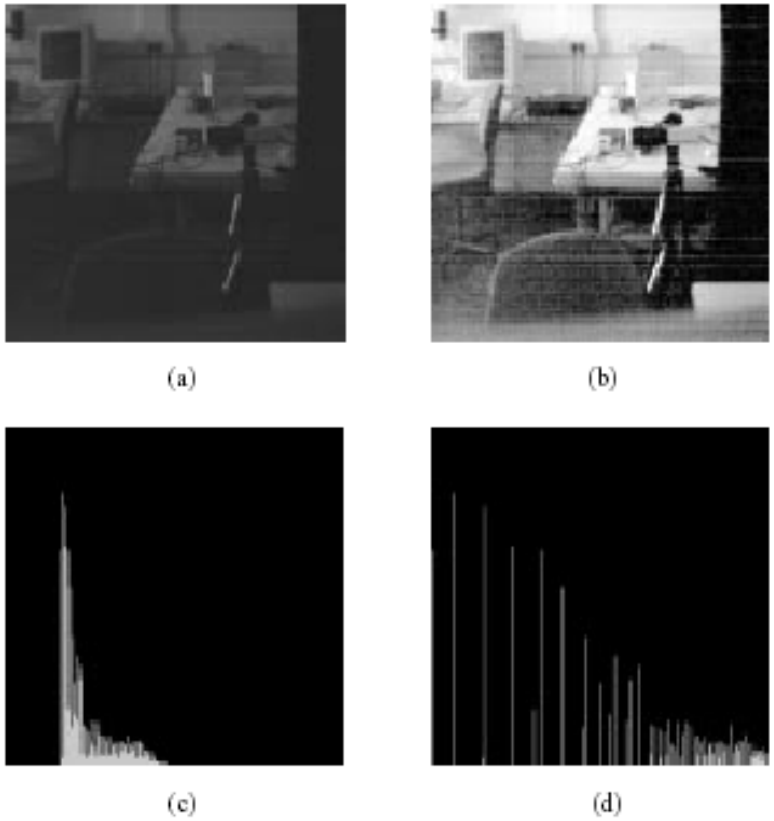


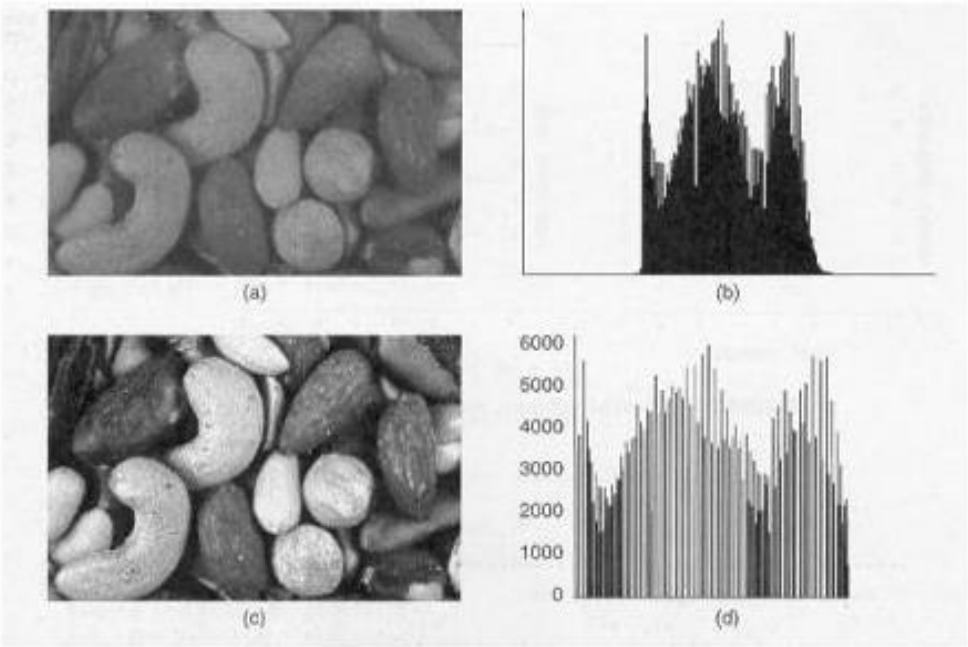
Before equalization



After equalization

Histogram Equalization: Examples





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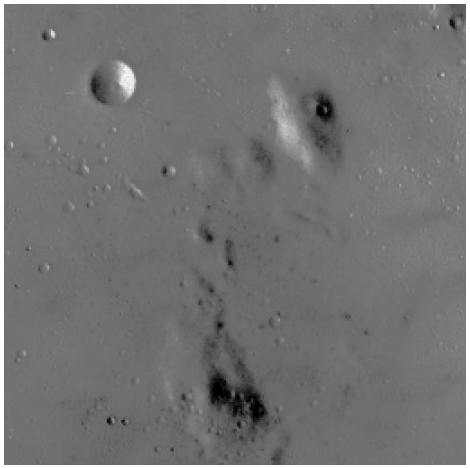
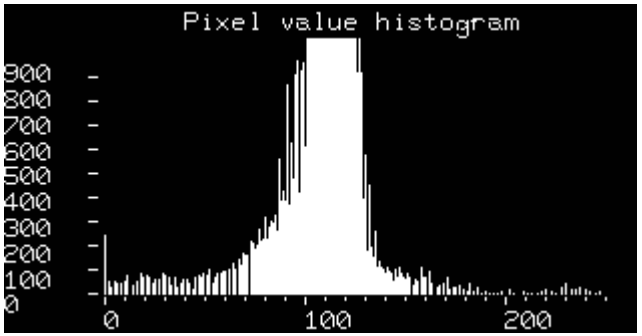
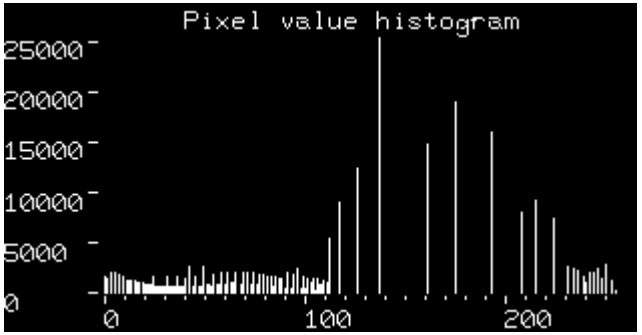
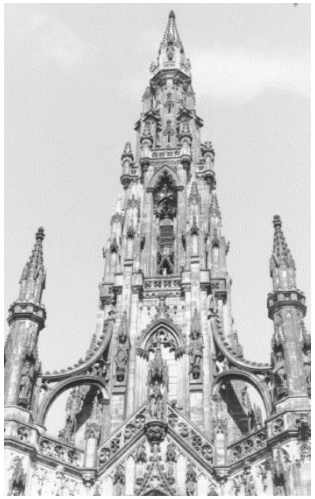
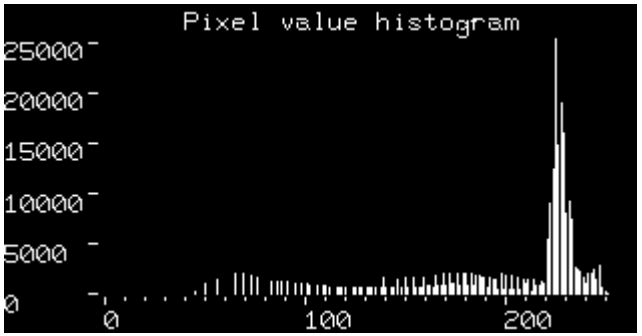
Histogram Equalization: Examples

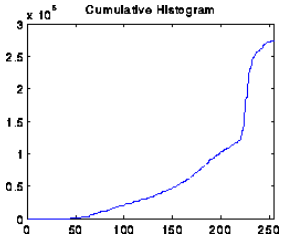












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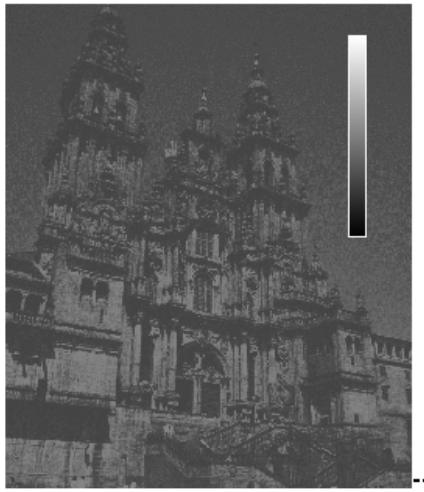
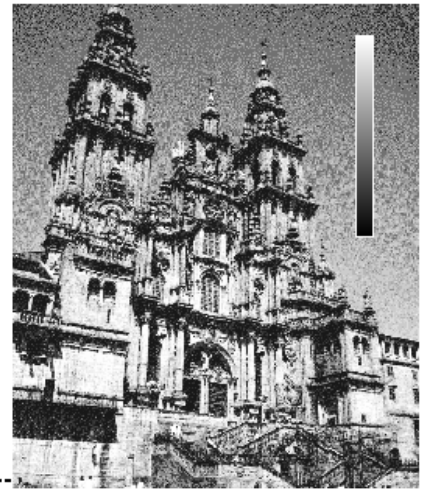
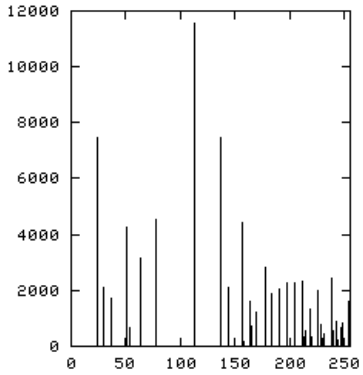
Histogram Equalization: Examples

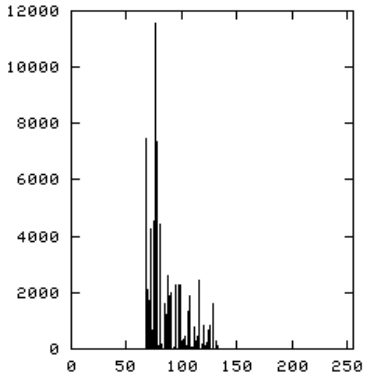


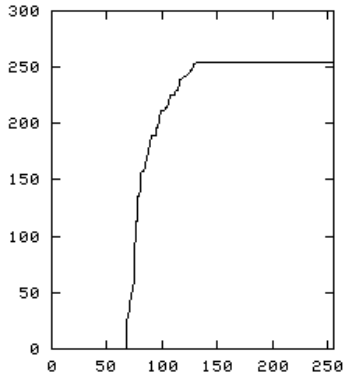


Left: original image, Right: histogram equalized image



Left: histogram equalized image, Right: histogram of the histogram equalized image





Left: histogram of the original image, Right: normalized cumulative histogram of the original image

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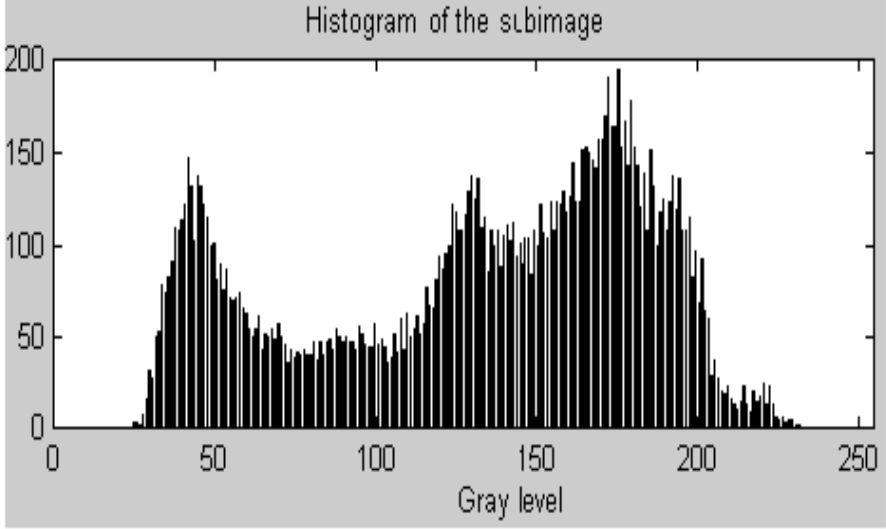
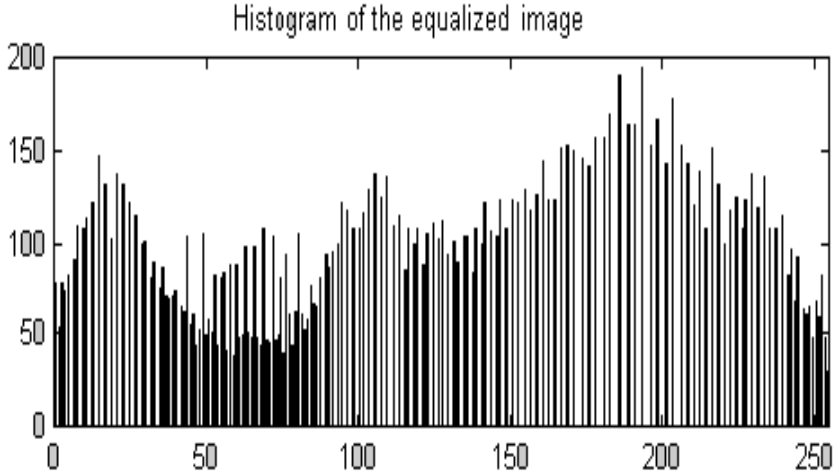
Histogram Equalization: Examples





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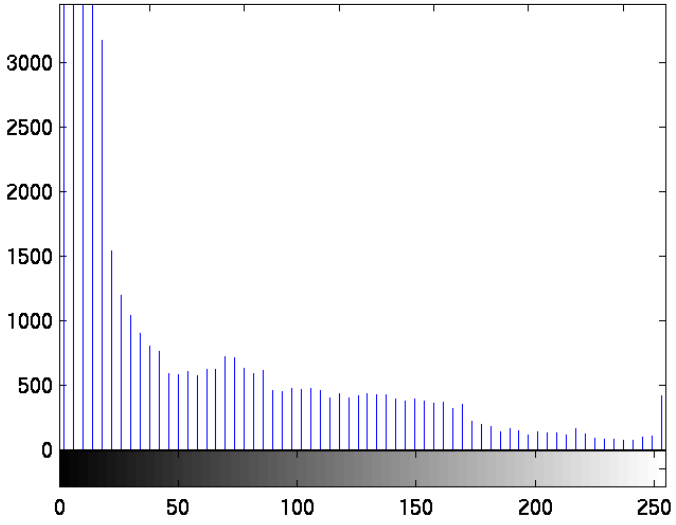
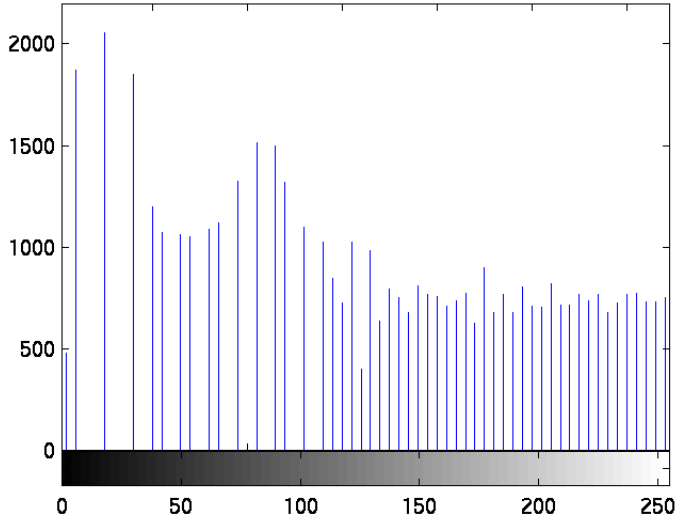
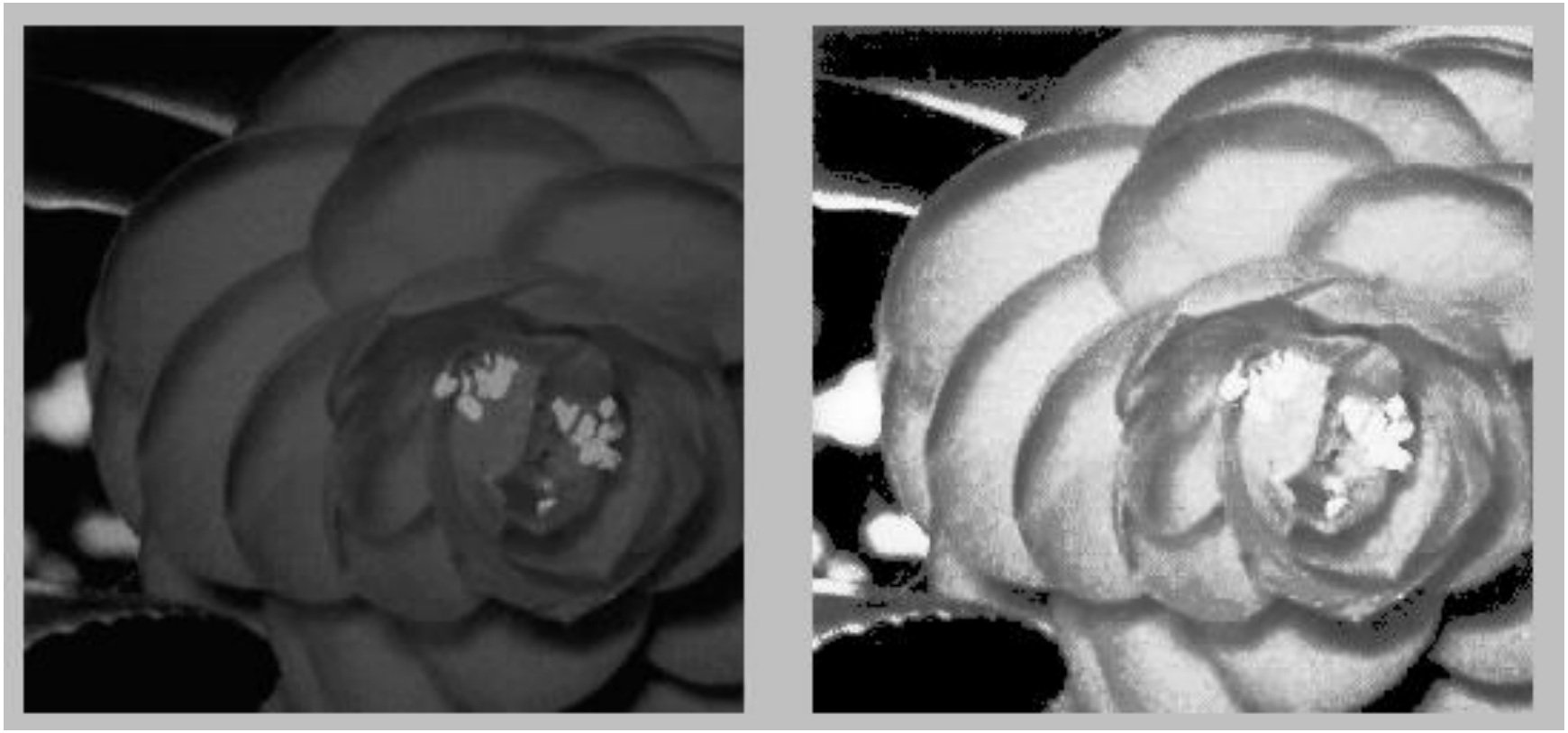
Histogram Equalization: Examples

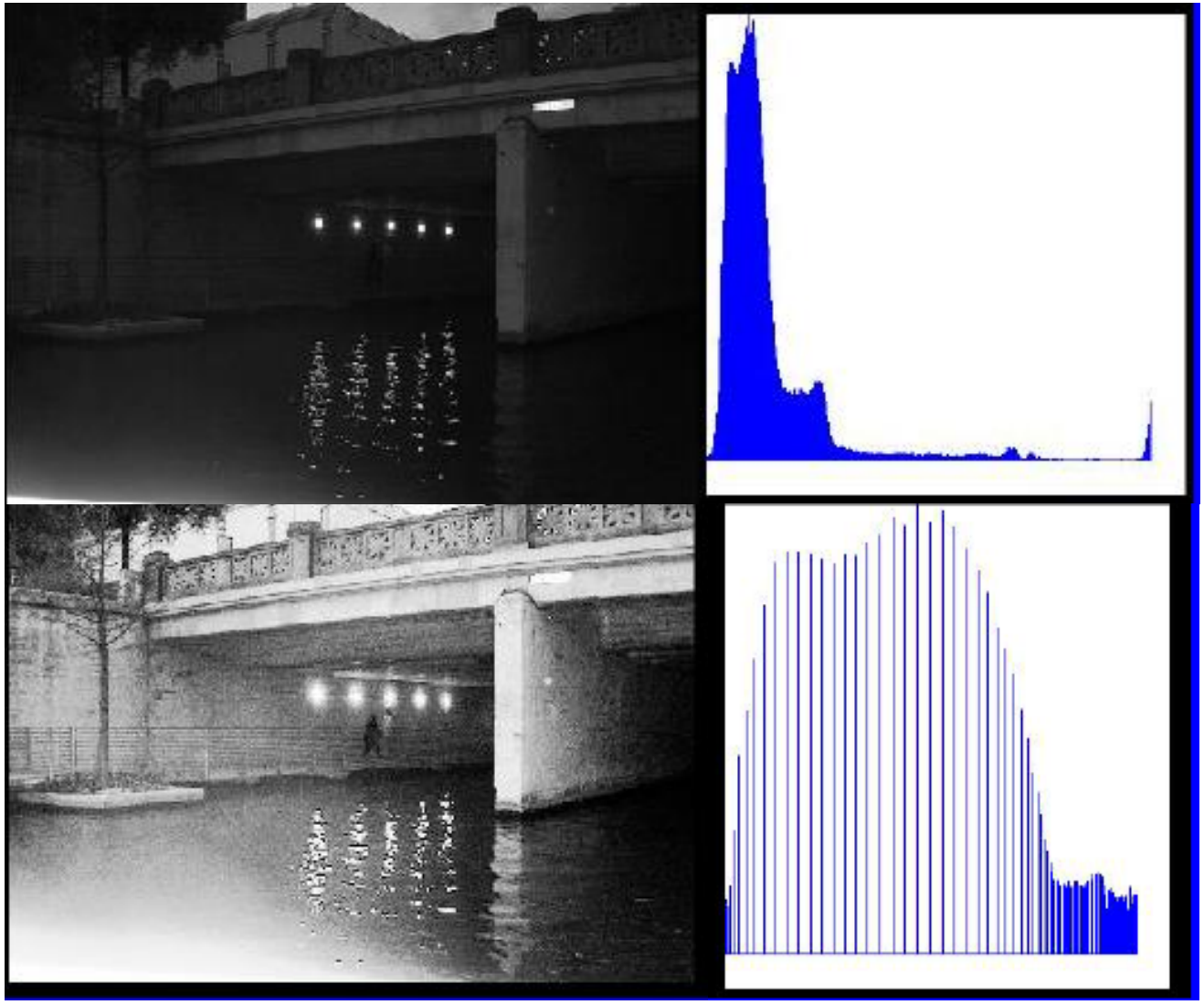




Histogram Equalization: Examples



Examples of histogram equalization



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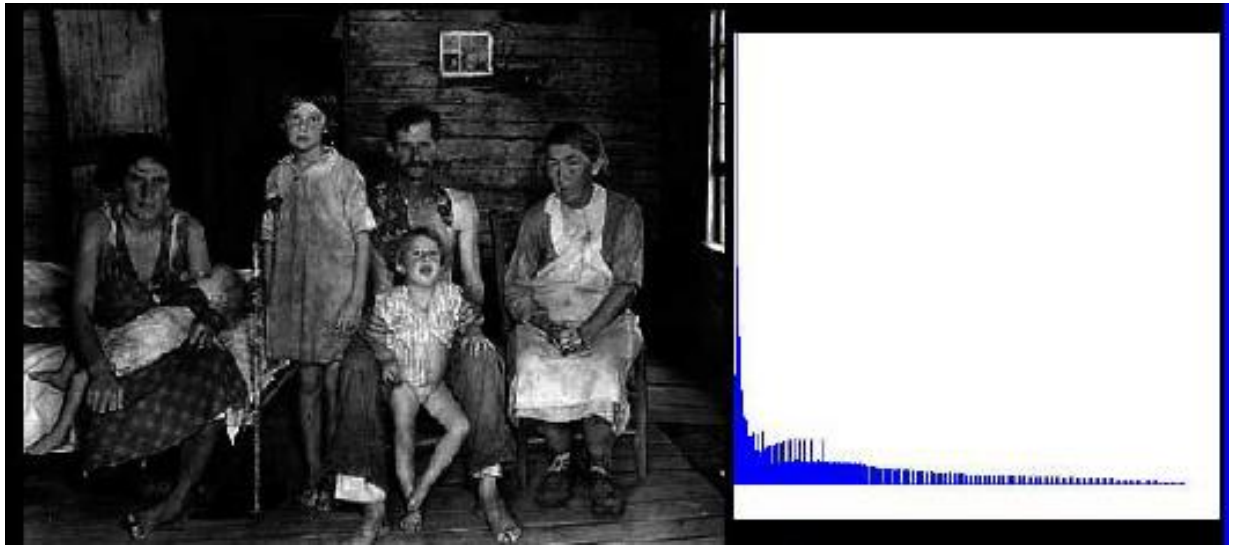
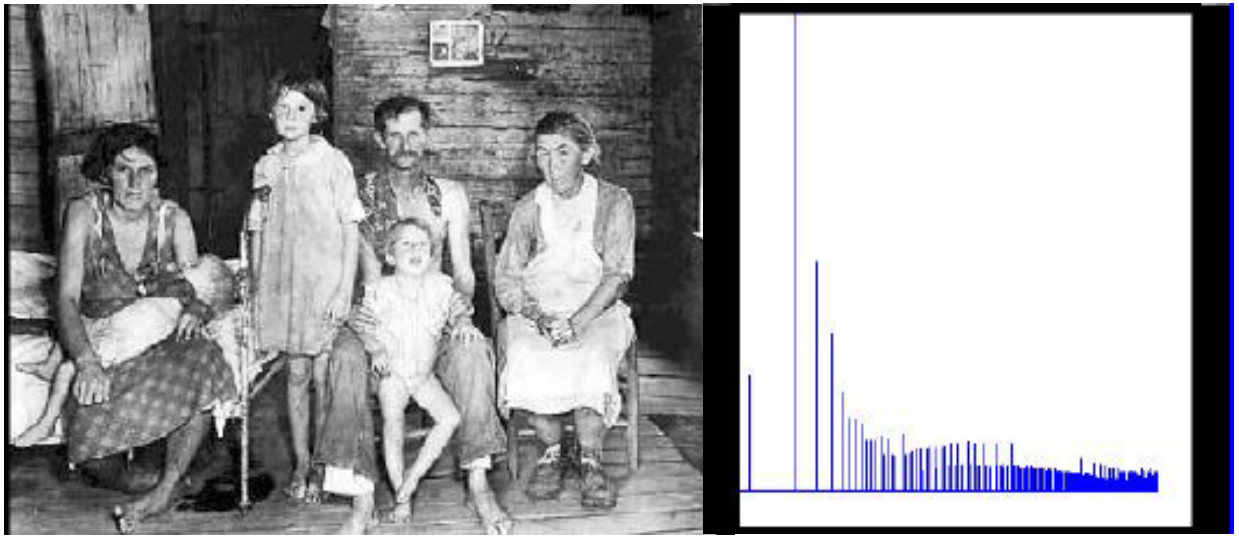
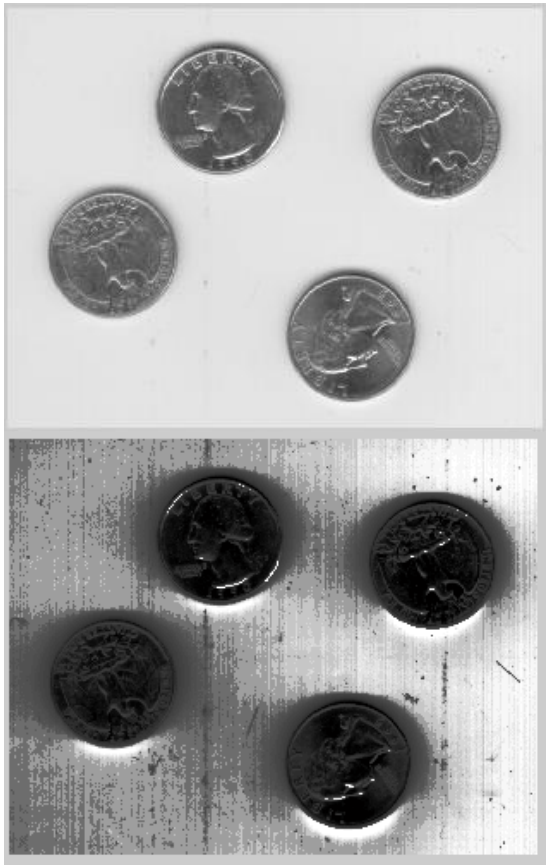
Examples

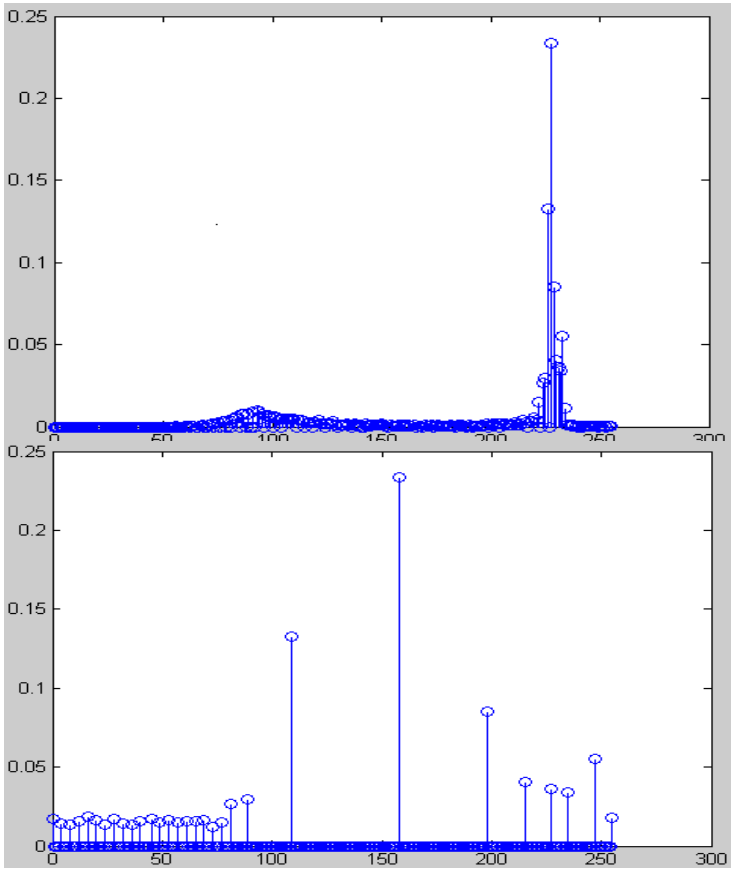




Histogram Equalization is not always desirable

Histogram equalization may not always produce desirable results, particularly if the given histogram is very narrow. It can produce false edges and false regions. It can also increase image “graininess” and “patchiness.”





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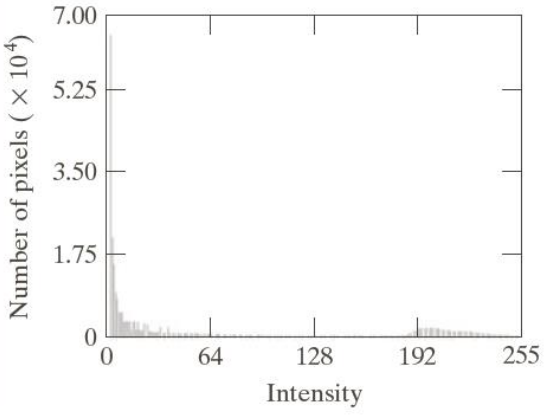
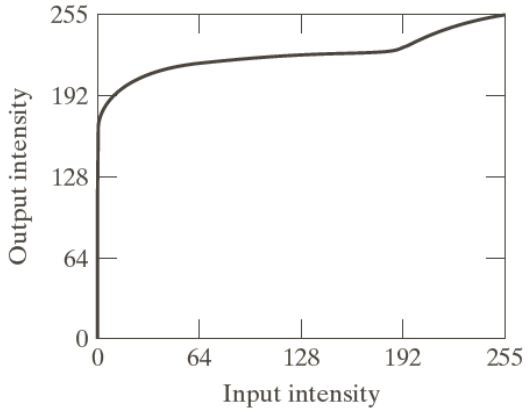
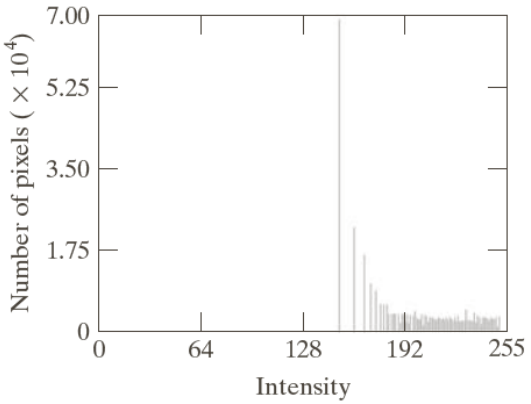
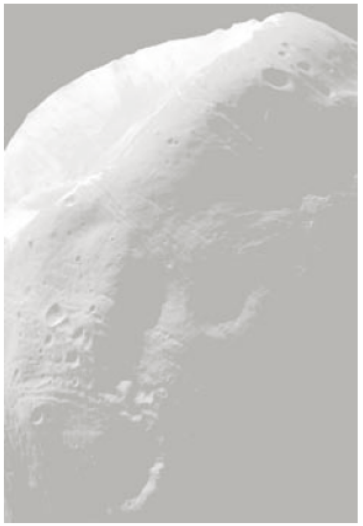
Another example of an unfortunate histogram equalization

- Example of image of Phobos (Mars moon) and its histogram.
- Histogram equalization (bottom of right image) does not always provide the desirable results.









Example: discrete case

- A 3 –bit 64×64 image with 8 intensities is described in the table.
- Discrete histogram equalised intensity levels are obtained through:

$$s_k = T(r_k) = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j.$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

After applying histogram equalisation:

$$s_0 = T(r_0) = \frac{(L-1)}{MN} \sum_{j=0}^{0} n_j = \frac{7}{4096} n_0 = \frac{7 \cdot 790}{4096} = 1.33$$

$$s_1 = T(r_1) = \frac{7}{4096} \sum_{j=0}^{1} n_j = \frac{7}{4096} (790 + 1023) = 3.08$$

$$s_2 = T(r_2) = \frac{7}{4096} \sum_{j=0}^{2} n_j = \frac{7}{4096} (790 + 1023 + 850) = 4.55$$

$$s_3 = T(r_3) = \frac{7}{4096} \sum_{j=0}^{3} n_j = \frac{7}{4096} (790 + 1023 + 850 + 656) = 5.67$$

$$s_4 = T(r_4) = \frac{7}{4096} \sum_{j=0}^{4} n_j = \frac{7}{4096} (790 + 1023 + 850 + 656 + 329) = 6.23$$

$$s_5 = T(r_5) = \frac{7}{4096} \sum_{j=0}^{5} n_j = \frac{7}{4096} (790 + 1023 + 850 + 656 + 329 + 245) = 6.65$$

$$s_6 = T(r_6) = \frac{7}{4096} \sum_{j=0}^{6} n_j = \frac{7}{4096} (790 + 1023 + 850 + 656 + 329 + 245 + 122) = 6.86$$

$$s_6 = T(r_6) = \frac{7}{4096} \sum_{j=0}^{6} n_j = \frac{7}{4096} (790 + 1023 + 850 + 656 + 329 + 245 + 122 +) = 7$$

Example: discrete case cont.

By rounding to the nearest integer we get:

$s_0 = 1.33 \rightarrow 1, s_1 = 3.08 \rightarrow 3, s_2 = 4.55 \rightarrow 5, s_3 = 5.67 \rightarrow 6$

$s_4 = 6.23 \rightarrow 6, s_5 = 6.65 \rightarrow 7\ s_6 = 6.86 \rightarrow 7, s_7 = 7 \rightarrow 7$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

The histogram of the new variable is found as follows:

$p(0) = 0$

$p(1) = p(s_0) = p(r_0) = 0.19$

$p(2) = 0$

$p(3) = p(s_1) = p(r_1) = 0.25$

$p(4) = 0$

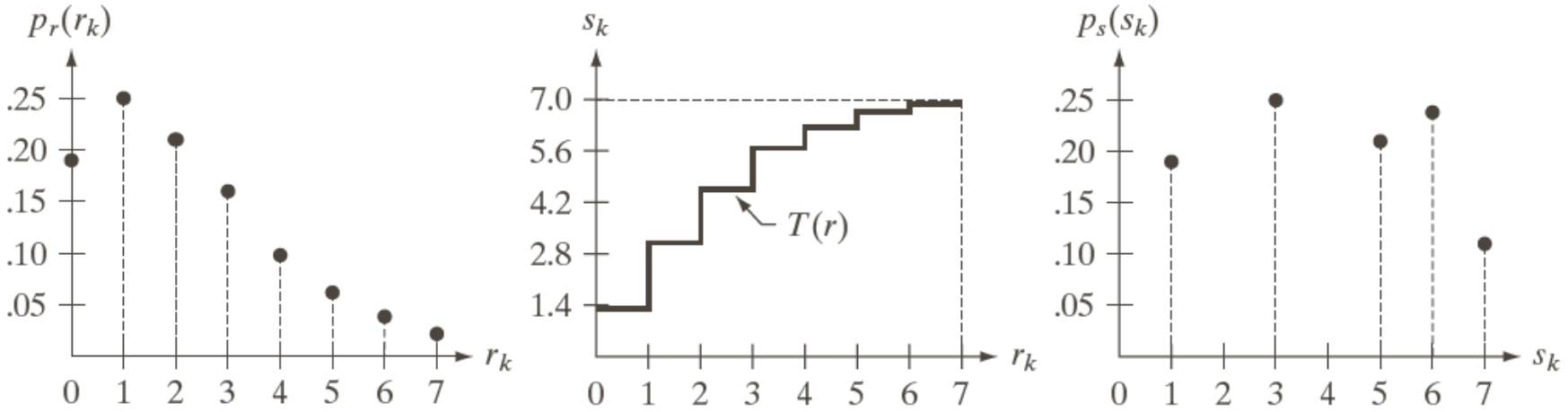
$p(5) = p(s_2) = p(r_2) = 0.21$

$p(6) = p(s_3) + p(s_4) = p(r_3) + p(r_4) = 0.24$

$p(7) = p(s_5) + p(s_6) + p(s_7) = p(r_5) + p(r_6) + p(r_7) = 0.1$

Example: discrete case cont.

Refer to the following figures for original histogram, transformation function and new histogram.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

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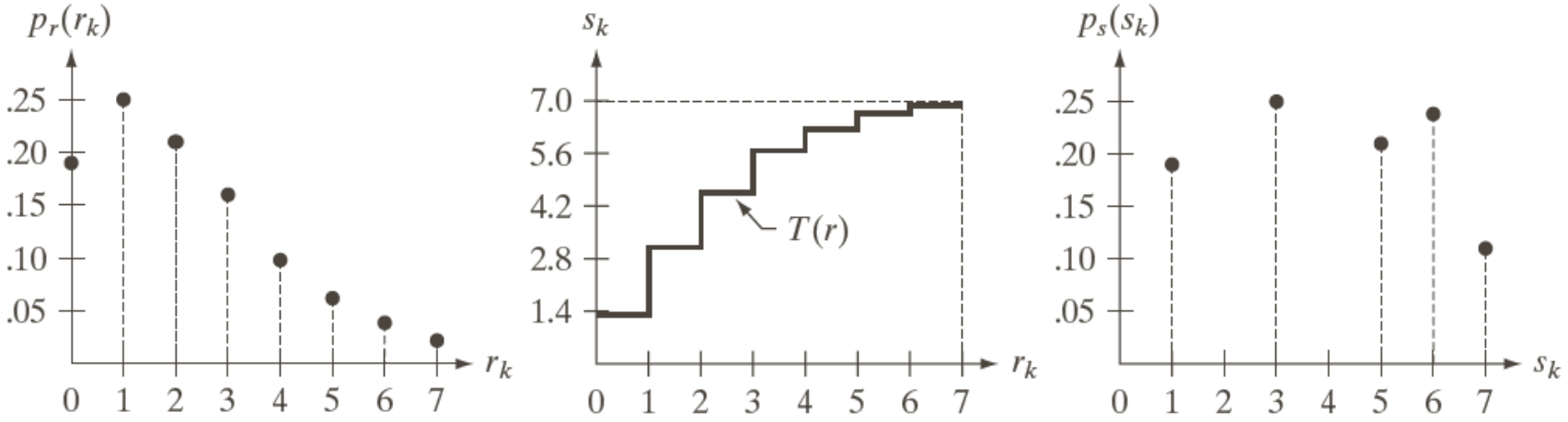
Example: discrete case cont.

- Refer to the following figures for original histogram, transformation function and new histogram.
- Notice that due to discretization, the resulting histogram will rarely be perfectly flat.
- However, it will more “extended” compared to the original histogram.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

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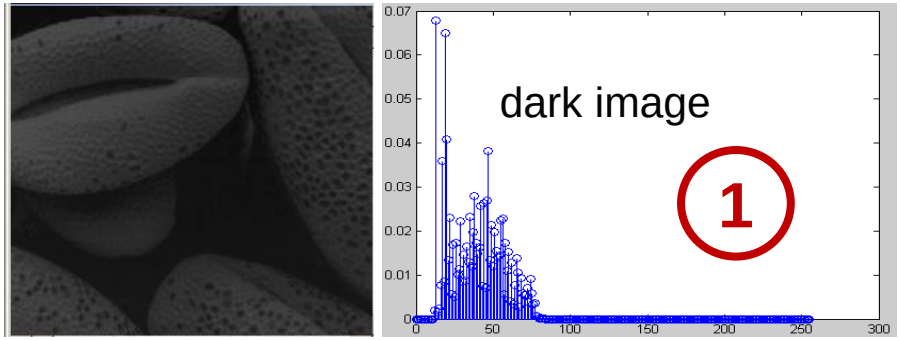
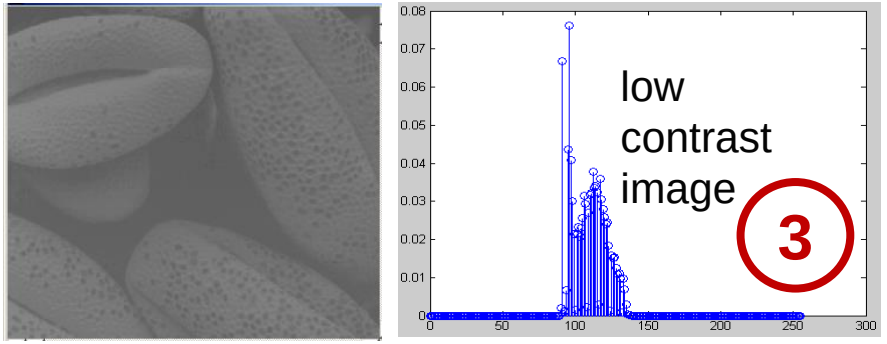
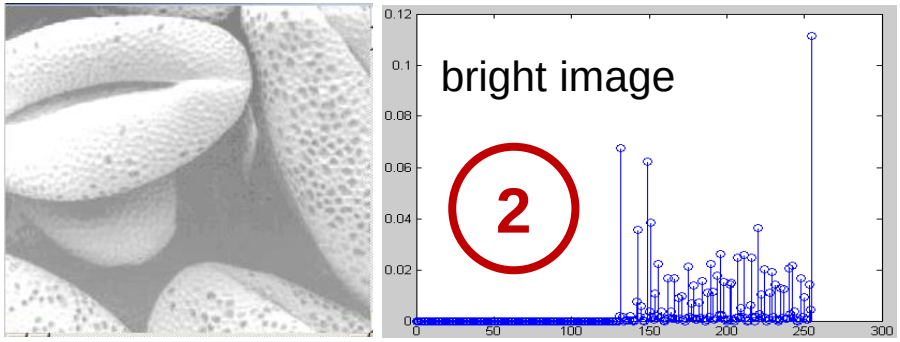
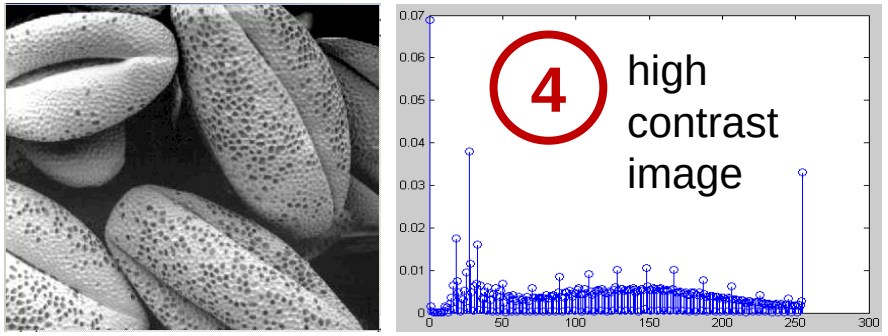
A set of images with same content but different histograms









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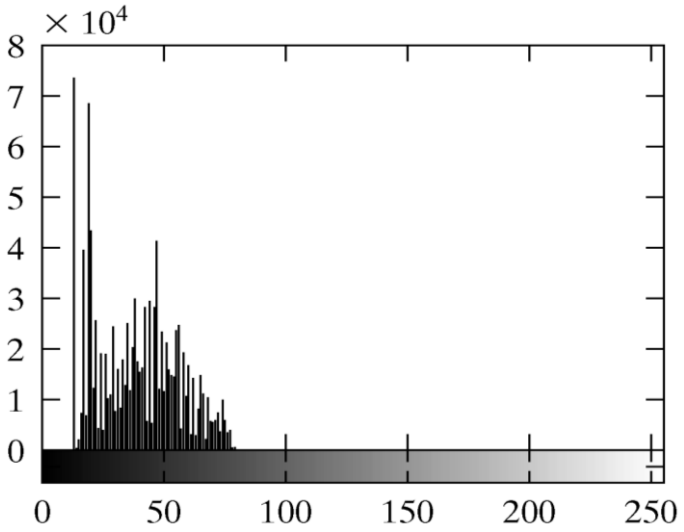
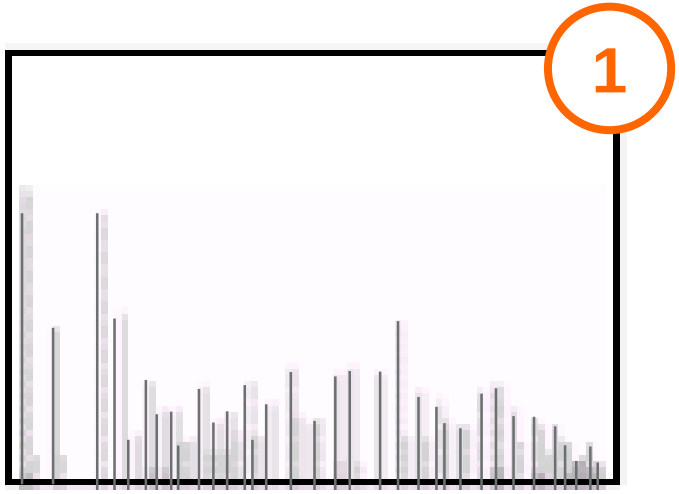
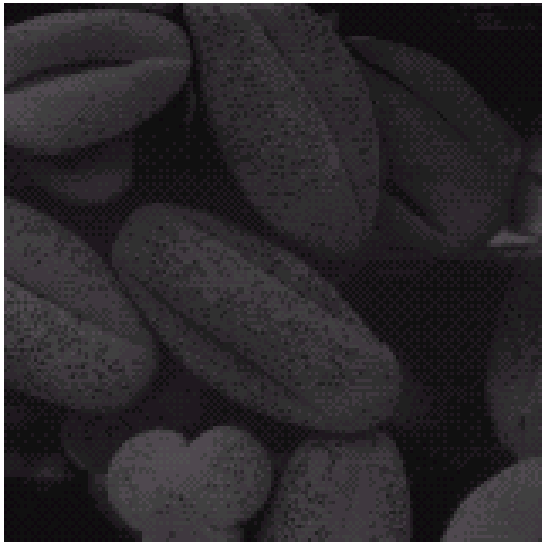
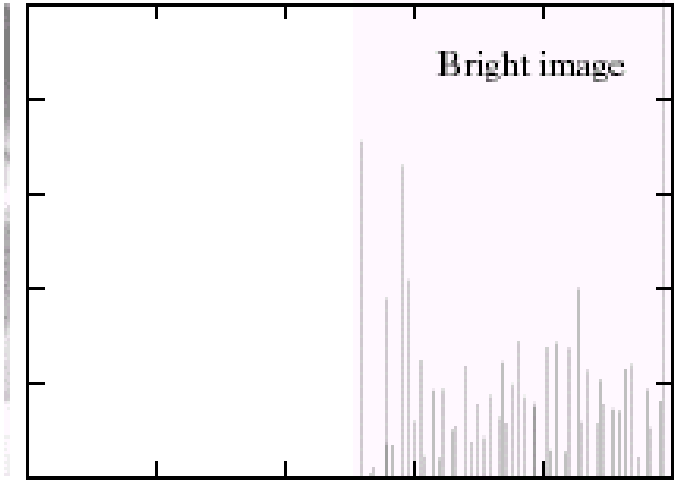
Histogram equalization applied to the dark image

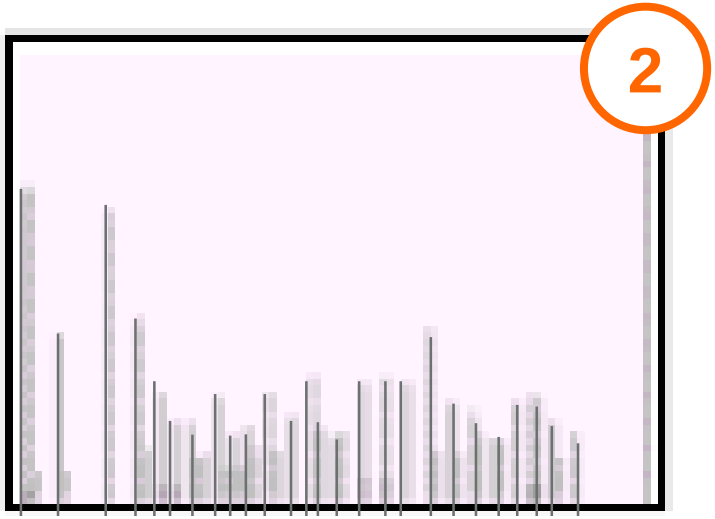






Histogram equalization applied to the bright image





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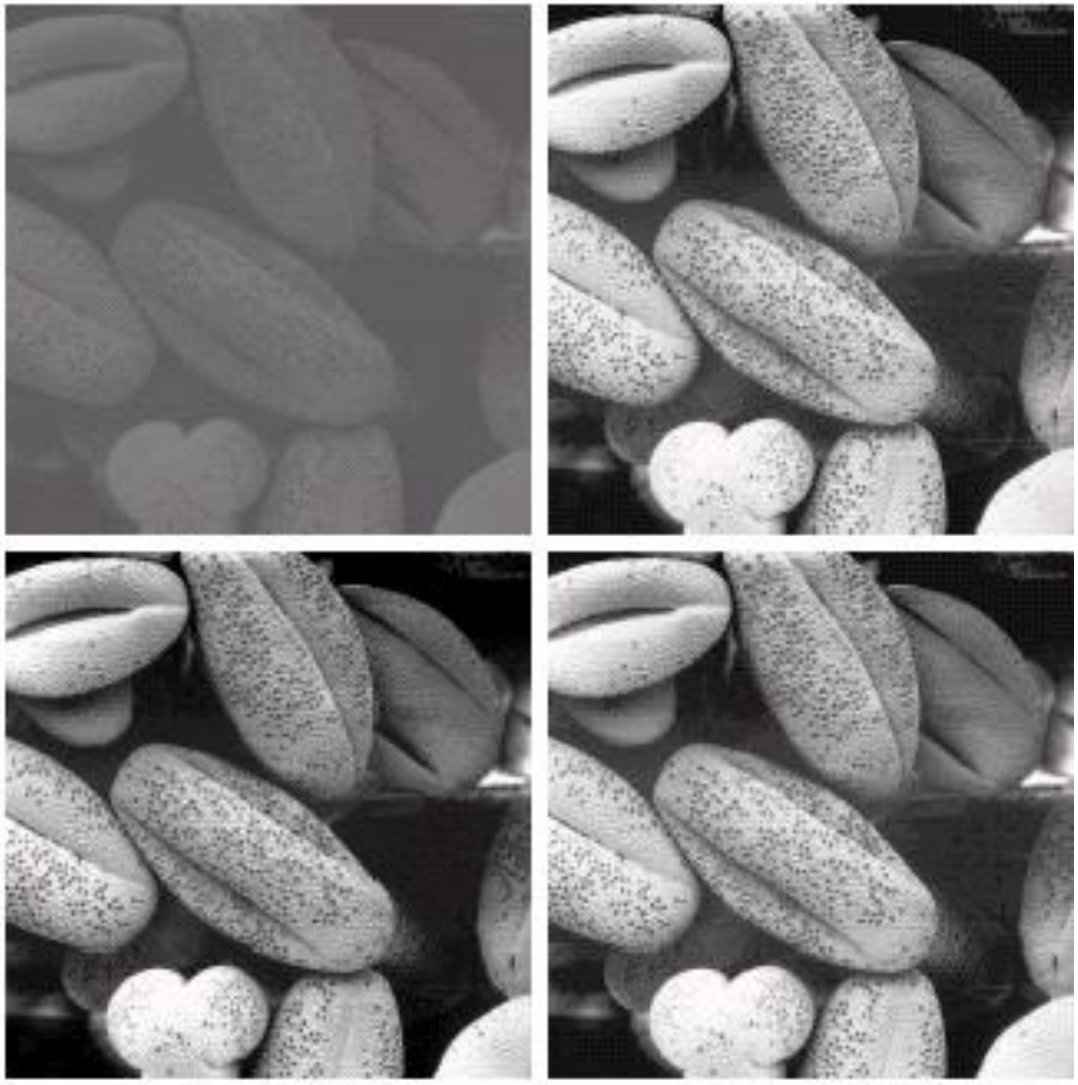
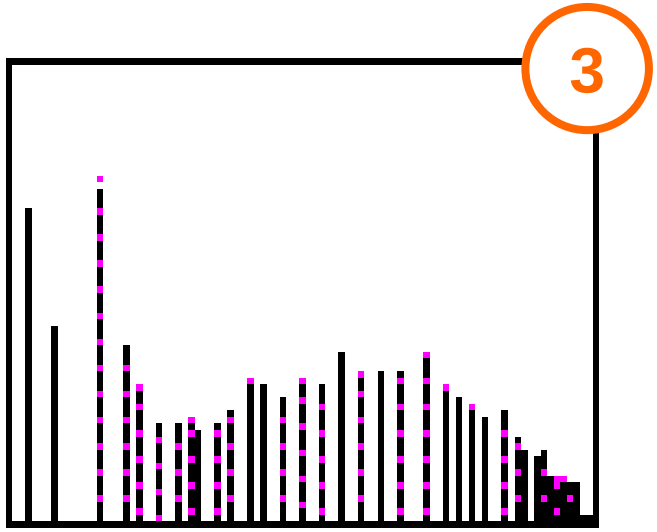
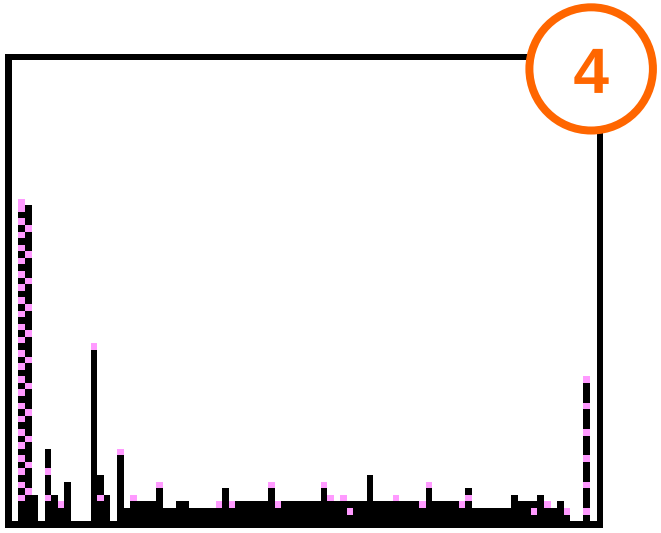
Histogram equalization applied to the low and high contrast images







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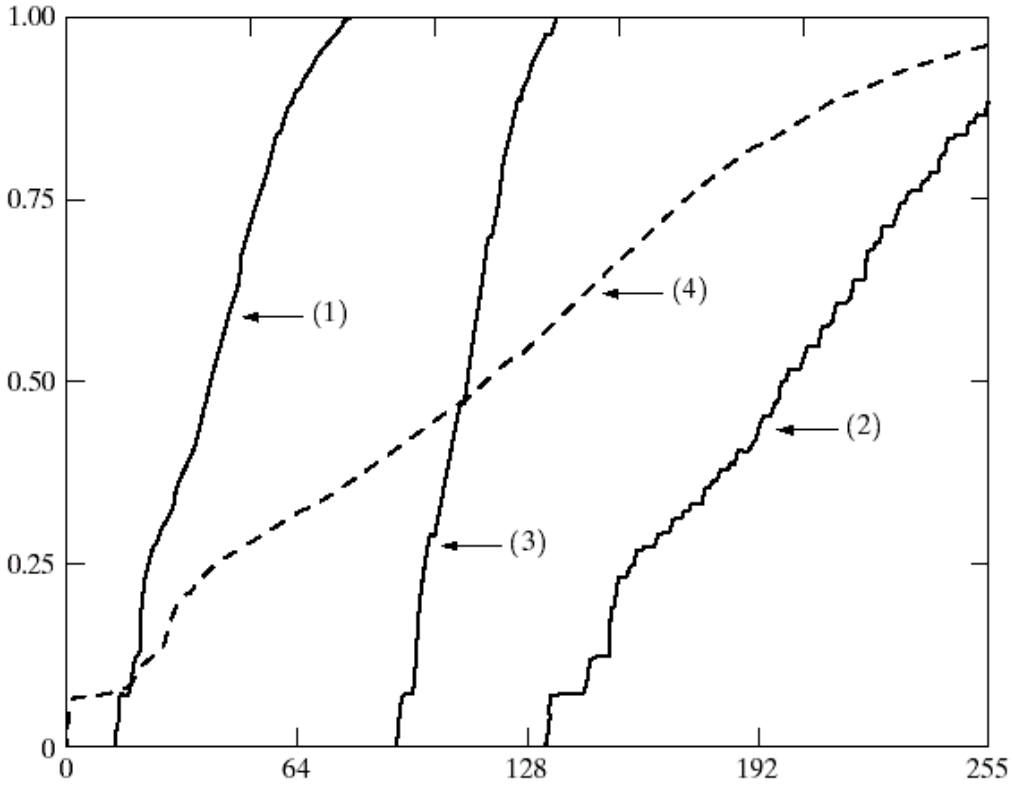
Transformation functions for histogram equalization for the previous example

- The function $T(r)$ used to equalize the four images of the previous example is shown below.
- Observe that the transformation function in cases 1,2,3 maps a small range of intensities to the entire range of intensities.
- Observe that for an image which is already bright, the transformation is an almost diagonal line.



Histogram Specification

- We are looking for a technique which can provide an image with any pre-specified histogram.
- This is called **histogram specification**.
- We assume that the original image has pdf $p_r(r)$.
- We are looking for a transformation $z = T(r)$ which provides an image with a specific pdf $p_z(z)$.
- This technique will use histogram equalization as an intermediate step.

Histogram Specification cont.

- We first equalize the given image

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

- If we had the desired image we could equalized it and obtain

$$s = T(z) = (L-1)\int_0^z p_z(w)dw$$

- Based on the above we can assume that

$$G(z) = T(r) \Rightarrow z = G^{-1}(T(r))$$

- In the case of continuous variables, if $p_r(r)$ and $p_z(z)$ are given we can obtain z after formulating the functions T, G and G^{-1}.

Histogram Specification: continuous form

- In the discrete case we first equalize the initial histogram of the image:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j.$$

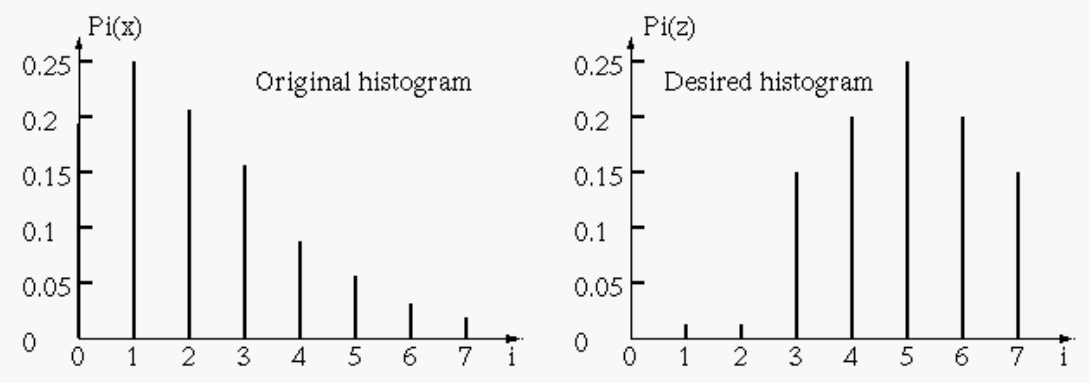
- Then we equalize the target histogram

$$s_k = G(z_q) = (L-1)\sum_{i=0}^{q} p_z(r_i) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j.$$

- Finally, we try to obtain some type of inverse transform

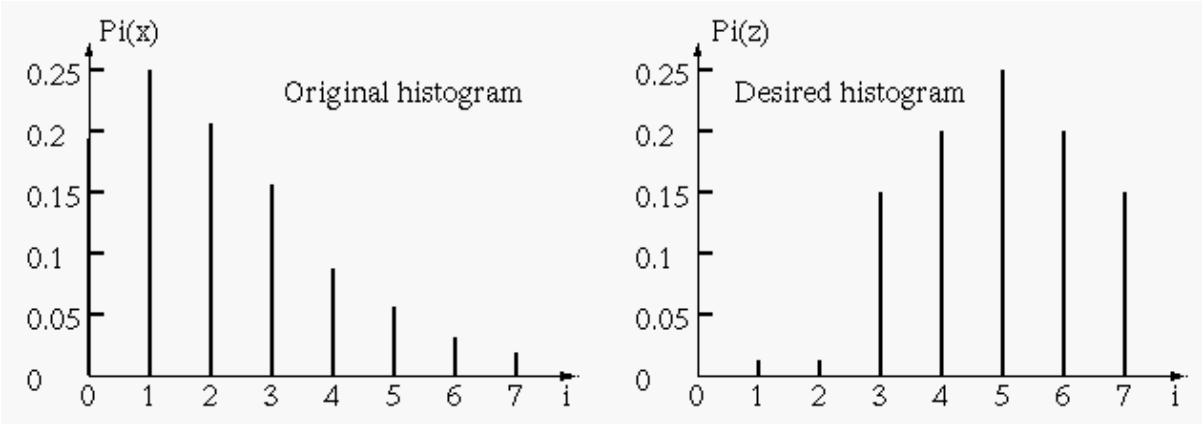
$$z_q = G^{-1}(s_k) = G^{-1}\big(T(r_k)\big)$$

Histogram Specification: Example



original intensities	number of pixels	probability	cumulative probability CM	equalised intensities CM x 7	normalised equalised intensities
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7

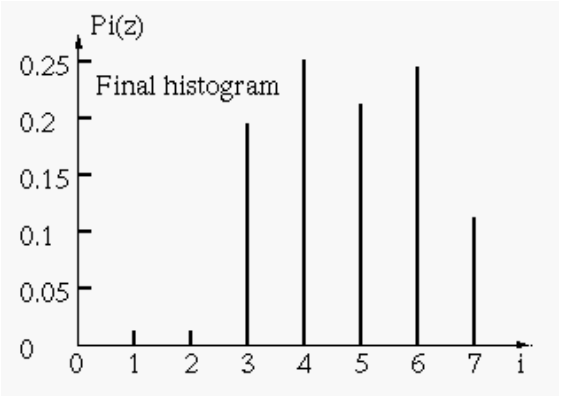
Histogram Specification: Example cont.



desired intensities	probability	cumulative probability CM	equalised intensities CM x 7	normalised equalised intensities
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0.15	0.15	1.05	1
4	0.2	0.35	2.45	2
5	0.3	0.65	4.55	5
6	0.2	0.85	5.95	6
7	0.15	1	7	7

Histogram Specification: Example cont.

original intensities	equalised intensities (AVAILABLE)	desired intensities	equalised intensities (NOT AVAILABLE!!!)	equalised intensities (available)	NEW intensities (available)
0	1	0	0	1	3
1	3	1	0	3	4
2	5	2	0	5	5
3	6	3	1	6	6
4	6	4	2	6	6
5	7	5	5	7	7
6	7	6	6	7	7
7	7	7	7	7	7



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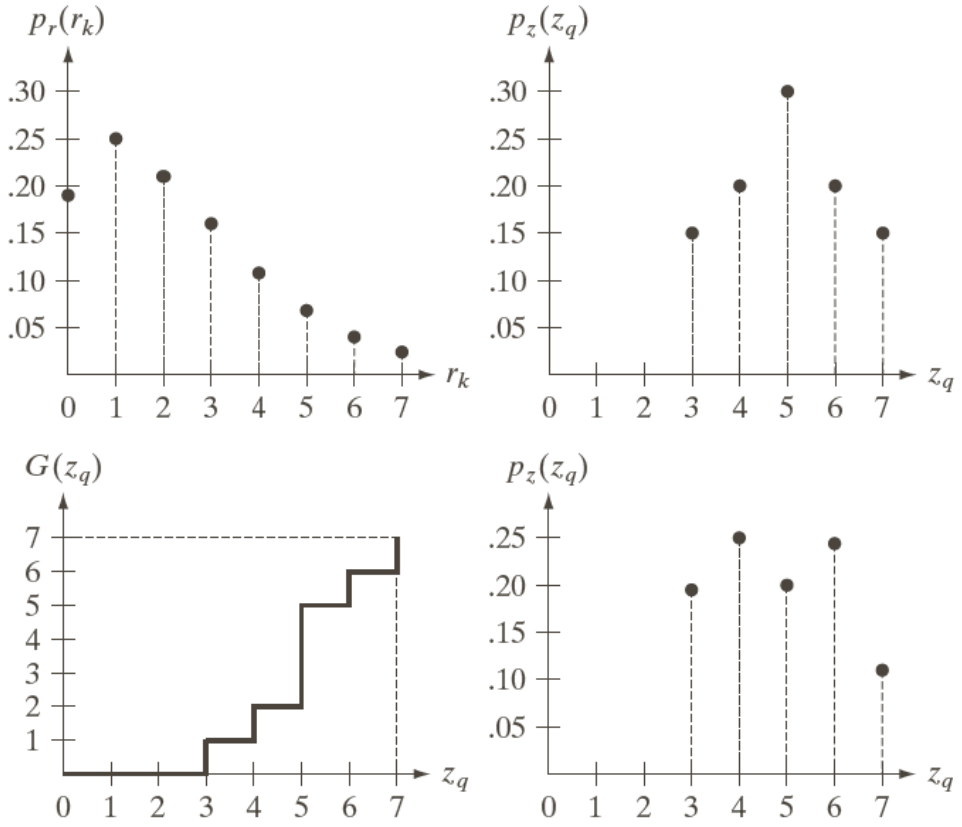
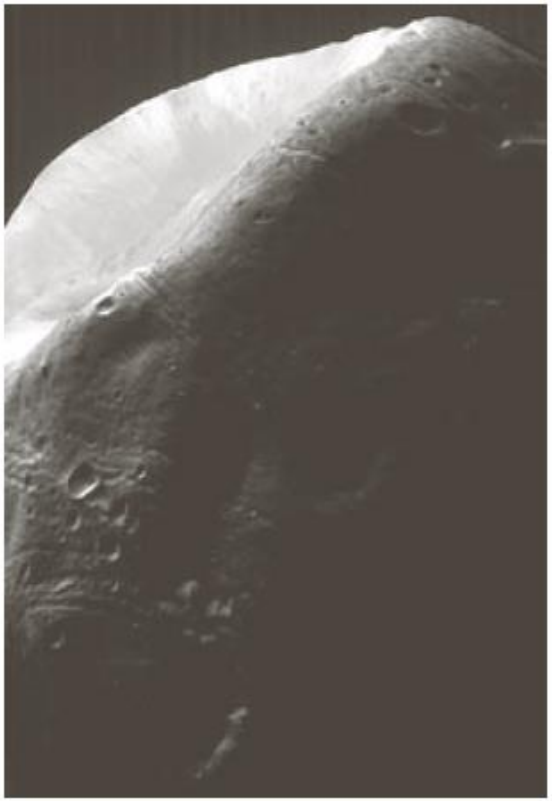
Histogram Specification: Example cont.

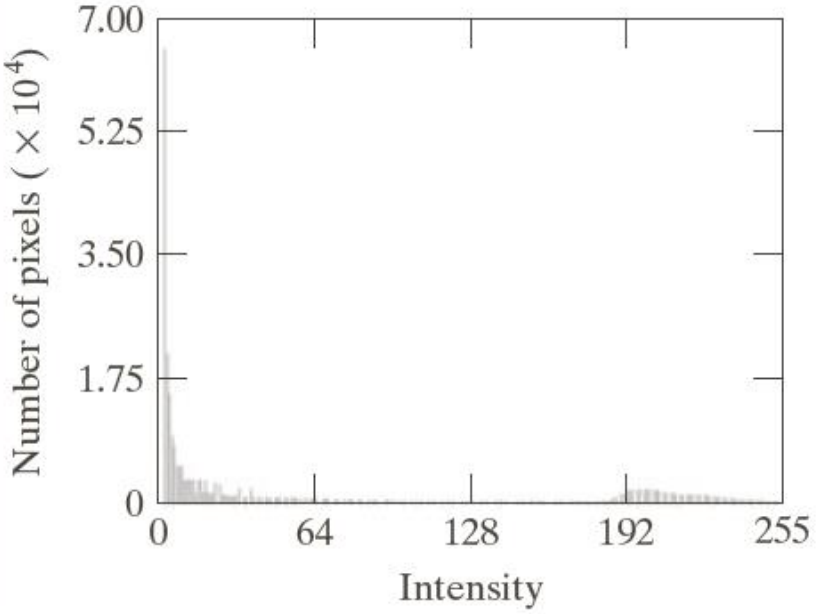
Notice that due to discretization, the resulting histogram will rarely be exactly the same as the desired histogram.

- Top left: original pdf
- Top right: desired pdf
- Bottom left: desired CDF
- Bottom right: resulting pdf



Histogram Specification: Example





a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

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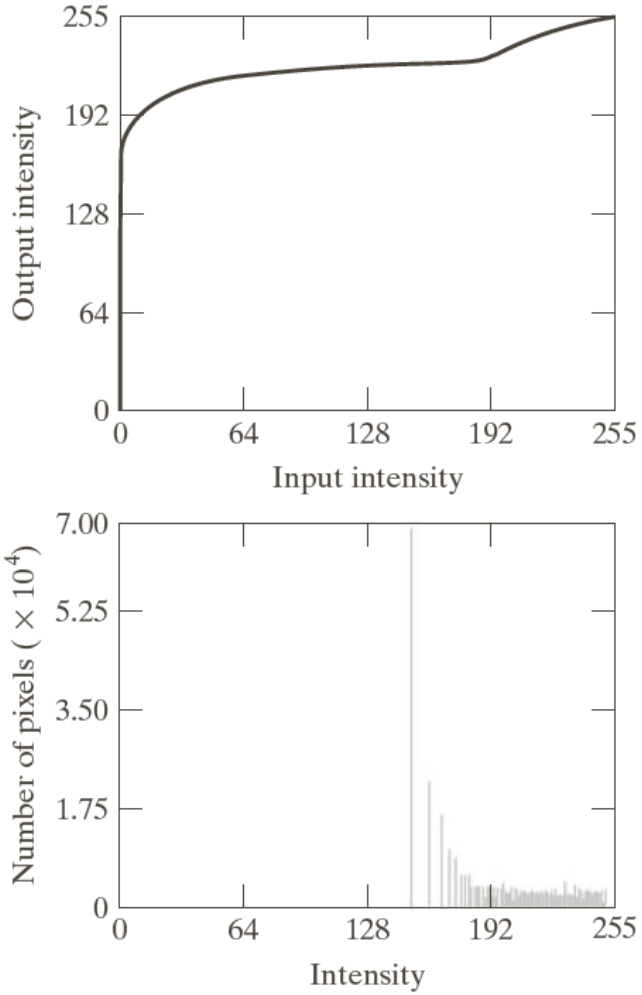
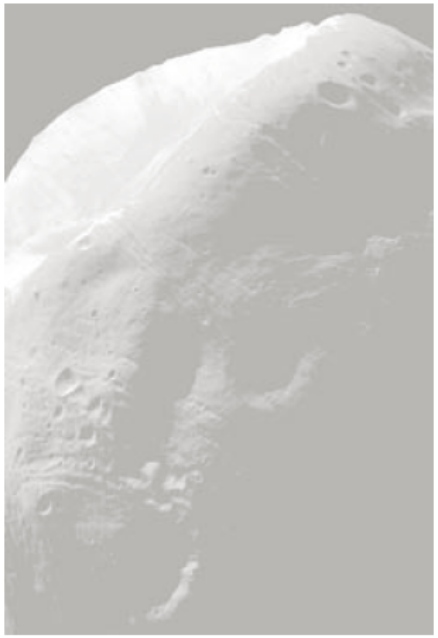
Histogram Specification: Example cont.





a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

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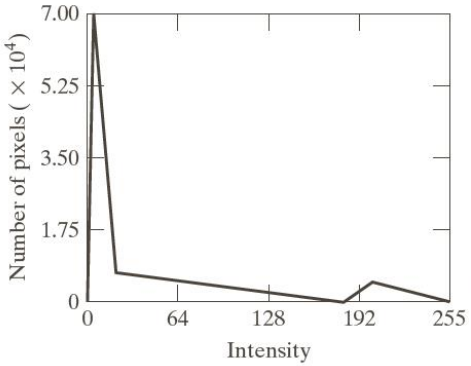
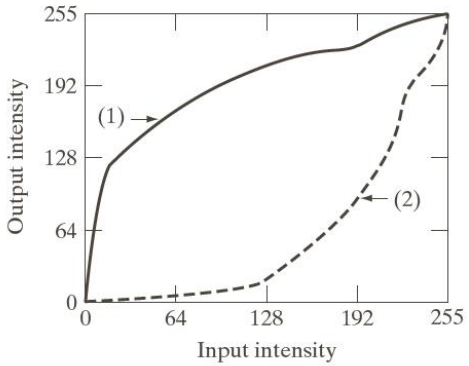
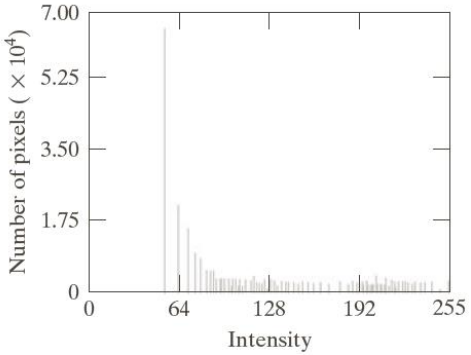
Histogram Specification: Example cont.







a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

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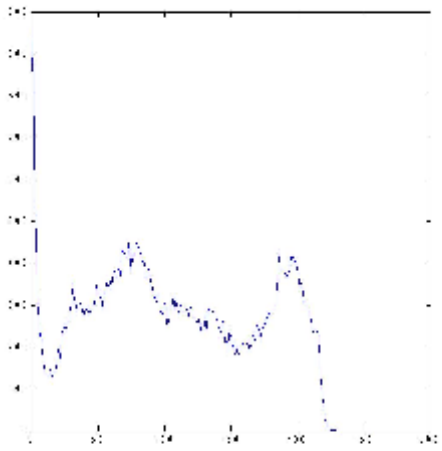
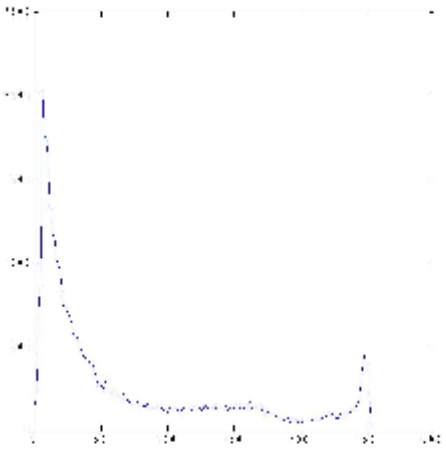
Histogram Specification: Example

Original Image

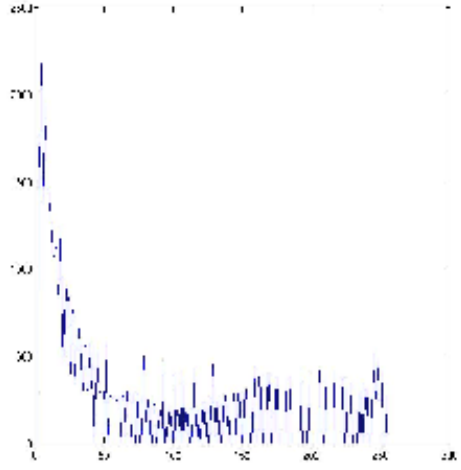
Image after histogram modification



Original Histogram



Specified Histogram



Histogram attained on modification

Local Histogram Equalization

- In many cases histograms are needed for local areas of an image.
- Possible applications could be:
  - Pattern detection based on histogram.
  - Adaptive enhancement.
  - Adaptive thresholding.
  - Object tracking based on histogram.

Local Histogram Equalization for local image enhancement

- The histogram processing methods discussed previously are global (transformation is based on the intensity distribution of the entire image).
- This global approach is suitable for overall enhancement.
- There are cases in which it is necessary to enhance details over small areas in an image.
- The number of pixels in these areas may have negligible influence on the computation of a global transformation.
- The solution is to devise transformation functions based on the intensity distribution in a neighbourhood around every pixel.
- carry other tasks such as detection, tracking and spatially adaptive thresholding.

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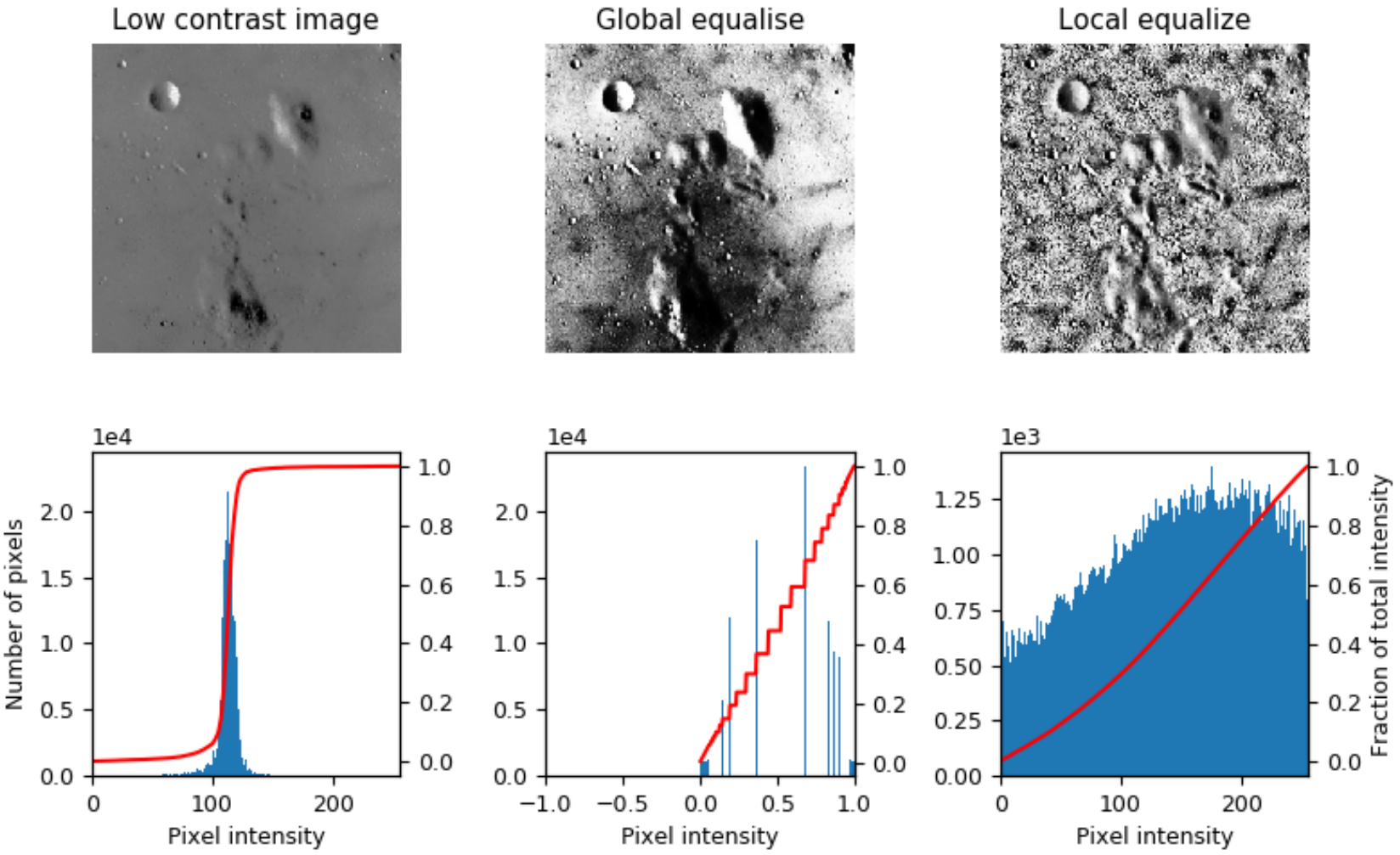
Example: Local Histogram Equalization



Example: Local Histogram Equalization

- Observe the details revealed with local (spatially adaptive) histogram equalization.

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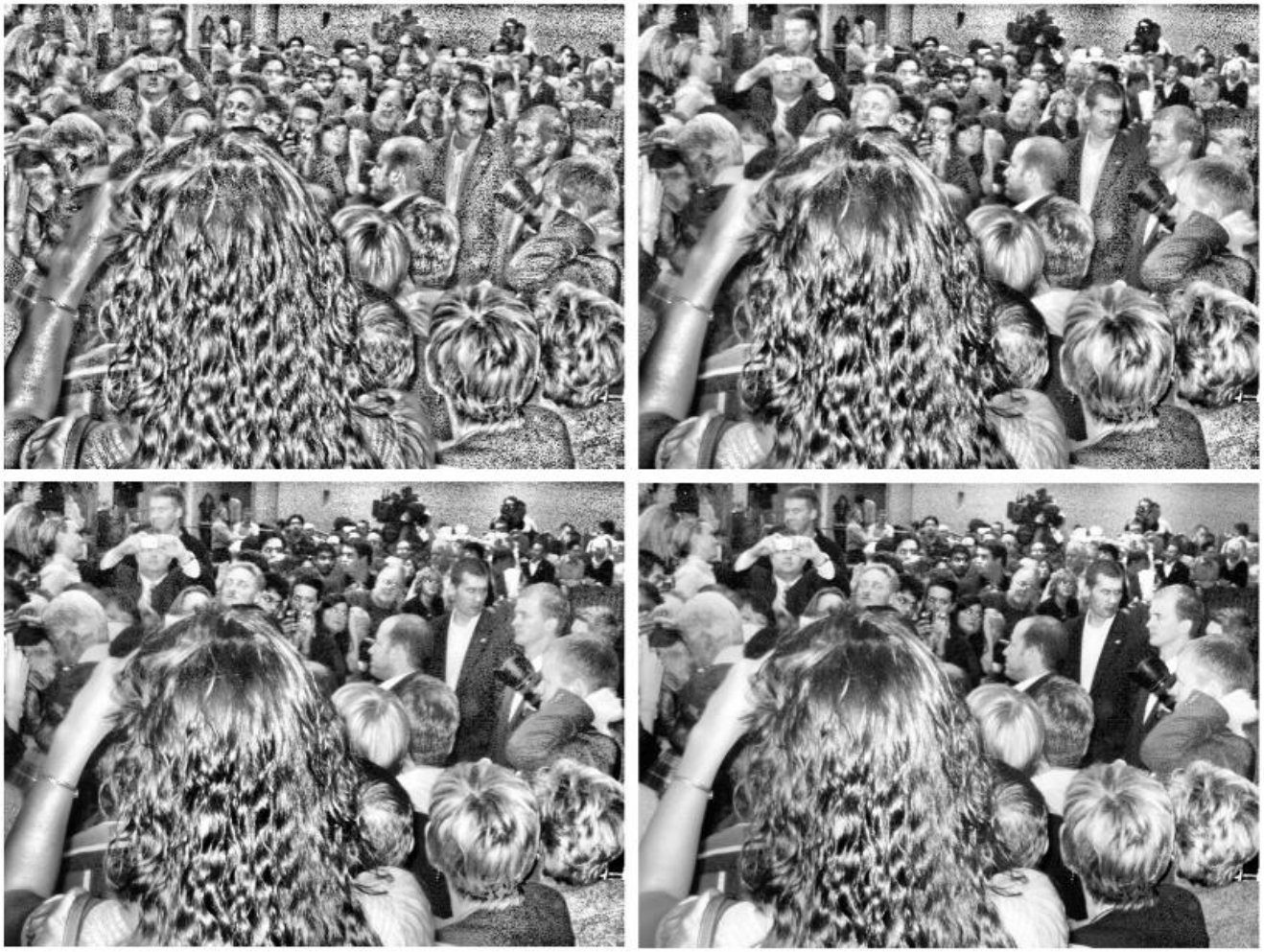
Example: Local Histogram Equalization



Top left: A window size of [25 25]. Top right: A window size of [64 64].
Bottom left: A window size of [100 100]. Bottom right: A window size of [200 200].

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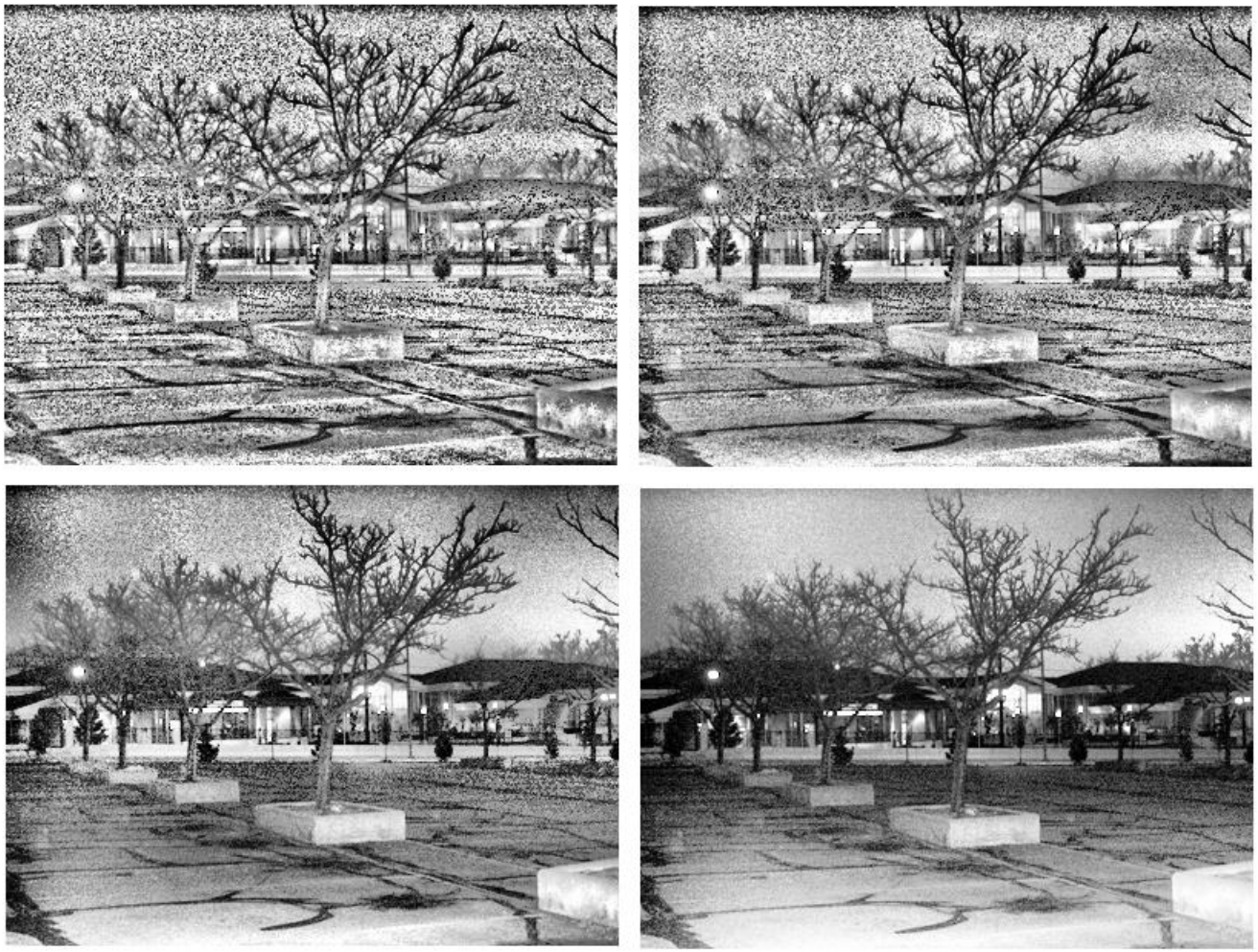
Example: Local Histogram Equalization



Top left: A window size of [15 15]. Top right: A window size of [30 30].
Bottom left: A window size of [75 75]. Bottom right: A window size of [150 150].

Example: Local Histogram Equalization

Example: Local Histogram Equalization drawbacks

- Image is split into smaller regions and the traditional histogram equalization is applied to each region.
- The smaller equalized images are combined into one to obtain a final resultant image.
- The final image appears to be very blocky in nature and has different contrast levels for each individual region.
- Post-processing is required to remove the blocking artifacts.