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Digital Image Processing

Image Transforms
The 2D Discrete Cosine Transform

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What is this lecture about?

- **Welcome back to the Digital Image Processing lecture!**
- In this lecture we will learn about one of the so called Discrete Cosine Transforms (DCT).
- From the above bullet point you might already have suspected that the DCT is not a single transform but a family of transforms.
- We will call the one that we will see here, DCT. In various textbooks different versions of the DCT have names such as Type I DCT, Type II DCT etc. Let us ignore these names and call the transform just DCT.
- We will start with the one-dimensional Discrete Cosine Transform (1D DCT) and show how this transform can be extended into two dimensions.
- The 1D DCT is also a member of the family of **unitary transforms**.

One-dimensional Discrete Cosine Transform (1D-DCT)

- The one-dimensional Discrete Cosine Transform (DCT) is defined as:

$C(u) = a(u)\sum_{x=0}^{N-1} f(x)\cos\left[\frac{(2x+1)u\pi}{2N}\right],\ 0 \le u \le N-1$

$$a(u) = \begin{cases} \sqrt{\dfrac{1}{N}} & u = 0 \\ \sqrt{\dfrac{2}{N}} & u = 1, \dots, N-1 \end{cases}$$

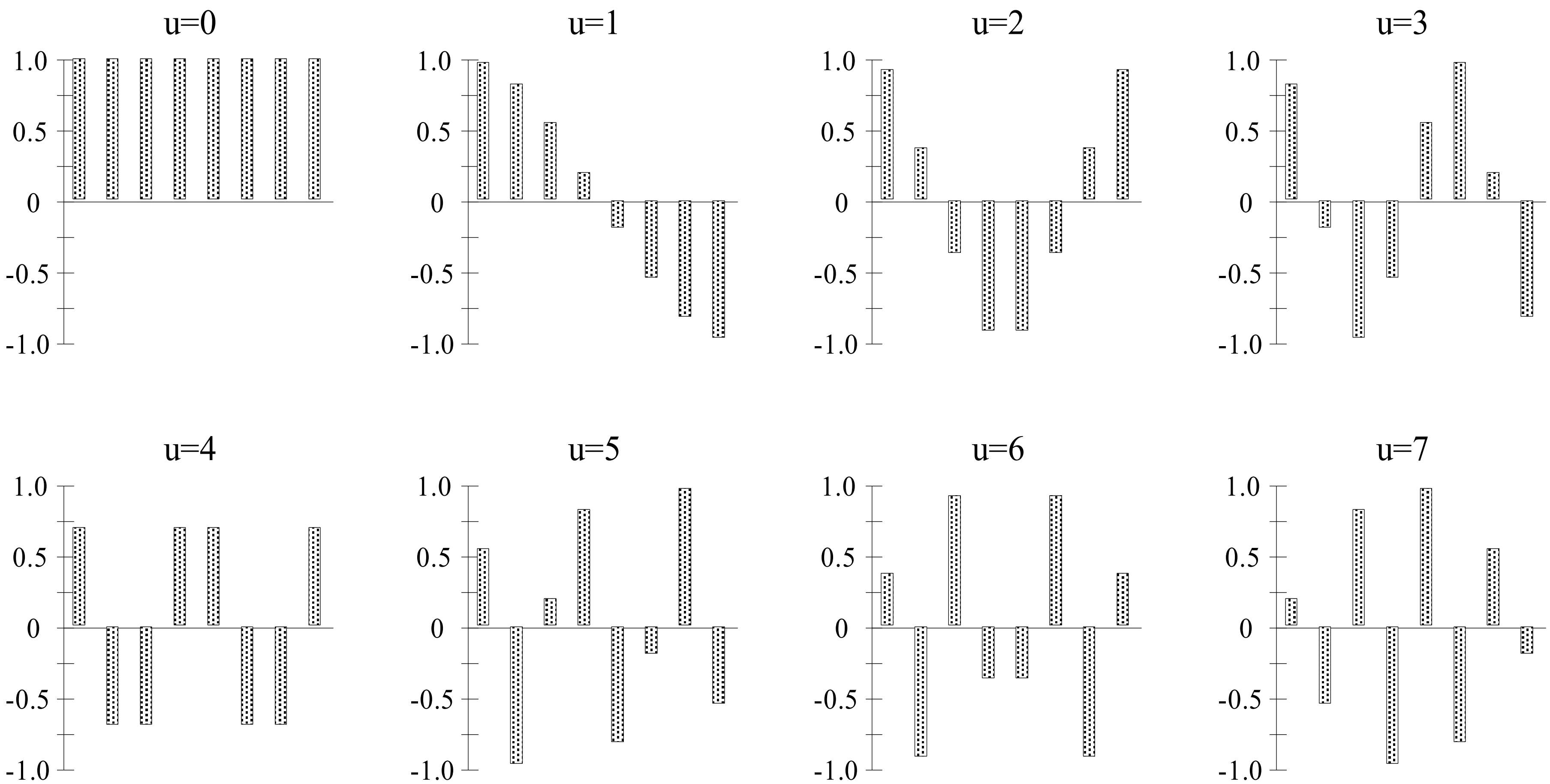
- The inverse transform is:

$$f(x) = \sum_{u=0}^{N-1} a(u)C(u)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

- It can be shown that DCT is a unitary transform.
- The difference with DFT is that the signal is projected onto real sinewaves instead of complex. It is a real transform.

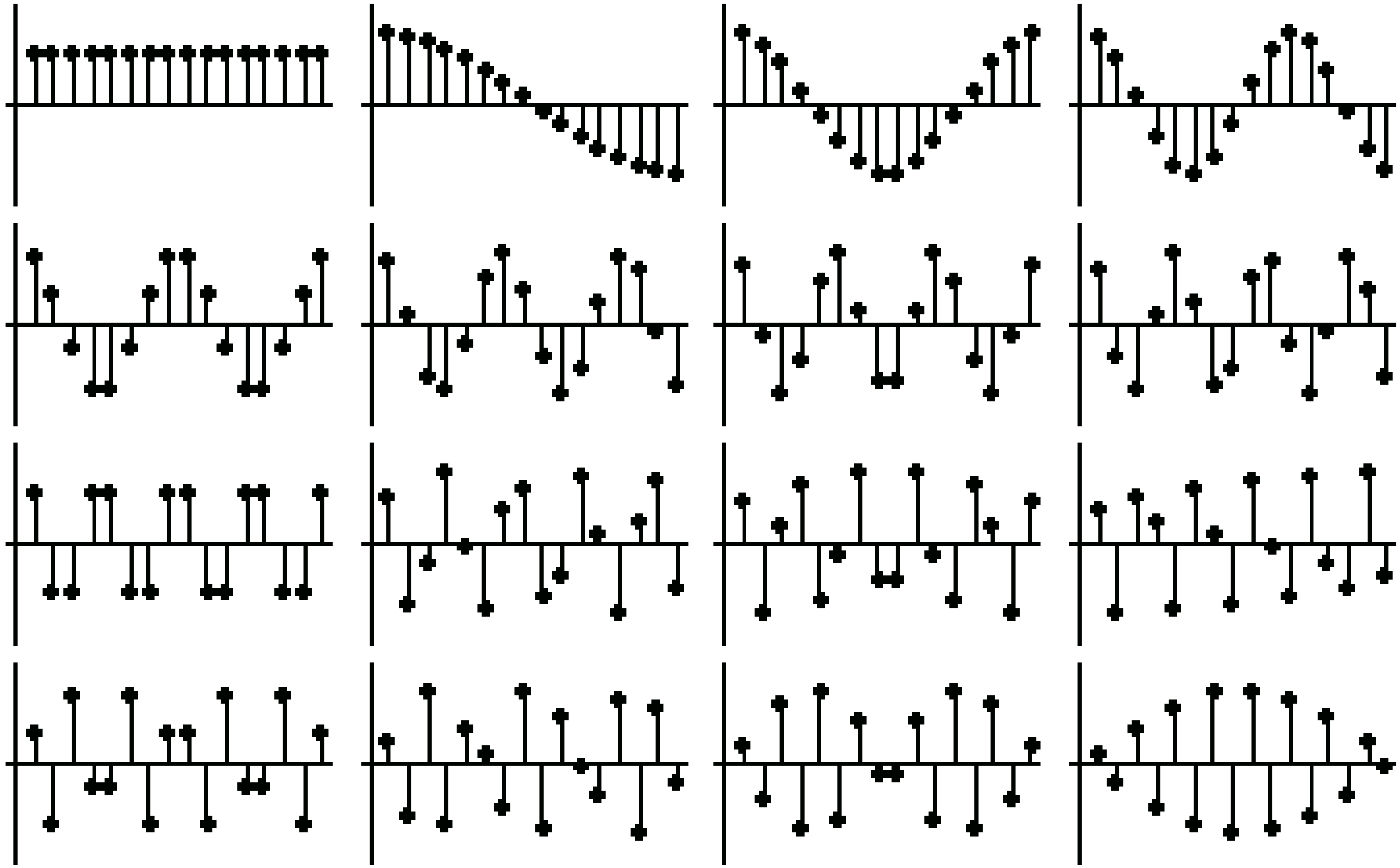
1-D Basis Functions N=8

- For a signal $f(x)$ with 8 samples, the rows of the 8×8 transformation matrix of the DCT are depicted below.



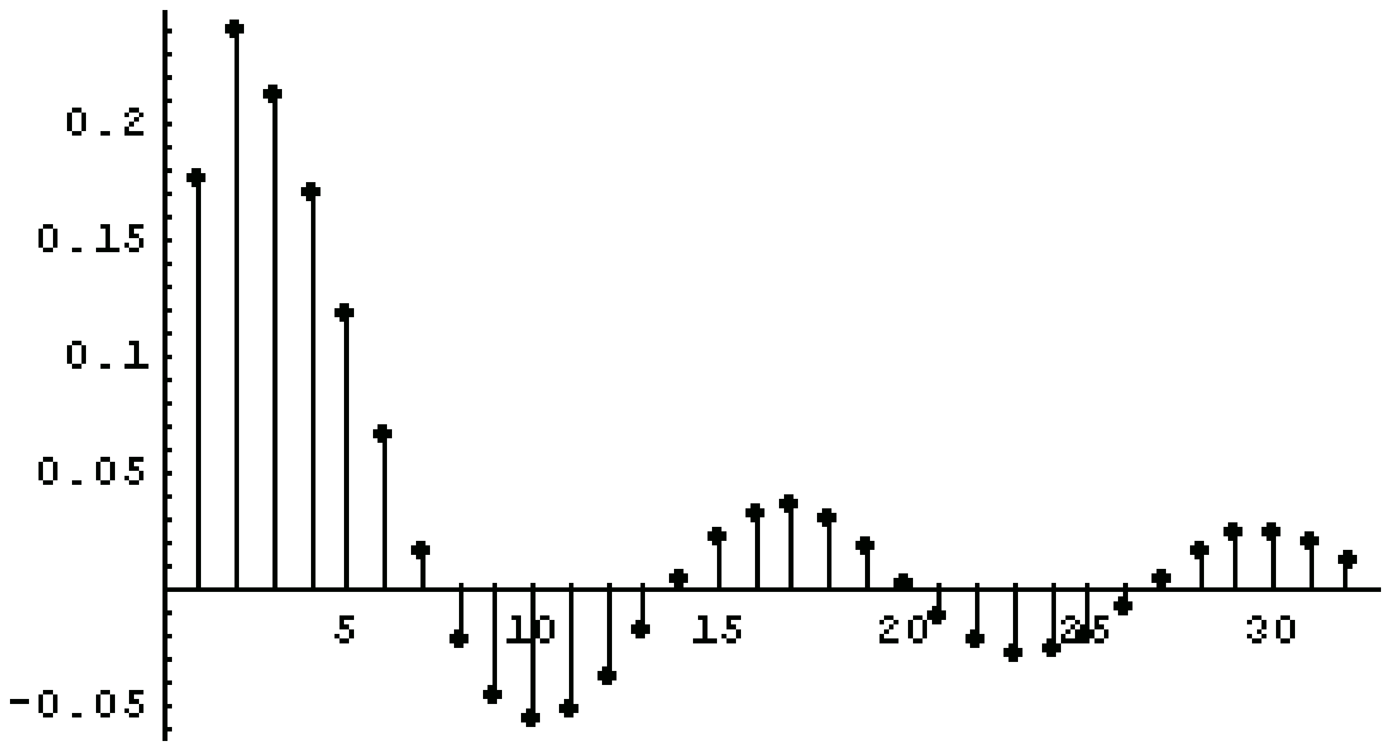
1-D Basis Functions $N=16$

- For a signal $f(x)$ with 16 samples, the rows of the 16×16 transformation matrix of the DCT are depicted below.



Example: One-dimensional Discrete Cosine Transform (DCT)

- Consider the signal $x[n] = \begin{cases} \frac{1}{5} & 0 \leq n \leq 4 \\ 0 & \text{elsewhere} \end{cases}$
- Its DCT is shown in the figure below.



Two-dimensional Discrete Cosine Transform (2D-DCT)

- Consider an image $f(x,y)$ of size $M \times N$.

- The two-dimensional Discrete Cosine Transform (DCT) is defined as:
$C(u,v) = a(u)a(v)\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{(2x+1)u\pi}{2M}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$,
$0 \le u \le M-1,\ 0 \le v \le N-1$

- The inverse transform is:
$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} a(u)a(v)C(u,v)\cos\left[\frac{(2x+1)u\pi}{2M}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$
$0 \le x \le M-1,\ 0 \le y \le N-1$

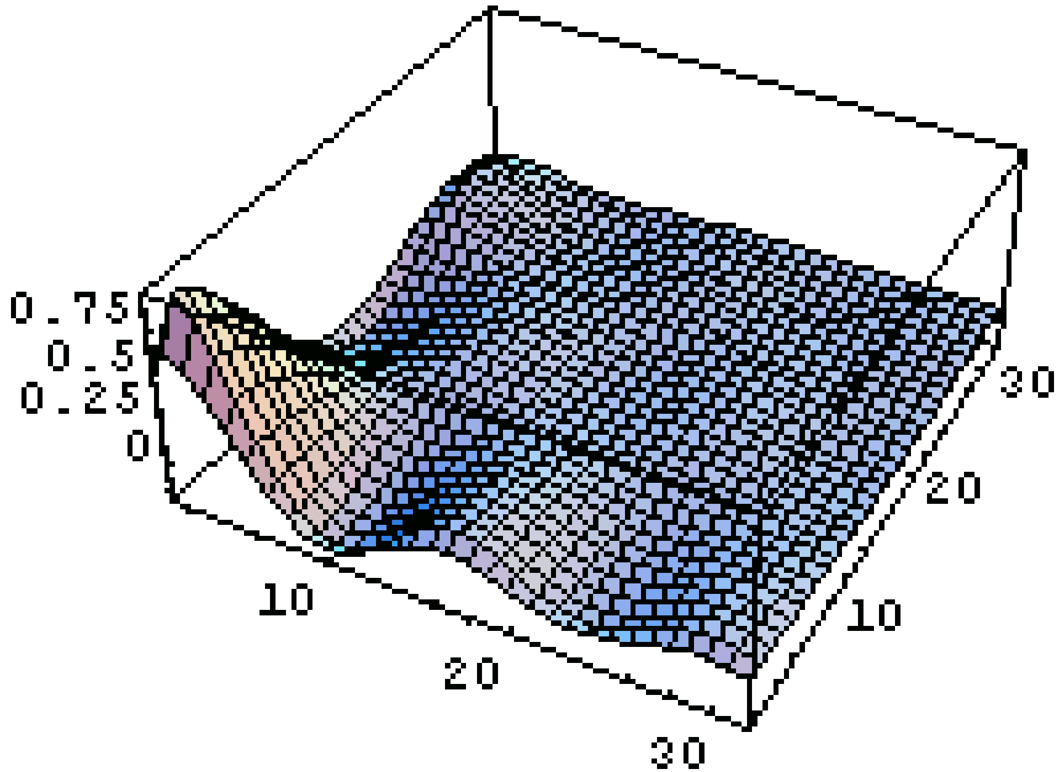
- $a(u)$ is defined as previously.

Example: Two-dimensional Discrete Cosine Transform (DCT)

- Consider the two-dimensional signal

$$f(x,y) = \begin{cases} 1 & 0 \le x \le 2, 0 \le y \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

- Its DCT is shown in the figure below.



Advantages of the Discrete Cosine Transform

- The DCT is a real transform.
- The DCT has excellent energy compaction properties.
- There are fast algorithms to compute the DCT similar to the FFT.

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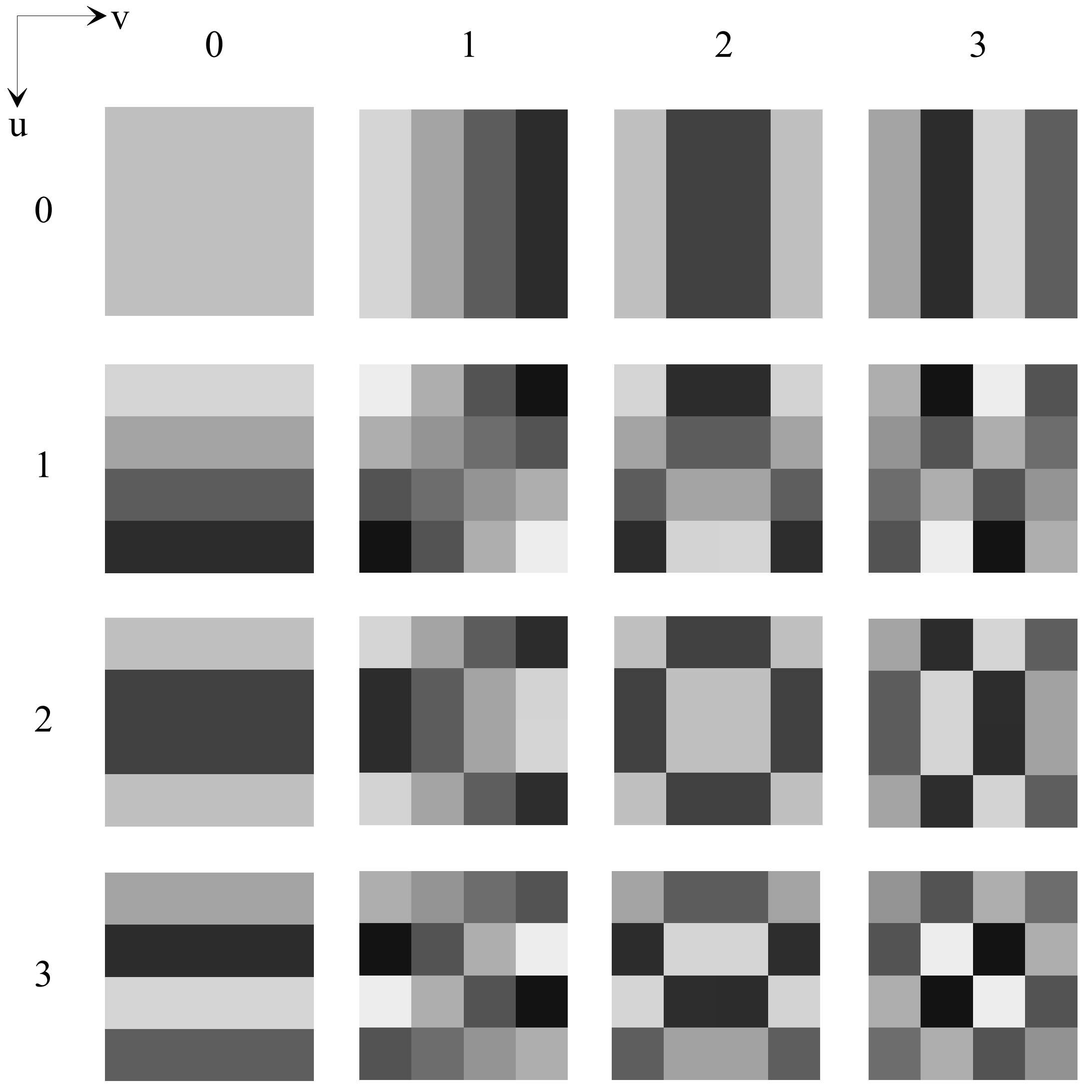
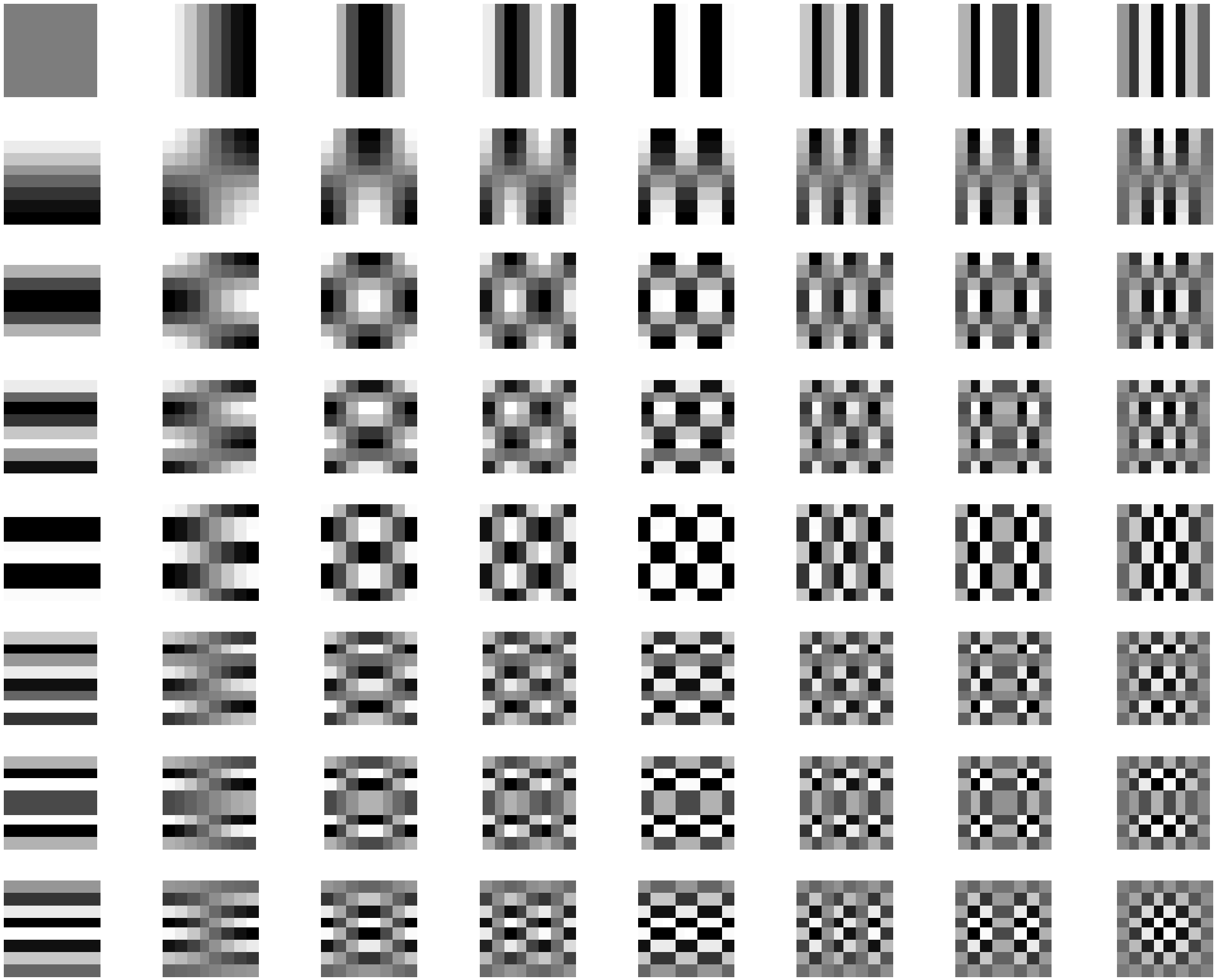
How to visualise 2D Basis Functions $N = 4$

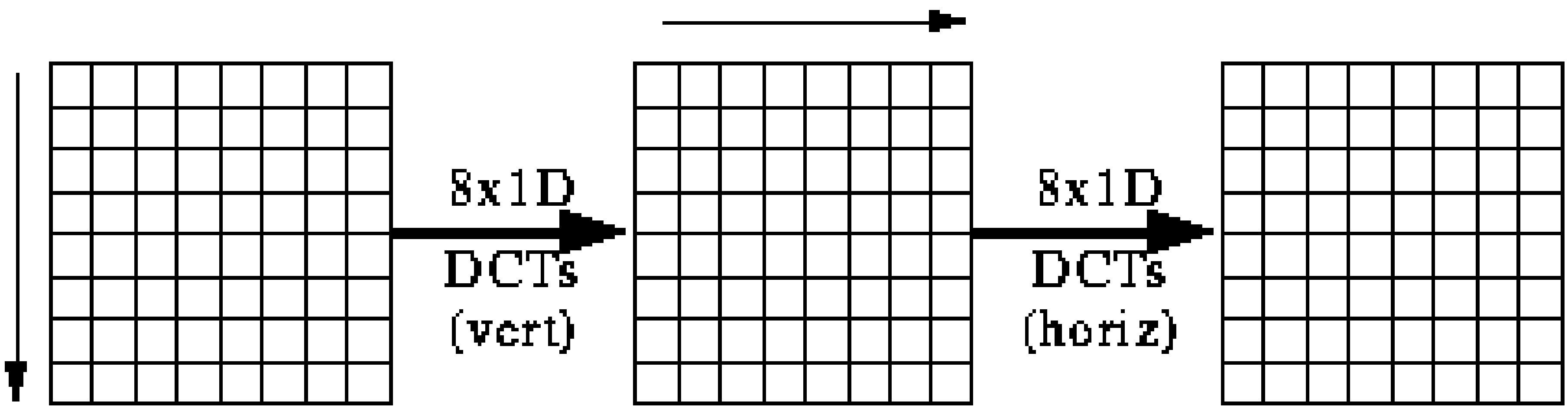


2-D Basis Functions $N = 8$



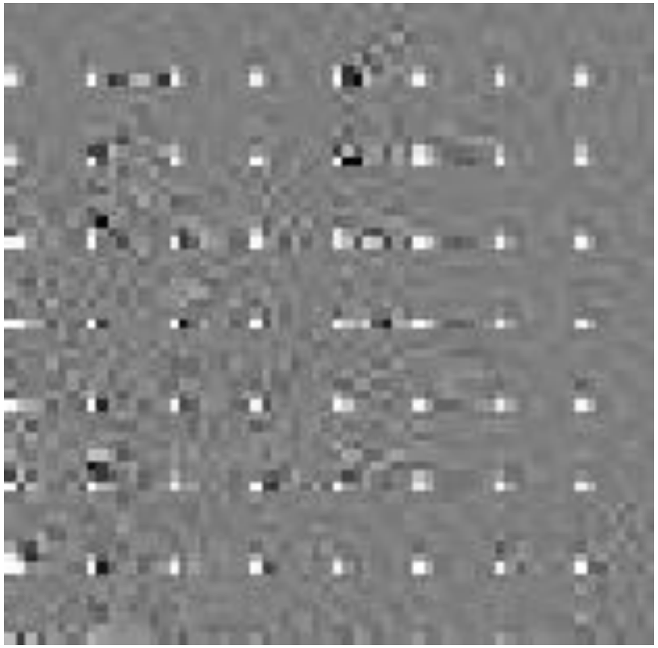
Separability of DCT

- The implementation 2D-DCT requires the sequential implementation of the corresponding one-dimensional transform row-by-row and then column-by-column (or the inverse), as with the case of 2D-DFT.



Example: 8×8 Block DCT

- The image below left is divided in patches (blocks) of size 8×8 pixels.
- The 2D-DCT is applied in each block.
- The result is depicted in the image below right.
- This is a standard way to use 2D-DCT in Image Compression Standards (JPEG). Details will be presented in Part 4 of the course.



Example: Energy Compaction

- Observe the excellent compaction property of 2D-DCT.
- Lena is shown on the left and its 2D-DCT on the right.