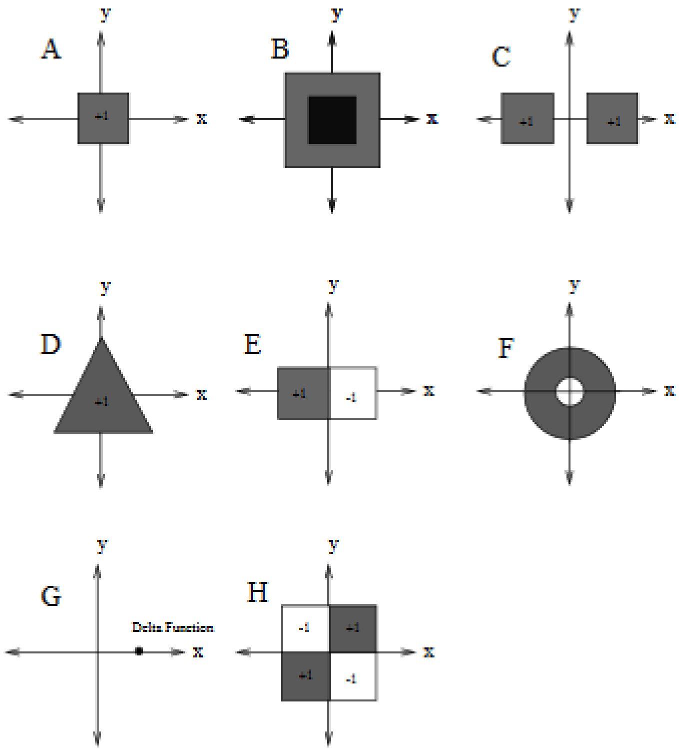
DFT Sample exam problems

1. Consider an $M \times M$ -pixel gray level real image $f(x, y)$ which is zero outside $-M \leq x \leq M$ and $-M \leq y \leq M$. Show that:
   (i) $F(-u,-v) = F^*(u,v)$ with $F(u,v)$ the two-dimensional Discrete Fourier Transform of $f(x,y)$.
   (ii) In order for the image to have the imaginary part of its two-dimensional Discrete Fourier Transform equal to zero, the image must be symmetric around the origin.
   (iii) In order for the image to have the real part of its two-dimensional Discrete Fourier Transform equal to zero, the image must be antisymmetric around the origin.

2. Consider the images shown below (A to H). Using knowledge of properties of the two-dimensional Discrete Fourier Transform symmetry and not exact calculation of it, list which image(s) will have a two-dimensional Discrete Fourier Transform $F(u,v)$ with the following properties:
   (i) The imaginary part of $F(u,v)$ is zero for all u, v.
   (ii) $F(0,0) = 0$
   (iii) $F(u,v)$ has circular symmetry.
   (iv) The real part of $F(u,v)$ is zero for all u, v.



3. Consider an $M \times M$ -pixel gray level image $f(x, y)$ which is zero outside $0 \leq x \leq M-1$ and $0 \leq y \leq M-1$. The image intensity is given by the following relationship

$$f(x,y) = \begin{cases} c, & x = x_1, x = x_2, 0 \leq y \leq M-1 \\ 0, & \text{otherwise} \end{cases}$$

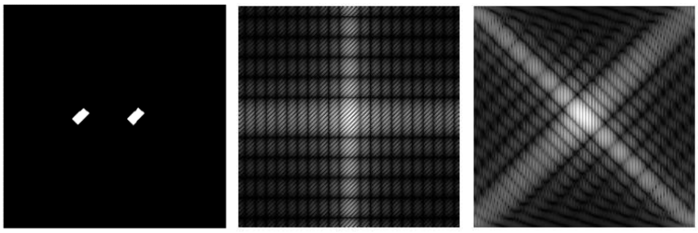
   where c is a constant value between 0 and 255 and x_1, x_2 $x_1 \neq x_2$ are constant values between 0 and $M-1$.
   (i) Plot the image intensity.

(ii) Find the $M \times M$ -point Discrete Fourier Transform (DFT) of $f(x, y)$.

(iii) Compare the original image and its Fourier Transform.

Hint: The following result holds: $\sum_{k=0}^{N-1} a^x = \frac{1-a^N}{1-a}, \; |a| \leq 1$.

4. Consider the image shown in **Figure 1.1(a)** below. Two plots of magnitude of Two-Dimensional Discrete Fourier Transform (2D DFT) are shown in **Figure 1.1(b)** and **1.1(c)** below. Discuss which one is the magnitude of the 2D DFT of the image of **Figure 1.1(a)**. Justify your answer.



(a) **(b)** **(c)**

Figure 1.1

5. Consider an $M \times N$ -pixel image $f(x, y)$ which is zero outside $0 \leq x \leq M-1$ and $0 \leq y \leq N-1$. In transform coding, we discard the transform coefficients with small magnitudes and code only those with large magnitudes. Let $F(u,v)$ denote the $M \times N$ -point Discrete Fourier Transform (DFT) of $f(x, y)$. Let $G(u,v)$ denote $F(u,v)$ modified by

$$G(u,v) = \begin{cases} F(u,v), & \text{when } |F(u,v)| \text{ is large} \\ 0, & \text{otherwise} \end{cases}$$

Let

$$\frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |G(u,v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u,v)|^2} = \frac{9}{10}$$

We reconstruct an image $g(x, y)$ by computing the $M \times N$ -point inverse DFT of $G(u,v)$. Express $\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (f(x, y) - g(x, y))^2$ in terms of $\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)^2$.