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Digital Image Procesing

Discrete CosineTrasform (DCT) in Image Processing

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1-D Discrete Cosine Transform

$$C(u) = a(u)\sum_{x=0}^{N-1} f(x)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

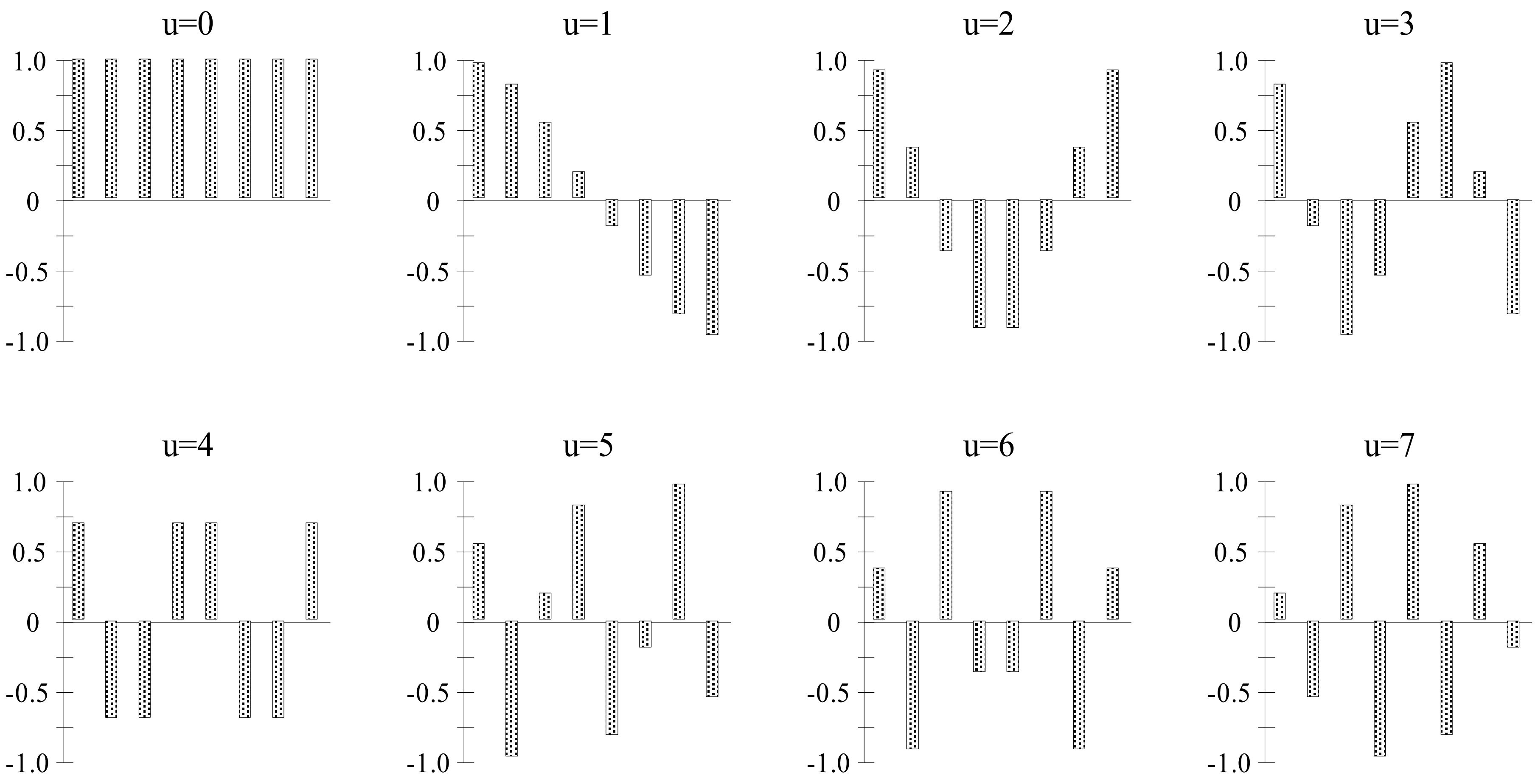
$$u = 0,1,\ldots,N-1$$

$$a(u) = \begin{cases} \sqrt{\dfrac{1}{N}} & u = 0 \\ \sqrt{\dfrac{2}{N}} & u = 1,\ldots,N-1 \end{cases}$$

1-D Inverse Discrete Cosine Transform (IDCT)

$$f(x) = \sum_{u=0}^{N-1} a(u)C(u)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

1-D Basis Functions N=8



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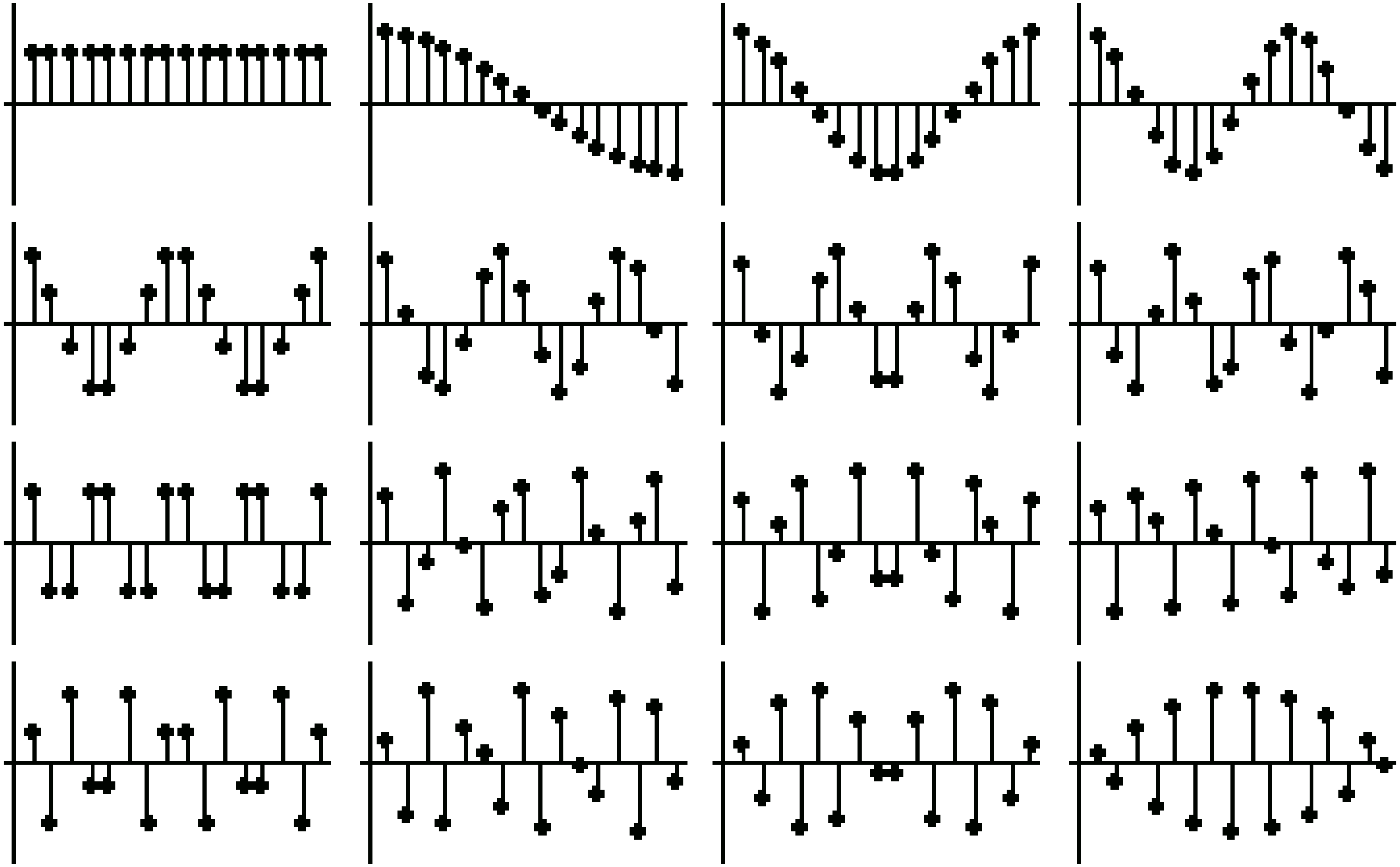
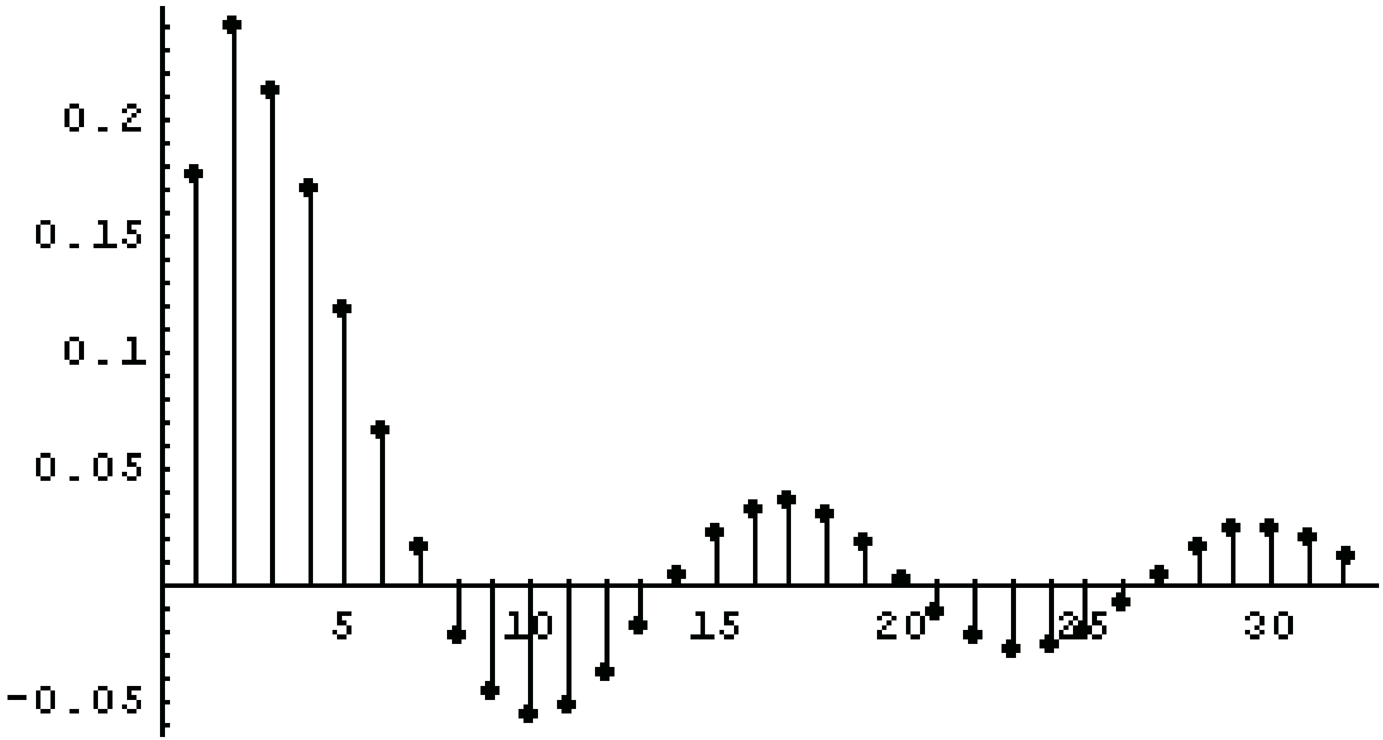
1-D Basis Functions $N=16$

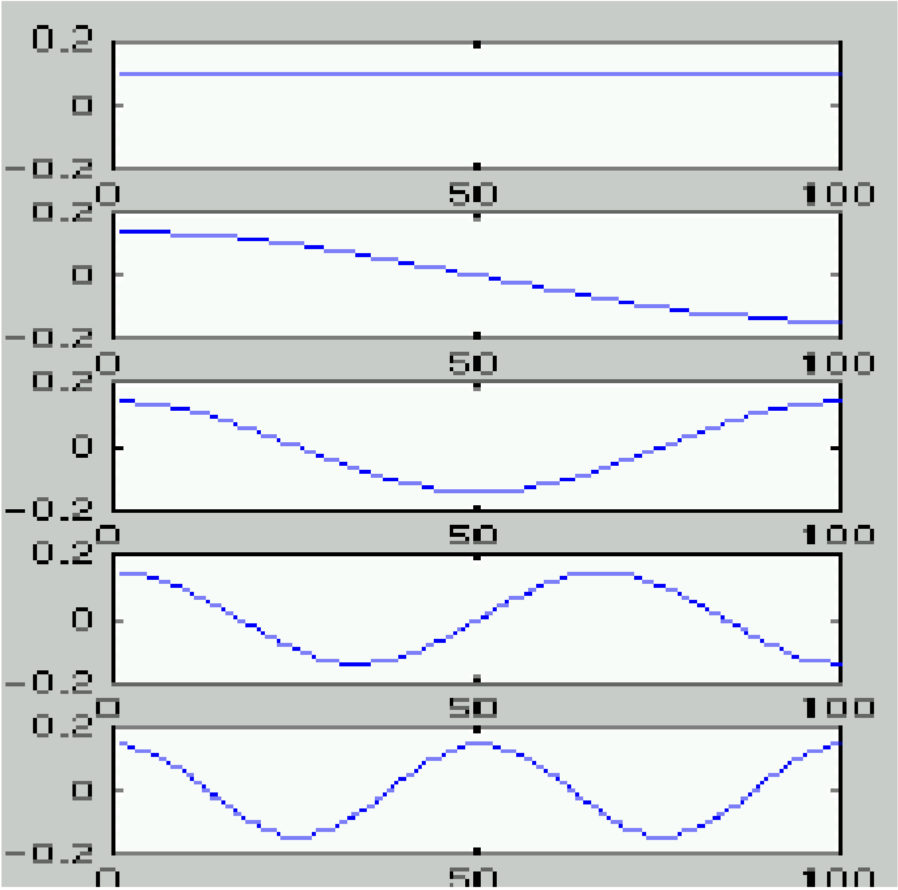


Example: 1D signal

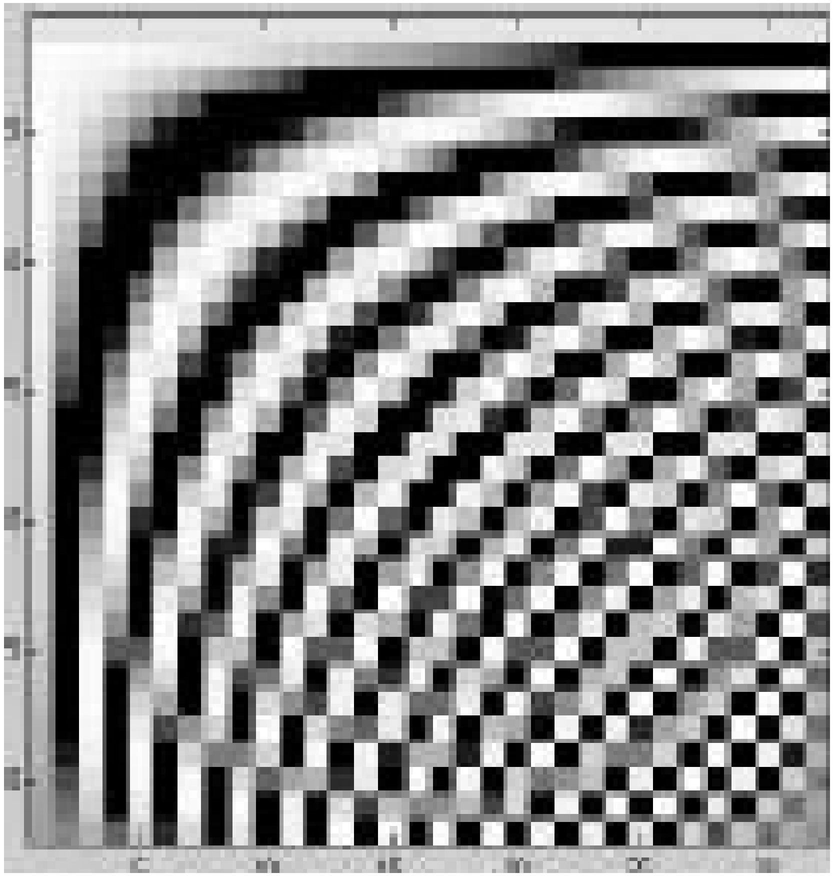
$$x[n] = \begin{cases} \frac{1}{5}, & \text{for } 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$



- First 5 vectors:



- Image of full 32x32:



2-D Discrete Cosine Transform (IDCT)

$$C(u,v) = a(u)a(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{(2x+1)u\pi}{2N}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} a(u)a(v)C(u,v)\cos\left[\frac{(2x+1)u\pi}{2N}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

$$u,v = 0,1,\ldots,N-1$$

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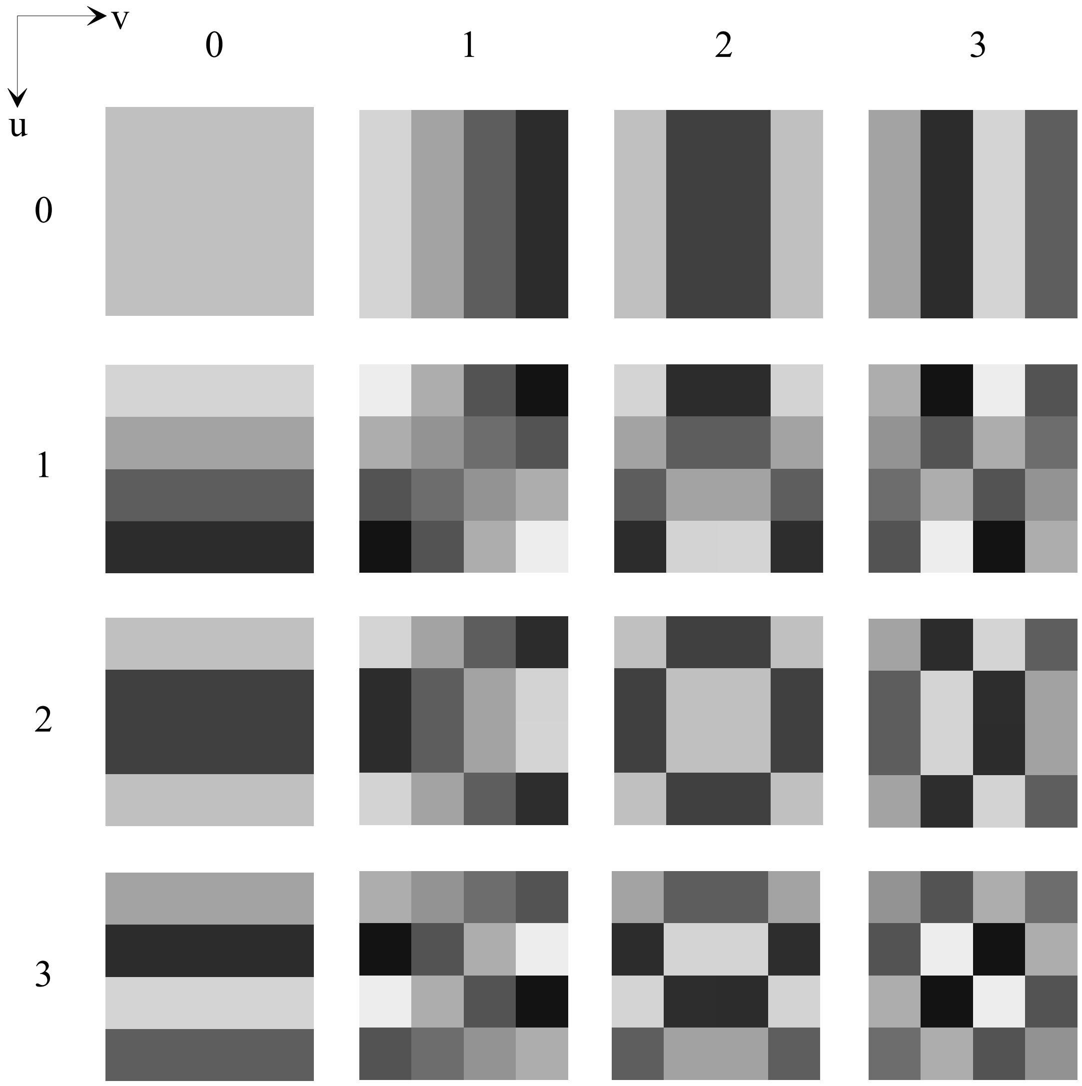
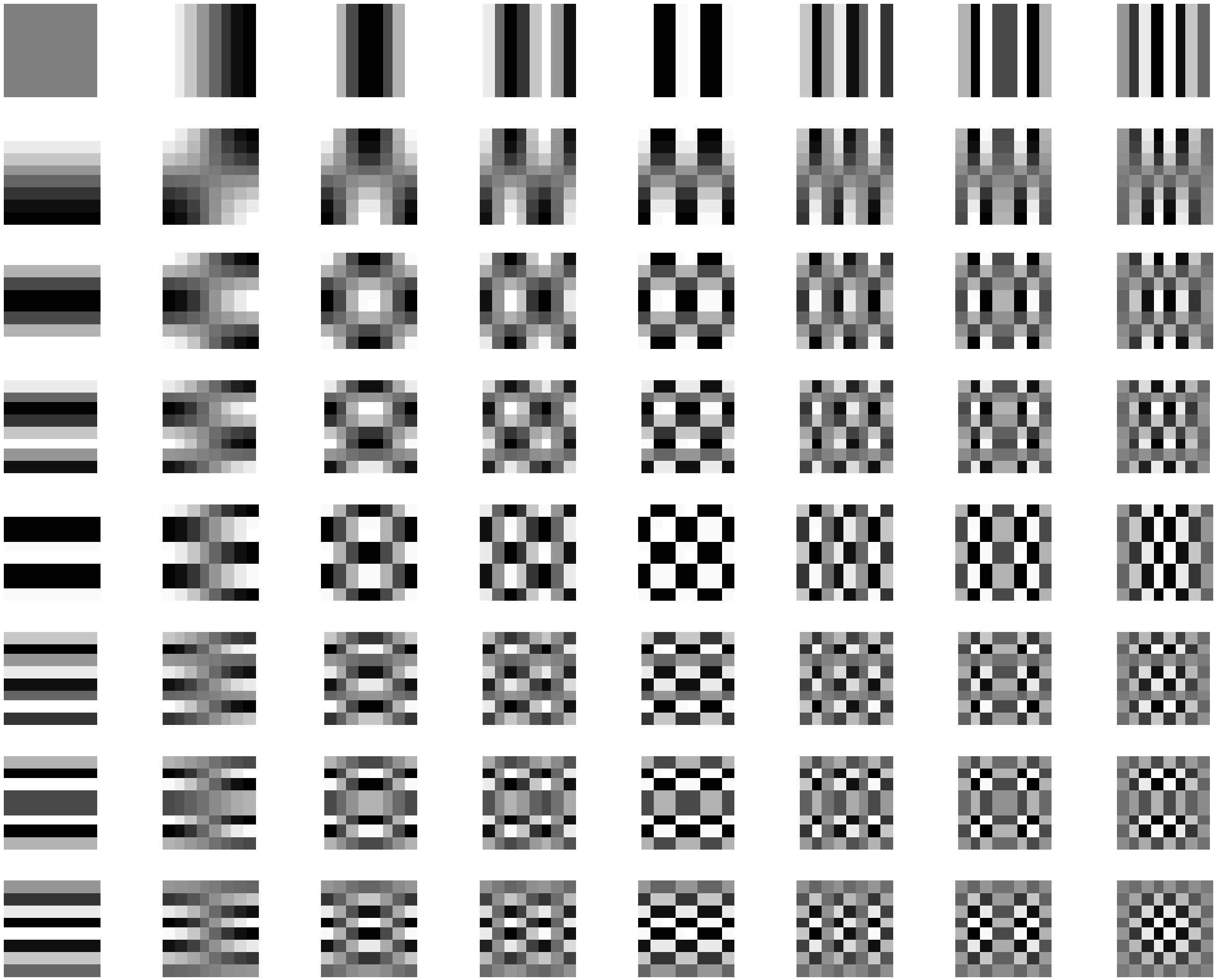
Advantages of the Discrete Cosine Transform

- Notice that the DCT is a real transform.
- The DCT has excellent energy compaction properties.
- There are fast algorithms to compute the DCT similar to the FFT.

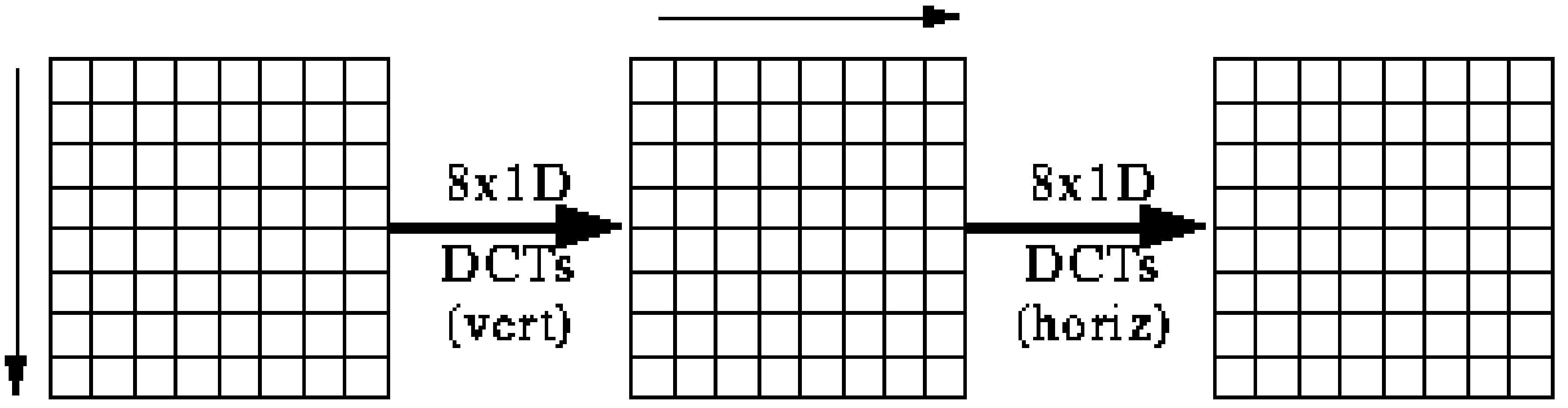
2-D Basis Functions $N=4$



2-D Basis Functions $N=8$

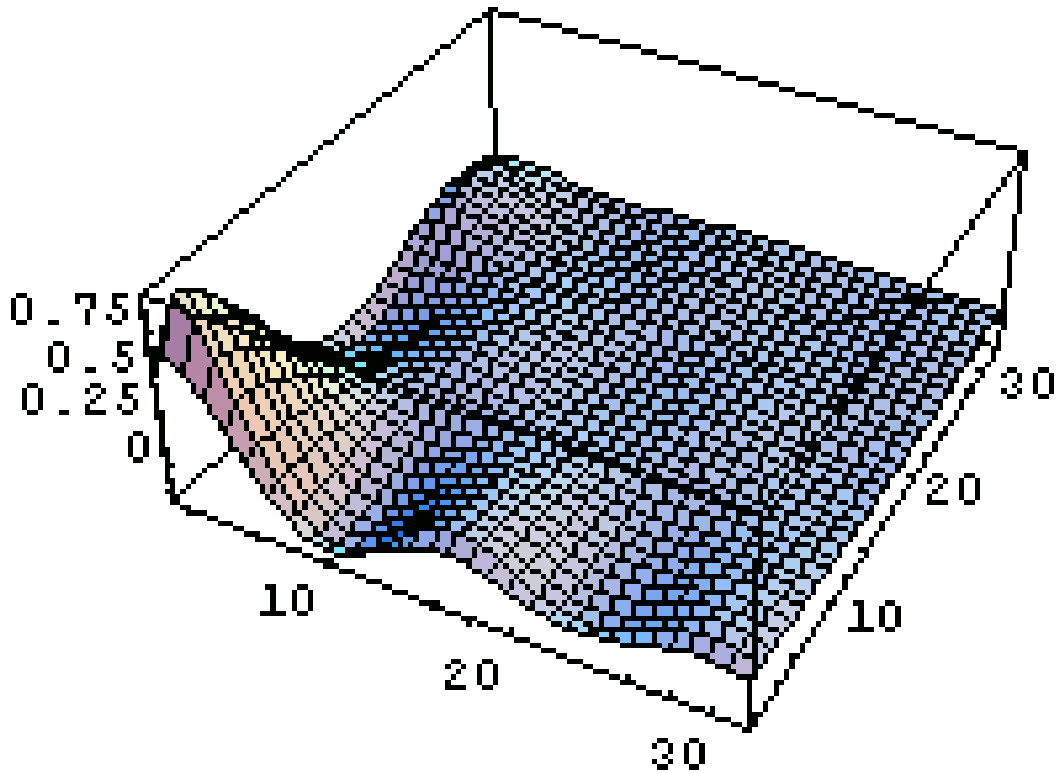


Separability of DCT

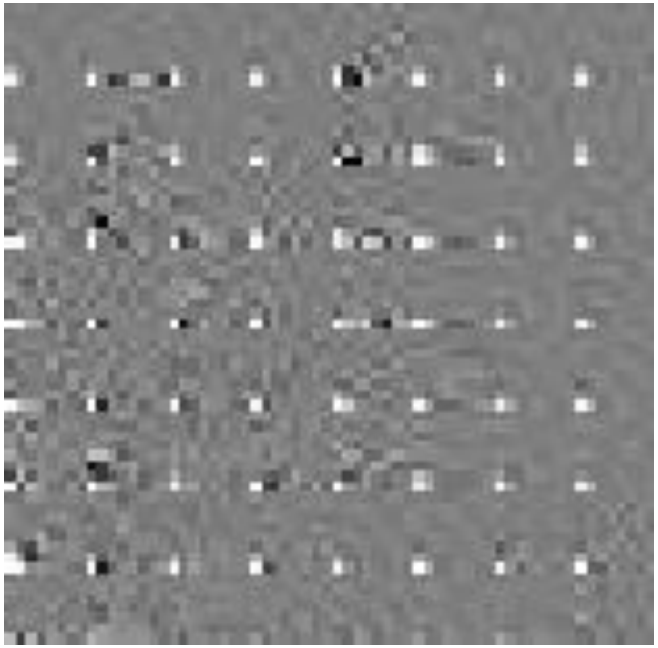


Example: 2D signal

$$x[n_1, n_2] = \begin{cases} 1, & 0 \le n_1 \le 2,\ 0 \le n_2 \le 4 \\ 0, & \text{otherwise} \end{cases}$$



Example: 8x8 Block DCT

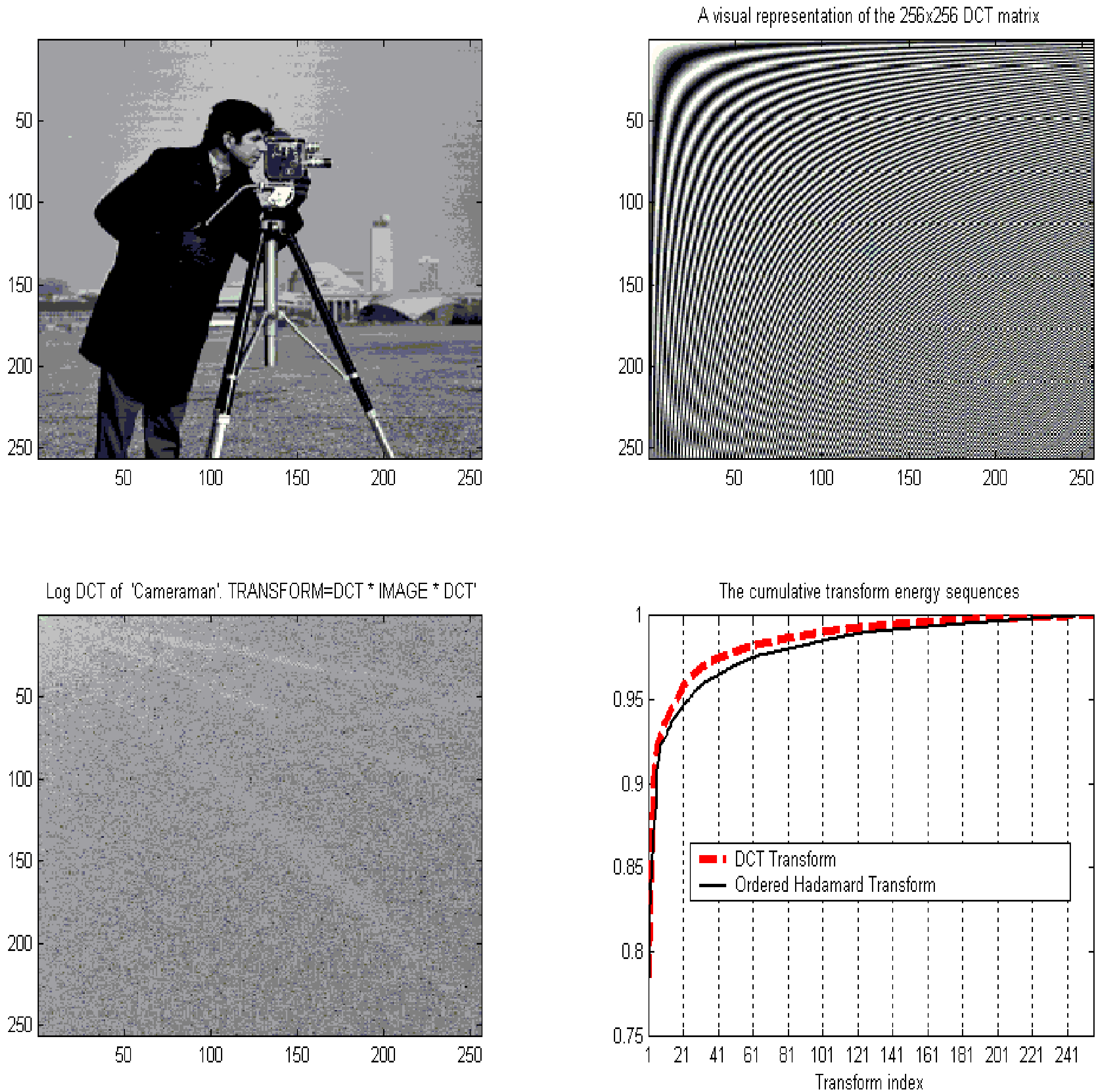


Example: Energy Compaction

- Original Lena image

- 2D DCT

Experiment that demonstrates the superiority of DCT in terms of energy compaction



Relation between DCT and DFT

Define

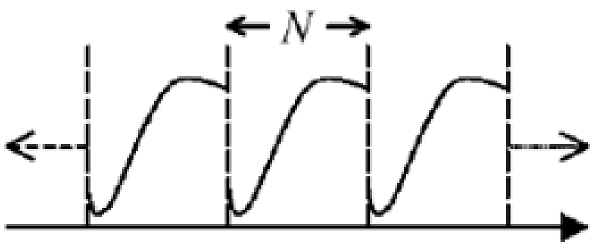
$$g(x) = f(x) + f(2N-1-x)$$
$$= \begin{cases} f(x), & 0 \le x \le N-1 \\ f(2N-1-x), & N \le x \le 2N-1 \end{cases}$$

$$\begin{array}{ccccccc} N-\text{point} & & 2N-\text{point} & \text{DFT} & 2N-\text{point} & & N-\text{point} \\ f(x) & \rightarrow & g(x) & \rightarrow & G(u) & \rightarrow & C_f(u) \end{array}$$

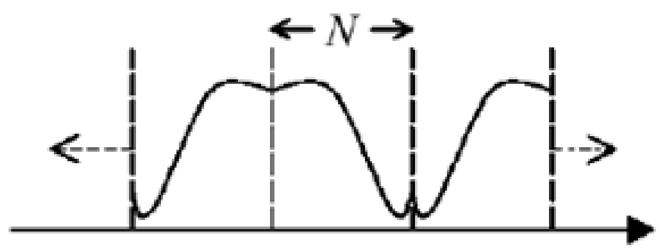
Relation between DCT and DFT

DCT has a higher compression ration than DFT

- DCT avoids the generation of spurious spectral components



DFT periodicity



DCT periodicity

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Using DCT for Image Compression