Double Auction and Negotiation for Dynamic Resource Allocation with Elastic Demands

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Abstract—Resource allocation is an important topic with a wide range of applications. In many practical cases, users and resource suppliers are players in the market. As a result, much effort has been made in applying market mechanisms (such as auction and game-theoretic results) to resource allocation. The conventional approach in such studies is to consider cases where users’ resource demands are fixed. However, in practice, resource demands are often elastic, which can be related to the quality of experience (QoE) that the user receives. We consider elastic resource demands in this paper, and propose a double auction and negotiation (DAN) scheme, which includes a conventional auction stage as well as a negotiation stage, where the latter allows users to dynamically adjust their demands. The proposed DAN scheme not only allows more users to get access to some amount of resource (thereby avoiding users becoming completely disconnected), but also increases the payoff of resource suppliers, as is confirmed by simulations. We also discuss the conditions of having Nash equilibrium in the users’ resource demands and suppliers’ pricing in the negotiation stage.

Index Terms—Auction, economics, Nash equilibrium, negotiation, resource allocation

I. INTRODUCTION

The information technology (IT) sector is exhibiting a growing trend of developing integrated systems that traditionally span over a wide range of technological areas. For example, the Industry 4.0 vision proposes Internet-connected smart factories which can make manufactural decisions autonomously [1]. The emerging concept of mobile edge computing (which is regarded as part of the 5G vision [2]) equips cellular network entities (such as basestations) with servers, so that users can run cloud services with significantly lower latency [3]. The introduction of femtocells allows users to buy their own cellular network devices based on their needs, thereby reducing the operators’ expenditure and improving the network coverage [4]. All these trends point to the same direction. That is, in the future, there will be more businesses operating on a single networked platform, and the interactions between them will become increasingly complex. The tradition that does not change however is that, due to physical limitations, the amount of resources (such as communication bandwidth, computational capability of servers, electricity, etc.) in the system remains limited. Therefore, there is a growing need of studying how to efficiently allocate resources to different business entities.

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One fast and effective method of allocating resources at market value is using auctions. The classical sealed-price double auction1 is used in [5] and [6], where bids are made privately and truthfulness can be ensured. References [7], [8], and [9] formulated the cloud resource pricing problem as a multi-unit combinatorial auction problem, which considers multiple types of computational resources for sale. They apply the Vickrey-Clarke-Groves (VCG) mechanism that maximizes social payoff while preserving truthfulness. Dynamic pricing for double auction was proposed in [10] and [11], aiming at achieving a game-theoretic equilibrium through price control among resource suppliers. Auction approaches are also seen in real platforms. For instance, it is used in Amazon EC2’s Spot Instance pricing mechanism [12]. Compared to fixed pricing mechanisms, the Spot Instance pricing mechanism can better capture market dynamics and encourage users to use the service during off-peak times when the price is low.

However, the above existing approaches have some drawbacks. For example, when the available resource is limited, users bidding at a lower prices may lose access to the resource unexpectedly. Although some users may finally obtain the resource, they have to pay a high price, which reduces the revenue of users and is not able to relieve the suppliers’ resource shortage. The main reason for causing these issues is that existing approaches focus on cases where users specify a fixed amount of resource demand, whereas they do not consider the fact that in many practical scenarios, the user’s resource demand can be elastic and vary with the price.

In this paper, we consider elastic resource demands from users. We assume that the amount of resource that each user obtains is related to its quality of experience (QoE). Each user has a desired amount of resource, but it can reduce this amount in case of resource shortage. We present a two-stage mechanism called double auction and negotiation (DAN). The first stage is called the auction stage, where users bid for the amount of resource that is initially requested by themselves. The second stage is called the negotiation stage, where the resource supplier asks for a fixed price (called reserve price) that each user has to pay for unit resource. The reserve price is determined based on the bidding price submitted by users in the auction stage. With this mechanism, each user has an extra

1 Double auction is an auction with multiple sellers and multiple buyers.
chance to obtain resources when its bid fails in the competition with other buyers. The user can adjust its resource demand according to the reserve price and its own payoff function. Accordingly, we divide the total amount of resource into two subsets, which are assigned to users respectively during the auction and negotiation stages.

The proposed DAN scheme has several advantages. It allows more users to get access to some amount of resource, and thereby avoiding users becoming completely disconnected. It also increases the payoff of resource suppliers, which is confirmed by simulations. We also discuss the conditions of having Nash equilibrium in the users’ resource demands and suppliers’ pricing in the negotiation stage.

The remainder of this paper is organized as follows. Section II presents the proposed DAN scheme. The Nash equilibrium of users and suppliers in the negotiation stage is discussed in Section III. Section IV presents the simulation results, and Section V draws conclusions.

II. THE DAN SCHEME

We consider a time-slotted framework where resource allocation is performed at the beginning of each slot. After the resource is assigned, users can use the resource for the remainder of the slot. Such a framework is suitable for applications that normally do not require instantaneous access to resources, such as batch processing, so that the users can wait until the next slot to get resource access. More sophisticated cases where users may require instantaneous resource access can be studied in the future.

At the beginning of a slot, we perform resource allocation using the proposed DAN scheme. We treat each user as a buyer and each resource supplier as a seller, and let \( \mathcal{N} \) denote the set of users and \( \mathcal{M} \) denote the set of suppliers. Each user \( i \in \mathcal{N} \) can request resource from supplier \( j \in \mathcal{M} \). We use \( \mathcal{N}_j \) to denote the set of users that request resource from supplier \( j \), and use \( \mathcal{N}_{j,\text{auc}} \subseteq \mathcal{N}_j \) (correspondingly, \( \mathcal{N}_{j,\text{neg}} \subseteq \mathcal{N}_j \)) to denote the subset of users that have obtained resource during the auction stage (correspondingly, negotiation stage).

A. Auction and Negotiation Procedure

The procedure of the DAN scheme for one transaction round is shown in Fig. 1, which includes the following steps.

Bid: Every user \( i \in \mathcal{N}_j \) offers a bid to one particular supplier \( j \), where we restrict that each user only bids to one supplier in each transaction round. A bid is in the form of \( (P_{ij}, W_{ij}) \), where \( P_{ij} \) is the price per unit resource and \( W_{ij} \) denotes the amount of requested resource. To help prevent malicious bids, \( P_{ij} \) should not be less than a pre-defined minimum price, otherwise the bid is rejected immediately.

Accept: After supplier \( j \) receives all bids from users, it first finds out the percentage of resource to be assigned in the auction stage. We denote this percentage by \( \epsilon (0 < \epsilon \leq 1) \), and the percentage assigned in the negotiation stage is then \( 1 - \epsilon \). The value of \( \epsilon \) is chosen by the supplier. Then, all users are ranked by their bids \( P_{ij} \), and supplier \( j \) assigns resources to users up to the limit specified by \( \epsilon \) and the total resource amount \( R_j \), starting from the user with the highest bid. We note that all user requests are accommodated if their total resource demand does not exceed \( \epsilon R_j \). Otherwise, the second-price sealed-bid auction is applied, which is a type of Vickrey auction that gives bidders incentive to bid their true value [13]. Suppose for a given \( j \), \( P_{ik,j} (k > 0) \) is the \( k \)-th highest price, i.e., \( P_{i_{k,j}} \geq P_{i_{k-1,j}} \geq P_{i_{k-2,j}} \geq ... \), then user \( i_k \) pays price \( P_{i_{k+1,j}} \), except for the lowest user \( i_{\lceil \epsilon N_j \rceil} \) who pays its own price \( P_{i_{\lceil \epsilon N_j \rceil,j}} \), where \( \lceil \cdot \rceil \) denotes the number of elements in the set.

Ask: When the ranking of user \( i \) is low and it is not assigned resource by supplier \( j \) in the auction stage, supplier \( j \) will send its current reserve price \( B_j \) to user \( i \). The value of \( B_j \) is determined by supplier \( j \) according to the strategy presented in Section II-C.

Demand Adjustment: When user \( i \) receives the reserve price \( B_j \) from supplier \( j \), it can adjust the amount of requested resource according to its own payoff function (defined later) that is related to \( B_j \). User \( i \) adjusts its requested resource amount from the original value \( W_{ij} \) to a new value \( Q_{ij} \), where \( Q_{ij} \) may be zero if the user decides not to request any resource from the supplier.

Accept/Fail: If the total amount of new requests does not exceed supplier \( j \)'s total available resource that is selected for negotiation, i.e., \( \sum_{j \in \mathcal{N}_j \setminus \mathcal{N}_{j,\text{neg}}} Q_{ij} \leq (1 - \epsilon) R_j \), supplier \( j \) will accept all users' adjusted demands. Otherwise, the users are ranked according to their bidding prices in the auction stage, and resources are assigned to users starting from the one with the highest bidding price, until the total amount of available resource is reached. This resource allocation policy is to encourage users to bid truthfully.

The detailed procedures of the supplier and the user are respectively shown in Algorithms 1 and 2, where only the subset of users that submit request to supplier \( j \) are considered.

B. Payoff of Users and Resource Suppliers

In the negotiation stage, the payoff function of user \( i \) when negotiating with supplier \( j \) is defined as follows:

\[
U_{ij} \triangleq \alpha_{ij} \log(1 + Q_{ij}) - B_j Q_{ij} - \varphi(x)
\]  

(1)
Algorithm 1 DAN procedure of supplier $j$

1: Define a variable $Y_j$ to record the current amount of resource that has been sold by supplier $j$
2: Initialize $Y_j \leftarrow 0$
3: loop
4: while $Y_j + W_{ij} \leq c R_j$ and $k \leq |N_j|$ do
5: $Y_j \leftarrow Y_j + W_{ij}$
6: Accept user $i_k$, charge user $i_k$ the price of $P_{\min(k+1,|N_j|)}$.
7: end while
8: Send $FAIL$ to remaining users $i_k$, $i_{k+1}, ..., i_{|N_j|}$.
9: end loop

Algorithm 2 DAN procedure of user $i \in N_j$

1: loop
2: while start of new timeslot do
3: Submit bid $(P_{ij}, W_{ij})$ to supplier $j$
4: while auction outcome received from supplier $j$ do
5: if bid accepted then
6: Pay price as specified by supplier $j$ to obtain resource in the amount of $W_{ij}$
7: else
8: receive supplier $j$’s reserve price $B_j$
9: Find new demand $Q_{ij}$ based on $B_j$, and send the request to supplier $j$
10: wait until received feedback from supplier $j$
11: Utilize resource $Q_{ij}$ if new demand is accepted, otherwise wait for next transaction round
12: end if
13: end loop

where “$\triangleq$” stands for ”is defined as”, and

$$x \triangleq \frac{1}{(1 - \epsilon) R_j - \sum_{i \in N_{j,\text{neg}}} Q_{ij}}$$

The first term in (1) is the user’s revenue in which $a_{ij} \geq 0$ is a weighting factor that indicates its relative importance compared to other economic factors [14]. The second term is the user’s direct cost paid for purchasing the resource. The third term is its indirect cost that is related to other users’ resource occupation, which affects issues such as queuing delay. The function $\varphi(x)$ is defined as a non-decreasing function satisfying

$$\varphi'(x) \triangleq \frac{d \varphi(x)}{dx} \geq 0$$

$$\varphi''(x) \triangleq \frac{d^2 \varphi(x)}{dx^2} \geq 0$$

For example, the conditions (3) and (4) can be satisfied when $\varphi(x)$ is a linear function. We also note that $a_{ij}$ and $\varphi(\cdot)$ both contain units for normalization purpose, so that the three terms in (1) can be expressed in one equation.

The user’s payoff in the auction stage can be defined in a similar way, by substituting $Q_{ij}, B_j, (1 - \epsilon)$, and $N_{j,\text{neg}}$ in (1) respectively with $W_{ij}, P_{ij}, \epsilon$, and $N_{j,\text{auc}}$.

The payoff of supplier $j$ is defined as follows:

$$S_j \triangleq \sum_{i \in N_{j,\text{auc}}} P_{ij} W_{ij} + B_j \sum_{i \in N_{j,\text{neg}}} Q_{ij} - X_j$$

where $X_j$ is the initial and upkeep cost of the supplier.

C. Strategies of Users and Resource Suppliers

Each user and each resource supplier respectively want to maximize their own payoff.

User’s Strategy: In the auction stage, as discussed earlier, the user’s best strategy is to bid the true value of the resource based on its own evaluation. In the negotiation stage, the reserve price $B_j$ is given to the user, and the user aims to choose $Q_{ij} \geq 0$ to maximize (1).

Supplier’s Strategy: The supplier first chooses the value of $\epsilon$. Given $\epsilon$, the only controllable part in its payoff function (5) is its reserve price $B_j$. The value of $B_j$ is determined based on the bids submitted by users in the auction stage, and $B_j$ is never lower than the highest price charged to the users who have won the bidding, i.e., $B_j \geq \max_{i \in N_{j,\text{auc}}} P_{ij}$. This gives users the incentive of participating in the auction stage and bid truthfully instead of waiting for negotiation directly. The bidding prices from users also reveal the current market value of resources to the resource supplier.

III. NASH EQUILIBRIUM IN THE NEGOTIATION STAGE

The strategy introduced in Section II-C is a non-cooperative game, because each user and supplier tries to maximize its own payoff independently. In the following, we focus on the negotiation stage, and discuss the conditions of having an equilibrium strategy for the user demand $Q_{ij}$ and the supplier’s reserve price $B_j$. When the equilibrium exists, an individual user cannot gain payoff by submitting aggressive demands, and an individual supplier also cannot gain payoff by submitting aggressive prices. We note that the auction procedure is standard, so we do not discuss about the auction stage here. The strategic choice of which supplier a user should apply for resources as well as the choice of $\epsilon$ is left for future work. For simplicity, we also ignore the fact that only a subset of negotiating users can obtain resources due to the resource capacity limit, and we focus on a fixed set of negotiating users (denoted by $N_{j,\text{neg}}$) and assume that all the users within this set can receive resources.

Definition 1. (Nash Equilibrium [15]) For a game with $H$ players, a strategy vector $\psi^* = (\psi_1^*, \psi_2^*, ..., \psi_H^*)$ is in Nash equilibrium if no single player $h$ can make profit by solely changing its own strategy. That means if player $h$ changes its equilibrium strategy $\psi_h^*$ to another strategy $\psi_h$, then for all
We note that $30 < h < H$, we have
\[
\Psi_h(\psi^1_1, ..., \psi^h_{i-1}, \psi^h_i, \psi^{h+1}_i, ..., \psi^H_i) 
\geq 
\Psi_h(\psi^1_1, ..., \psi^h_{i-1}, \psi^h_i, \psi^{h+1}_i, ..., \psi^H_i)
\]  \quad (6)
where $\Psi_h$ is the payoff of player $h$.

In the negotiation stage, different resource suppliers are isolated and do not have competition, because the set of users that each supplier may serve for is already fixed. Therefore, for an arbitrary resource supplier $j$, the players in this game include supplier $j$ and all the users in $\mathcal{N}_{j,\text{neg}}$. In the following, we first fix the reserve price $B_j$ and discuss the condition of having Nash equilibrium among all users with given $B_j$. Then, we consider the Nash equilibrium among all users and the supplier.

**Proposition 1.** In the negotiation stage, for a given $B_j$, the necessary and sufficient condition of the users’ resource demands $Q_{ij}$ attaining equilibrium state is that for all $i \in \mathcal{N}_{j,\text{neg}}$, the following satisfies:
\[
Q_{ij} = \max \left\{ \frac{a_{ij}}{B_j + \varphi'(x)} - 1; 0 \right\} \quad (7)
\]
where we recall that $x = \frac{1}{(1-\epsilon)H_j - \sum_{i \in \mathcal{N}_{j,\text{neg}}} Q_{ij}}$.

The Nash equilibrium exists if there exists at least one solution set $\{Q_{ij} : \forall i \in \mathcal{N}_{j,\text{neg}}\}$ that satisfies (7) for all $i$.

**Proof.** Sufficiency: We note that $Q_{ij} = \frac{a_{ij}}{B_j + \varphi'(x)} - 1$ is equivalent to
\[
\frac{\partial U_{ij}}{\partial Q_{ij}} = \frac{a_{ij}}{1 + Q_{ij}} - B_j - \varphi'(x) = 0
\]
We also note that
\[
\frac{\partial^2 U_{ij}}{\partial Q_{ij}^2} = \frac{-a_{ij}}{\left(1 + Q_{ij}\right)^2} - \varphi''(x) \leq 0
\]
where the inequality follows from (4).

We focus on user $i$ in the following and assume that other users do not change their strategies, i.e., $Q_{ij'}$ is fixed for $i' \neq i$. It follows that, when $\frac{a_{ij}}{B_j + \varphi'(x)} - 1 \geq 0$, the maximum value of $U_{ij}$ is attained at $Q_{ij} = 0$; when $\frac{a_{ij}}{B_j + \varphi'(x)} - 1 < 0$, $U_{ij}$ is non-increasing for $Q_{ij} \geq 0$ and the maximum value of $U_{ij}$ is attained at $Q_{ij} = 0$. Hence, the equilibrium is attained when (7) is satisfied for all users $i$.

Necessity: We prove by contradiction. Suppose that (7) is not satisfied. According to the above discussion, there exists at least one user $i$ that can change its value of $Q_{ij}$ so that $U_{ij}$ increases while other users’ strategies remain unchanged. Hence, this state is not in equilibrium.

**Proposition 2.** In the negotiation stage, assume that there exists a solution set $\{Q_{ij} : \forall i \in \mathcal{N}_{j,\text{neg}}\}$ satisfying (7) for all $i \in \mathcal{N}_{j,\text{neg}}$, then the game involving both supplier $j$ and users $i \in \mathcal{N}_{j,\text{neg}}$ has a Nash equilibrium.

**Proof.** Proposition 1 has already shown that, under the assumption of this proposition, the users reach equilibrium when $B_j$ is given. We only focus on $B_j$ in the following.

We note that $B_j$ only affects the term $B_j \sum_{i \in \mathcal{N}_{j,\text{neg}}} Q_{ij} \cong V_j$ in the supplier’s payoff function (5). According to (7), when $B_j \geq a_{ij}$, we have $Q_{ij} = 0$ for all $i$ (because $\varphi'(x) \geq 0$ according to (3)), yielding $V_j = 0$. When $B_j = 0$, we also have $V_j = 0$.

Because $V_j \geq 0$ is a continuous function of $B_j$ (with $Q_{ij}$ determined by (7)), and $V_j = 0$ when $B_j = 0$ or $B_j \geq a_{ij}$, there exists at least one maximum value of $V_j$ within the interval $B_j \in [0, a_{ij}]$ which is the global maximum of $V_j$. The equilibrium of supplier $j$ and users $i \in \mathcal{N}_{j,\text{neg}}$ is attained at this maximum point.

Equation (7) is intuitive because the quantity that consumers are willing to buy decreases as the price becomes high. Discussion on the existence of the solution set $\{Q_{ij} : \forall i \in \mathcal{N}_{j,\text{neg}}\}$ satisfying (7) is left for future work, while we note that the solution obviously exists when $\varphi'(x) = 0$. When the solution exists, $Q_{ij}$ and $B_j$ take finite values when in equilibrium, which means that any value of $Q_{ij}$ (or $B_j$) that is too low or too high is not beneficial for the user (or supplier).

**IV. SIMULATION RESULTS**

In this section, we evaluate the performance of the proposed DAN scheme and compare it to the traditional double auction scheme without negotiation. We set $|\mathcal{M}| = 3$, $\epsilon = 0.6$, and $R_j = 500$ for all $j$. In every transaction round in the simulation, each user $i$ is randomly assigned to a supplier $j$, from which user $i$ requests resources. The assignment is performed in such a way that the number of users at each supplier is evenly distributed. In user $i$’s initial request $(P_{ij}, W_{ij})$, $W_{ij}$ and $P_{ij}$ are respectively uniformly distributed in intervals $(20, 40)$ and $(1, 2)$. We ignore the indirect cost $\varphi'(x)$ in (7). For a given $B_j$, the resource demand submitted by the user in the negotiation stage is $Q_{ij} = a_{ij} - 1$, where we set $a_{ij} = P_{ij}(1 + W_{ij})$, which is also obtained from (7) by ignoring $\varphi'(x)$ and substituting $B_j$ and $Q_{ij}$ respectively with $P_{ij}$ and $W_{ij}$. We set the supplier’s reserve price as $B_j = 1.2 \cdot \max_{i \in \mathcal{N}_{j,\text{neg}}} P_{ij}$ unless otherwise specified. We show the instantaneous results in Fig. 2 and consider the overall results of 50 transaction rounds in Fig. 3.

We first fix $|\mathcal{N}| = 90$. Fig. 2 shows the instantaneous payoff values of supplier $j = 1$ in different transaction rounds. We can see that the payoff of the proposed DAN scheme is...
almost always larger than the traditional auction scheme. The average payoffs of the DAN scheme and the traditional auction scheme are respectively 1063.132 and 895.349. In Fig. 3(a), we evaluate how the reserve price affects the payoff of the suppliers. We vary the reserve price of one particular supplier, while the other suppliers take the default value as discussed earlier in this section. We can see that each supplier’s payoff has a maximum point, which confirms our analysis in Section III. The different shapes of suppliers’ payoff curves is mainly due to randomness in the simulation.

We then conduct simulations with different market sizes where the total number of users $|N| \in \{10, 30, 50, 70, 90, 110, 130, 150\}$. The average payoffs of all suppliers are shown in Fig. 3(b). In Fig. 3(c), we consider the performance from the users’ perspective and plot the average number of users that have obtained a non-zero amount of resource (we call such users as winning users). We can see from Figs. 3(b) and 3(c) that the proposed DAN scheme performs better particularly when the number of users is large. This is because the negotiation stage in the proposed DAN scheme encourages more users to buy a small amount of resource even when the unit price is high.

V. CONCLUSIONS

We have proposed the DAN scheme which makes use of the fact that users can have elastic resource demands. The DAN scheme contains two stages, an auction stage and a negotiation stage. We have analytically shown the conditions of existence of Nash equilibrium. Simulation results also confirm that the DAN scheme can improve the performance from both the suppliers’ and users’ perspective, especially under high market demands. We regard this paper as an initial attempt toward this problem. Future work can investigate the optimal choice of $\epsilon$, the class of $\phi(x)$ that guarantees existence of Nash equilibrium in the negotiation stage, as well as the interplay between the auction and negotiation stages.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China (91438110, 61302072, 61471109), the Fundamental Research Funds for the Central Universities (N140405007), the Program for New Century Excellent Talents in University (NCET-12-0102), and the Specialized Research Fund for the Doctoral Program of Higher Education (2012042120049).

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Fig. 3. Overall results: (a) average payoff of suppliers at different reserve prices $B_j$, (b) average payoff of suppliers at different number of users, (c) average number of winning users under different number of users.