Rate and Power Adaptation for Physical-Layer Network Coding with M-QAM Modulation

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***Abstract*—Physical-layer network coding (PNC) is an effective strategy for increasing the throughput of wireless networks. In the current literatures, PNC without rate and power adaptation is mainly focused. Realizing that the transmission efficiency can be improved through rate and power adaptation in wireless networks, this paper focuses on developing a rate and power adaptation scheme for PNC. Through formulating how the data rate and transmission power affect the bit error rate (BER) of involved links in PNC, we observe that with a given data rate, the transmission power has to satisfy some constraints. Using these power constraints, we obtain a candidate set of optimal transmission power. By traversing the candidate set and the data rates supported by nodes, a rate and power adaptation scheme is developed. To test its performance, we apply the proposed scheme into an existing PNC-supported MAC protocol. Simulation results demonstrate that the proposed scheme can improve the throughput and delay performance in various scenarios.**

***Index Terms*—Denoise-and-forward (DNF), physical-layer network coding (PNC), rate and power adaptation, wireless networks.**

I. Introduction

Network coding can essentially improve the throughput performance of wireless networks. In conventional network coding (CNC) [1], relay nodes encode the packets separately received from source nodes, and then broadcast the encoded packet. Physical-layer network coding (PNC) extends CNC to further increase network throughput [2]. In PNC, a relay node encodes the packets simultaneously received from source nodes and forwards the encoded packet over a broadcasting channel. Two relaying methods of PNC are generally used: amplify-and-forward (AF) and denoise-and-forward (DNF) [3]. PNC with AF method is also known as analog network coding (ANC). We focus on PNC with DNF in this paper, and simply refer to it as PNC. An example topology of PNC is shown in Fig. 1, where two flows (i.e. $s_1 \to r \to d_1$ and $s_2 \to r \to d_2$) exist, which means s_i wants to send packets to d_i ($i = 1, 2$). This example topology is also known as X-shaped topology. The PNC based on X-shaped topology is named overhearing PNC because d_1 (or d_2) in X-shaped topology needs to overhear packets from s_2 (or s_1) to successfully decode.

In wireless networks, channel conditions vary with the fading condition and location of nodes. To optimize the network

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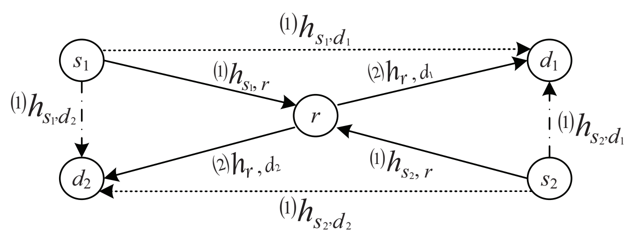

Fig. 1. PNC based on X-shaped topology with two flows: $s_1 \to r \to d_1$ and $s_2 \to r \to d_2$. The number in the brackets indicates the transmission timeslots. The variable h_{n_i,n_j} is the channel gain from node n_i to node n_j.

throughput, it is necessary to adaptively switch data rates to match the channel conditions. Many existing works pay much attention to CNC with rate adaptation [4], [5], while the rate adaptation scheme for PNC is not widely studied. In [6], we have proposed a rate adaptation scheme for ANC with the X-shaped topology, and observed that maximum transmission power does not mean maximizing the achievable data rate, due to self-interference and unequal distances between source nodes and the relay r. However, to the best of our knowledge, a rate and power adaptation scheme for DNF-based PNC has not been proposed in existing literatures.

In this paper, we focus on designing a practical rate and power adaptation scheme for PNC with M-QAM modulation. In our scheme, dynamic data rates are selected by switching the modulation level of M-QAM (i.e. the value of M). Our goal is to find the maximum M and optimal transmission power to ensure the maximum bit error rate (BER) of all the involved links in PNC smaller than a given threshold. Compared with the rate and power adaptation scheme for ANC in [6], the challenges in this paper are to formulate the relationship between BER, modulation level M, and transmission power (while [6] only considered the channel capacity) and perform optimization based on this formulation.

This paper tackles the above challenges. We first formulate how data rate and transmission power affect BER. Afterwards, we find that given a data rate, the optimum power must satisfy some constraints. Subject to these constraints, we obtain a candidate set of optimal transmission power. Then, using the candidate set, we develop a practical rate and power adaptation scheme for PNC. In the proposed scheme, we traverse the transmission rates supported by nodes, and at each given rate, we obtain the optimal transmission power (that minimizes the maximum BER of the involved links in PNC) from the candidate set. Finally, to test the performance of the proposed scheme, we apply it into an existing PNC-supported MAC protocol proposed in [7].

The remainder of this paper is organized as follows. Section II formulates the relationship between BER, data rate, and transmission power. Section III shows the details of the proposed scheme. With applying the proposed scheme into an existing PNC-supported MAC protocol, the simulation results are given in Section IV. Section V draws conclusions.

II. Problem Formulation

In this section, we first model the received signals in PNC. Then, we present the formula of the relationship between BER, data rate, and transmission power. Although the following analysis is for PNC based on X-shaped topology (i.e. overhearing PNC), it can be easily extended to cover PNC for other topologies where two flows exist and either one or two destination nodes do not need overhearing, by assuming that a virtual overhearing link exists and its channel gain is infinity.

A. Received Signals

When performing PNC, we have to consider signal superposition and self-interference. We define z_n as noise and h_{n_i,n_j} as the channel gain from node n_i to node n_j, where n_i and n_j can be either s_1, s_2, d_1, d_2, or r. The signal sent by s_i is x_i. In the first transmission timeslot, x_1 and x_2 sent by s_1 and s_2 are superposed at the relay. The received signal at the relay can be written as

$$y_r = h_{s_1,r}x_1 + h_{s_2,r}x_2 + z_n \,. \tag{1}$$

The overheard signals at d_1 and d_2 are

$$y_{d_1} = h_{s_2,d_1}x_2 + (h_{s_1,d_1}x_1 + z_n)\,, \tag{2}$$

$$y_{d_2} = h_{s_1,d_2}x_1 + (h_{s_2,d_2}x_2 + z_n)\,. \tag{3}$$

In the second transmission timeslot, r broadcasts the encoded packet to destinations. The received signals at d_1 and d_2 are given by

$$y'_{d_1} = h_{r,d_1}C_{\text{PNC}}(y_r) + z_n\,, \tag{4}$$

$$y'_{d_2} = h_{r,d_2}C_{\text{PNC}}(y_r) + z_n\,, \tag{5}$$

where $C_{\text{PNC}}(\cdot)$ is the encoding function at the relay r, i.e. the DNF operation.

B. Relationship between BER, Data Rate, and Transmission Power

The symbol error rate (SER) and BER performances of PNC are evaluated over additive white Gaussian noise (AWGN) channel in this subsection. Each node transmits signals with M-QAM modulation and decodes signals based on the minimum distance decision rule [8]. Let $L=\sqrt{M}$, $Q(\cdot)$ denote the tail probability of the standard normal distribution, and σ_n^2 denote the power of noise. Similar to [9] and [10, Appendix A], assuming that the baseband signal is shaped with a raised cosine pulse with a roll-off factor $\beta=1$, we can write the SER with respect to average received signal-to-interference-noise ratio (SINR):

$$p_{\text{s}} = 1-\left(1-\frac{2(L-1)}{L}Q\left(\sqrt{\frac{3E_{\text{s}}}{(M-1)(\sigma_n^2+I_{n_i})T_{\text{s}}}}\right)\right)^2, \tag{6}$$

where E_s is the received energy per symbol, I_{n_i} is the total destructive interference power at the receiver node n_i excluding self-interference, and T_{s} is the time length of each symbol. Let P_{T,n_i} be the transmission power of node n_i. We have $E_{\text{s}} = P_{\text{T},n_i}\left|h_{n_i,n_j}\right|^2 T_{\text{s}}$. We consider the interference from other nodes because our analysis is for wireless multi-hop networks.

Let p_{b} denote the BER for square M-QAM, which is constrained by

$$\frac{p_{\text{s}}}{\log_2 M} \le p_{\text{b}} \le p_{\text{s}}. \tag{7}$$

We consider the upper bound where $p_{\text{b}} = p_{\text{s}}$ in the following analysis and simulations.

In the second timeslot of PNC, the relay r transmits its signal with the general M-QAM modulation. The corresponding SER can be directly obtained from (6).

In the first timeslot of PNC, let $p_{\text{b},r}$ denote the BER of the superposed signal received by the relay r. Similarly with (6), we express the variable $p_{\text{b},r}$ as

$$\begin{aligned} p_{\text{b},r} =& 1-\Bigg(1-\frac{2(L-1)}{L} \\ & Q\left(f_M\left(\min\left\{c_1\sqrt{P_{\text{T},s_1}}, c_2\sqrt{P_{\text{T},s_2}}\right\}\right)\right)\Bigg)^2, \end{aligned} \tag{8}$$

where $c_1 = \sqrt{\frac{3\left|h_{s_1,r}\right|^2}{(M-1)(\sigma_n^2+I_r)}}$, $c_2 = \sqrt{\frac{3\left|h_{s_2,r}\right|^2}{(M-1)(\sigma_n^2+I_r)}}$ and $f_M(\cdot)$ denotes the relationship between the SINR of the superposed signal and the minimum among P_{T,s_1} and P_{T,s_2}. The exact formula of $f_M(\cdot)$ depends on the data rate and transmission power at source nodes, the used coding and mapping method at the relay node [11], the accuracy of the synchronization method used in PNC, etc. As a whole tendency, the function $f_M(\cdot)$ becomes bigger with the increase of the minimum among P_{T,s_1} and P_{T,s_2} [12]. Thus, we assume that $f_M(\cdot)$ is a monotonically increasing function to derive the constraints of optimal transmission power.

According to [13], the BERs of overheard signals received at d_1 and d_2 in the first transmission timeslot are

$$\begin{aligned} p_{\text{b},d_1} = 1-\Bigg(1-\frac{2(L-1)}{L}\,Q\Big(\max\Big\{0, a_1\sqrt{P_{\text{T},s_2}} \\ -b_1\sqrt{P_{\text{T},s_1}}\Big\}\Big)\Bigg)^2, \end{aligned} \tag{9}$$

$$\begin{aligned} p_{\text{b},d_2} = 1-\Bigg(1-\frac{2(L-1)}{L}\,Q\Big(\max\Big\{0, a_2\sqrt{P_{\text{T},s_1}} \\ -b_2\sqrt{P_{\text{T},s_2}}\Big\}\Big)\Bigg)^2, \end{aligned} \tag{10}$$

where $a_1=\sqrt{\frac{3\left|h_{s_2,d_1}\right|^2}{(M-1)(\sigma_n^2+I_{d_1})}}$, $b_1=\sqrt{\frac{3\left|h_{s_1,d_1}\right|^2(L-1)^2}{(M-1)(\sigma_n^2+I_{d_1})}}$, $a_2=\sqrt{\frac{3\left|h_{s_1,d_2}\right|^2}{(M-1)(\sigma_n^2+I_{d_2})}}$, and $b_2=\sqrt{\frac{3\left|h_{s_2,d_2}\right|^2(L-1)^2}{(M-1)(\sigma_n^2+I_{d_2})}}$.

If $a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}}\le 0$ or $a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}}\le 0$, relay nodes can directly select the plain routing (PR) relaying method because it has a better performance than PNC. In our analysis, we only consider the cases that $a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}}>0$ and $a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}}>0$. Thus, the equalities (9) and (10) can also be written as

$$p_{\mathrm{b},d_1}=1-\left(1-\frac{2(L-1)}{L}Q\left(a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}}\right)\right)^2, \quad (11)$$

$$p_{\mathrm{b},d_2}=1-\left(1-\frac{2(L-1)}{L}Q\left(a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}}\right)\right)^2. \quad (12)$$

According to (11) and (12), it is easy to see that simultaneously transmitted signals can interfere with each other. To achieve the best BER performance for both transmissions, the transmission powers should be suitably controlled.

III. Optimizing Data Rate and Transmission Power

In this section, we discuss the constraints that the optimum power should obey for a given data rate, and present the details of the proposed rate and power adaptation scheme.

A. Constraints for Power Control

Let p'_{b,d_i} denote the BER of PNC in the second timeslot, which can be optimized with traditional methods (a brief description is presented in Subsection III-B), because in the second timeslot, PNC is executed with the conventional M-QAM modulation.

The BER at destination nodes after decoding is $1-(1-p_{b,r})(1-p_{b,d_i})(1-p'_{b,d_i})$. Because the BER of PNC in the second timeslot is independent from that in the first timeslot and can be optimized individually, we treat p'_{b,d_i} as a constant when analyzing the decoding BER. Thus, the decoding BER has the following upper bound

$$\begin{aligned}1-&(1-p_{b,r})(1-p_{b,d_i})(1-p'_{b,d_i})\\ &=p_{b,r}+p_{b,d_i}+p'_{b,d_i}-p_{b,r}p_{b,d_i}\\ &\quad-p_{b,r}p'_{b,d_i}-p_{b,d_i}p'_{b,d_i}+p_{b,r}p_{b,d_i}p'_{b,d_i}\\ &\le p_{b,r}+p_{b,d_i}+p'_{b,d_i}\\ &\le 2\max\{p_{b,r},p_{b,d_i}\}+p'_{b,d_i}. \quad (13)\end{aligned}$$

According to (13), the upper bound of the decoding BER is determined by $\max\{p_{b,r},p_{b,d_i}\}$ with the assumption that p'_{b,d_i} is a constant. Jointly considering both the decoding processes of destination nodes, the transmission power at source nodes should be controlled to minimize the maximum of $\max\{p_{b,r},p_{b,d_1}\}$ and $\max\{p_{b,r},p_{b,d_2}\}$ with a given modulation level M. This can be formulated as

$$\min_{P_{T,s_1},P_{T,s_2}}\{\max\{p_{\mathrm{b},d_1},p_{\mathrm{b},d_2},p_{\mathrm{b},r}\}\}. \quad (14)$$

Because $Q(\cdot)$ is a monotonically decreasing function, this optimization problem can be simplified as

$$\begin{aligned}\max_{P_{\mathrm{T},s_1},P_{\mathrm{T},s_2}}\Big\{\min\Big\{&a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}},\\ &a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}},\\ &f_M\left(\min\left\{c_1\sqrt{P_{\mathrm{T},s_1}},c_2\sqrt{P_{\mathrm{T},s_2}}\right\}\right)\Big\}\Big\}. \quad (15)\end{aligned}$$

Because $f_M(\cdot)$ is a monotonically increasing function, the overall optimization problem is equivalent to

$$\begin{aligned}\max_{P_{\mathrm{T},s_1},P_{\mathrm{T},s_2}}\quad & g(P_{\mathrm{T},s_1},P_{\mathrm{T},s_2})\\ &=\max\Big\{\min\Big\{a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}},\\ &\quad a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}},\\ &\quad f_M\left(c_1\sqrt{P_{\mathrm{T},s_1}}\right),f_M\left(c_2\sqrt{P_{\mathrm{T},s_2}}\right)\Big\}\Big\}, \quad (16)\end{aligned}$$

s.t.

$$\begin{aligned}&a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}}>0,\\ &a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}}>0,\\ &0<P_{\mathrm{T},s_1}\le P_{\mathrm{T,max}},\\ &0<P_{\mathrm{T},s_2}\le P_{\mathrm{T,max}},\end{aligned}$$

where $P_{\mathrm{T,max}}$ denotes the maximum transmission power.

Let P^*_{T,s_1} and P^*_{T,s_2} be the optimal solution, i.e. $g(P_{\mathrm{T},s_1},P_{\mathrm{T},s_2})$ achieves its peak value at $\{P^*_{\mathrm{T},s_1},P^*_{\mathrm{T},s_2}\}$. For the optimal solution, there are the following lemmas.

Lemma 1: At least one of P^*_{T,s_1} and P^*_{T,s_2} has to be equal to $P_{\mathrm{T,max}}$.

Proof: Suppose that both P^*_{T,s_1} and P^*_{T,s_2} are smaller than $P_{\mathrm{T,max}}$, we prove this lemma by deriving a contradiction.

Let both P^*_{T,s_1} and P^*_{T,s_2} be smaller than $P_{\mathrm{T,max}}$, and $0<qP^*_{\mathrm{T},s_1},qP^*_{\mathrm{T},s_2}<P_{\mathrm{T,max}}$ with $q>1$. Because $a_1\sqrt{P_{\mathrm{T},s_2}}-b_1\sqrt{P_{\mathrm{T},s_1}}$ and $a_2\sqrt{P_{\mathrm{T},s_1}}-b_2\sqrt{P_{\mathrm{T},s_2}}$ are bigger than zero, we have

$$\begin{aligned}a_1\sqrt{P^*_{\mathrm{T},s_2}}-b_1\sqrt{P^*_{\mathrm{T},s_1}}&<a_1\sqrt{qP^*_{\mathrm{T},s_2}}-b_1\sqrt{qP^*_{\mathrm{T},s_1}},\\ a_2\sqrt{P^*_{\mathrm{T},s_1}}-b_2\sqrt{P^*_{\mathrm{T},s_2}}&<a_2\sqrt{qP^*_{\mathrm{T},s_1}}-b_2\sqrt{qP^*_{\mathrm{T},s_2}},\\ f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}})&<f_M(c_1\sqrt{qP^*_{\mathrm{T},s_1}}),\\ f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})&<f_M(c_2\sqrt{qP^*_{\mathrm{T},s_2}}).\end{aligned}$$

These inequalities indicate that $g(qP^*_{\mathrm{T},s_1},\ qP^*_{\mathrm{T},s_2})>g(P^*_{\mathrm{T},s_1},\ P^*_{\mathrm{T},s_2})$, which is contradictory with the assumption that $g(P_{\mathrm{T},s_1},P_{\mathrm{T},s_2})<=g(P^*_{\mathrm{T},s_1},P^*_{\mathrm{T},s_2})$. Thus, as the optimal solution, P^*_{T,s_1} and P^*_{T,s_2} cannot be both smaller than $P_{\mathrm{T,max}}$, and this lemma is proved. ∎

Lemma 2: When the best solution is obtained, at least one of the following equalities holds

1) $a_1\sqrt{P^*_{\mathrm{T},s_2}}-b_1\sqrt{P^*_{\mathrm{T},s_1}}=a_2\sqrt{P^*_{\mathrm{T},s_1}}-b_2\sqrt{P^*_{\mathrm{T},s_2}}$,

2) $a_1\sqrt{P^*_{\mathrm{T},s_2}} - b_1\sqrt{P^*_{\mathrm{T},s_1}} = f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}})$,
3) $a_2\sqrt{P^*_{\mathrm{T},s_1}} - b_2\sqrt{P^*_{\mathrm{T},s_2}} = f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})$,
4) $f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}}) = f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})$.

Proof: When $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}})$ or $f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})$, this lemma can be directly proved. If $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}})$, by deceasing P^*_{T,s_2}, there must exist P^*_{T,s_2} that makes equality 2) or 4) hold while keeping $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = f_M(c_1\sqrt{P^*_{\mathrm{T},s_1}})$. If $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})$, by decreasing P^*_{T,s_1}, we must have P^*_{T,s_1} that makes equality 3) or 4) hold while keeping $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = f_M(c_2\sqrt{P^*_{\mathrm{T},s_2}})$.

When $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = a_1\sqrt{P^*_{\mathrm{T},s_2}} - b_1\sqrt{P^*_{\mathrm{T},s_1}}$ or $a_2\sqrt{P^*_{\mathrm{T},s_1}} - b_2\sqrt{P^*_{\mathrm{T},s_2}}$, we prove this lemma by deriving contradictions. Suppose that $\{P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}\}$ is obtained and no equality holds. If $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = a_1\sqrt{P^*_{\mathrm{T},s_2}} - b_1\sqrt{P^*_{\mathrm{T},s_1}}$, we can further optimize this solution by decreasing P^*_{T,s_1} or increasing P^*_{T,s_2} while keeping $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = a_1\sqrt{P^*_{\mathrm{T},s_2}} - b_1\sqrt{P^*_{\mathrm{T},s_1}}$. It means that $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = a_1\sqrt{P^*_{\mathrm{T},s_2}} - b_1\sqrt{P^*_{\mathrm{T},s_1}}$ can further become bigger, which is contradictory with the assumption that $g(P_{\mathrm{T},s_1}, P_{\mathrm{T},s_2})$ arrives its peak value at $\{P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}\}$. If $g(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2}) = a_2\sqrt{P^*_{\mathrm{T},s_1}} - b_2\sqrt{P^*_{\mathrm{T},s_2}}$, by decreasing P^*_{T,s_2} or increasing P^*_{T,s_1}, we have similar result.

Thus, at least one of equalities holds when the optimal solution is obtained. ■

According to Lemmas 1 and 2, a candidate set of optimal transmission power can be obtained. Because the optimal transmission power is related with the modulation level M, the optimal data rate and transmission power are adaptively selected by traversing the candidate set of optimal transmission power and the offered modulation levels.

B. Rate and Power Adaptation

The proposed scheme consists of two optimization processes, i.e. the optimization processes for the first and second timeslots in PNC. In this subsection, we present the two processes, respectively. Let $M^*_k,\ k \in \{1, 2\}$ be the maximum M which makes sure that the BER of signals in the kth timeslot is smaller than a given threshold. Let R_k be the transmission rate in the kth timeslot. On the unit bandwidth, we have $R_k = \log_2(M^*_k)$.

In the first timeslot of PNC, s_1 and s_2 transmit their packets with the same modulation level. M^*_1 is equal to the biggest M which satisfies the constraint that the maximum BER of the involved links $s_1 \to r$, $s_2 \to r$, $s_1 \to d_2$ and $s_2 \to d_1$ is smaller than the given threshold. The Algorithm 1 describes the process of the rate and power adaptation in this timeslot. For a given M, we first obtain the candidate set of the optimal power solution according to the Lemmas 1 and 2. Then, from the candidate set, we derive the locally optimal transmission power under the condition that the modulation level M is given. Finally, by traversing the offered M, the Algorithm 1 can find M^*_1 and the corresponding globally optimal transmission power. Algorithm 1 completes within finite steps, hence its complexity is $O(1)$.

In the second timeslot of PNC, we define $M^{\max}_{n_i \to n_j}$ as the maximum M of link $n_i \to n_j$ which satisfies the BER constraint. Given a M, the BER of a link is evaluated by using (6) with the maximum transmission power supported by wireless nodes. Nodes can use the maximum transmission power for the reason that there is no signal superposition and self-interference in this timeslot of PNC. To ensure the successful receptions of both destination nodes in the second timeslot, we have $M^*_2 = \min\{M^{\max}_{r\to d_1}, M^{\max}_{r\to d_2}\}$.

Algorithm 1 Rate and Power Adaptation for the first transmission timeslot of PNC.

Initialization:
$P^*_{\mathrm{T},s_1} \leftarrow P^*_{\mathrm{T},s_2} \leftarrow P_{\mathrm{T,max}}$;
$M^*_1 \leftarrow 2$, $M \leftarrow \{256, 128, 64, 32, 16, 8, 4, 2\}$;
$p^*_b(P_{\mathrm{T},s_1}, P_{\mathrm{T},s_2}) \leftarrow p^*_b(P_{\mathrm{T,max}}, P_{\mathrm{T,max}})$; //the maximum BER of the involved links
Iteration:
for $j = 1$ to 8 **do**
  $L \leftarrow \sqrt{M(j)}$;
  for $i = 1$ to 4 **do**
    Get $P_{\mathrm{T},s_2,i}$ from equality i) in Lemma 2 by assuming $P_{\mathrm{T},s_1} = P_{\mathrm{T,max}}$;
    $P_{\mathrm{T},s_1,i} \leftarrow P_{\mathrm{T,max}}$;
  end for
  for $i = 1$ to 4 **do**
    Get $P_{\mathrm{T},s_1,i+4}$ from equality i) in Lemma 2 by assuming $P_{\mathrm{T},s_2} = P_{\mathrm{T,max}}$;
    $P_{\mathrm{T},s_2,i+4} \leftarrow P_{\mathrm{T,max}}$;
  end for
  for $i = 1$ to 8 **do**
    if $P_{\mathrm{T},s_1,i}$ and $P_{\mathrm{T},s_2,i}$ satisfy the constraints in (16) **then**
      if $p^*_b(P_{\mathrm{T},s_1,i}, P_{\mathrm{T},s_2,i}) < p^*_b(P^*_{\mathrm{T},s_1}, P^*_{\mathrm{T},s_2})$ **then**
        $P^*_{\mathrm{T},s_1} \leftarrow P_{\mathrm{T},s_1,i}$, $P^*_{\mathrm{T},s_2} \leftarrow P_{\mathrm{T},s_2,i}$;
      end if
    end if
  end for
  if $p^*_b(P^*_{T,s_1}, P^*_{T,s_2}) <$ BER-threshold **then**
    $M^*_1 \leftarrow M(j)$;
    return;
  end if
end for

IV. Simulation Results

To evaluate the performance of the proposed rate and power adaptation scheme, we apply it into an existing PNC-supported MAC protocol in [7], i.e. the overhearing-supported PNC MAC protocol (OPNC-MAC). For convenience, OPNC-MAC with the function of rate and power adaptation is named the Rate and Power adaptive and Overhearing-supported PNC-MAC protocol (RPOPNC-MAC). According to [7], network coding is not always efficient in practice, i.e. we need to switch the relaying method to match the given channel conditions. Therefore, RPOPNC-MAC should support the relaying methods of PNC, CNC, and plain routing. We compare two different protocol groups: 1) protocols *without* rate adaptation;

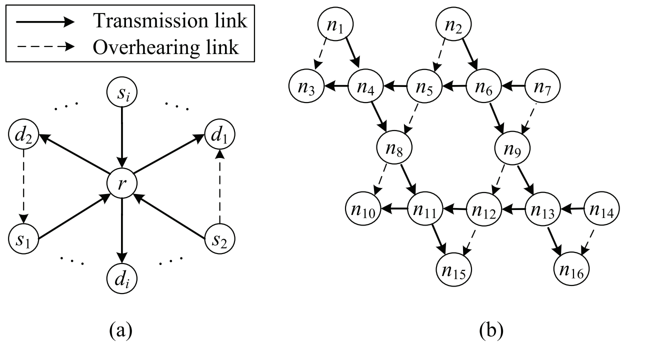

Fig. 2. Simulation topologies: (a) wheel topology, (b) #-shaped topology.

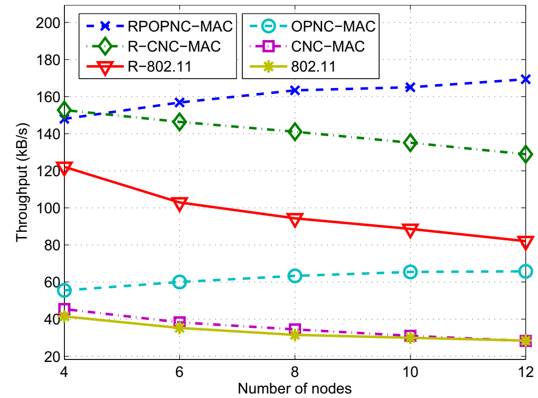

Fig. 3. Throughput vs. number of nodes in the wheel topology.

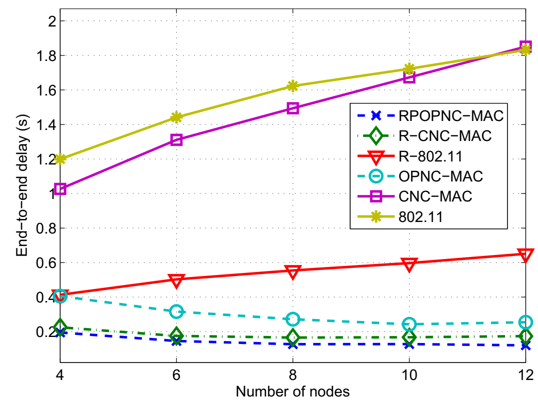

Fig. 4. End-to-end delay vs. number of nodes in the wheel topology.

2) protocols *with* rate adaptation. Protocols *without* rate adaptation include OPNC-MAC protocol, MAC protocol for CNC with reliable broadcasting [14] (referred to as CNC-MAC), and conventional IEEE 802.11 MAC protocol. CNC-MAC and 802.11 MAC with the function of rate adaptation are named R-CNC-MAC and R-802.11 (which was developed in [15]), respectively. In RPOPNC-MAC, the rate adaptation method is similar to that in R-CNC-MAC or R-802.11 when CNC or PR is used. Protocols with rate adaptation include RPOPNC-MAC, R-CNC-MAC and R-802.11.

RPOPNC-MAC is evaluated with our simulator jointly developed with MATLAB and C. To take into account both single-relay and multi-relay cases, two topologies are considered: a wheel topology and a #-shaped topology, as shown in Fig. 2. The application of PNC in more complicated networks is not considered, because PNC needs to be incorporated with coding-aware routing to achieve a good performance in these networks [16], which is beyond the scope of this paper. In RPOPNC-MAC, the maximum transmission power is 3 dBm and the BER threshold is 10^{-5}. We select $f_M(x) = x$, which is the simplest form that can satisfy the requirements described in Sections II and III. Readers can refer to [7] for the details of other parameter setup. Each simulation is run 50 s with 10 different random seeds to obtain overall performance.

A. Wheel Topology

The wheel topology is shown in Fig. 2(a), where nodes communicate with each other through the same relay node r. The sources s_1, s_2, $\cdots$, s_m and destinations d_1, d_2, $\cdots$, d_m are placed in a circle centered at r. The distance between r and each end node is 150 m. The source s_i and its destination d_i are opposite nodes, and cannot communicate with each other directly. The angle between two source-destination edges (e.g. the angle between $s_1 \to d_1$ and $s_2 \to d_2$) is a random value from 0 to 2π, and changes every second. The source nodes are backlogged, i.e. they always have packets to send.

The throughput under different number of nodes in the wheel topology is shown in Fig. 3. We can observe that compared with the protocols without rate adaptation, the throughput of the protocols with rate adaptation is significantly promoted. Specially, the throughput gain of RPOPNC-MAC over OPNC-MAC is 2.59 on the average. It is can be found that the throughput is approximately constant (with a slight increase) when the PNC-supported MAC protocols are performed. The reason is that the transmissions of the source nodes are coordinated by the relay node r, i.e. no contention exists between these source nodes (also due to the wait-for-PNC mechanism [10]). The throughput of the conventional CNC-supported and PR-supported protocols decreases when the number of end nodes increases, which is caused by severe channel contention when there are a lot of end nodes.

The end-to-end delay under different numbers of nodes is shown in Fig. 4. We can find that the end-to-end delay is greatly reduced in the protocols with rate adaptation. We can also observe that the delay of RPOPNC-MAC protocol is the lowest in all the considered protocols. The reason is that when PNC is performed, the channel contention is reduced and data packets are sent immediately (without entering the queue of the relay) after being received.

B. #-Shaped Topology

A #-shaped topology with 16 nodes is shown in Fig. 2(b), where the performance of the RPOPNC-MAC is evaluated. We set the distance between a node and each of its neighbors to 175 m. In the topology, we configure four flows that are $n_1 \to n_{15}$, $n_2 \to n_{16}$, $n_7 \to n_3$, and $n_{14} \to n_{10}$.

Fig. 5 shows the throughput under different packet rates. It is can be observed that RPOPNC-MAC still has the best throughput performance. We can also find that the average throughput gain of RPOPNC-MAC over OPNC-MAC is 1.76.

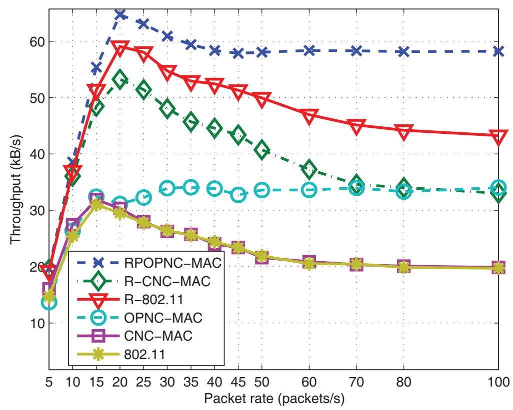

Fig. 5. Throughput vs. packet rate in the #-shaped topology.

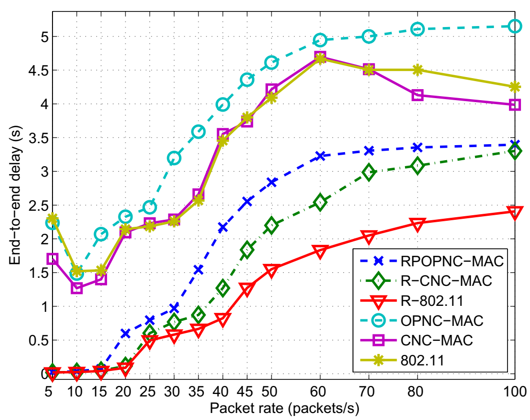

Fig. 6. End-to-end delay vs. packet rate in the #-shaped topology.

It is necessary to notice that R-CNC-MAC has the lowest throughput among the protocols with rate adaptation. The reason is that with the increase of the packet rate, the coding opportunities are greatly reduced by unbalanced flows at nodes n_4 and n_{13}, while the overhead used to sense coding opportunities still exists [14]. The performance RPOPNC-MAC is not reduced by the unbalanced-flow effect because relay nodes coordinate the transmissions of source nodes.

Fig. 6 shows the end-to-end delay of the #-shaped topology. We can observe that the end-to-end delay of the PNC-supported protocols is slightly higher than those of the CNC-supported protocols and PR-supported protocols. The phenomena is caused by the fact that the #-shaped topology actually consists of four X-shaped topologies and bottlenecks may be formed at some of the shared nodes (i.e. n_5, n_8, n_9, n_{12}) in any two neighboring X-shaped topologies. Unsuccessful PNC operations caused by the contention occurring at these bottlenecks result in the fact that the relays have to switch to other relaying methods at the cost of delay.

V. Conclusions

In this paper, we have proposed a rate adaptation and power control scheme for overhearing PNC with M-QAM modulation. By formulating the relationship between BER, data rate, and transmission power, we derive the constraints of optimal transmission power. Using these constraints, the proposed scheme selects the optimal data rate and transmission power by traversing data rate supported by wireless nodes. To test its performance, we have extended OPNC-MAC in [7] to RPOPNC-MAC to support the proposed rate and power adaptation scheme. The simulation results indicate that RPOPNC-MAC brings throughput improvement over conventional non-PNC-supported schemes and PNC-supported schemes with fixed data rate and power, while maintaining a reasonable delay. This demonstrates that the proposed scheme is beneficial for networks that require high and stable throughput performance.

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