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Rate and Power Adaptation for Analog Network Coding

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***Abstract*—Network coding is an emerging technique of packet forwarding that encodes the packets at the relay node, in order to increase the throughput of a relaying network. Analog Network Coding (ANC) encodes the packets by superposing the signals at the physical layer, which can bring further throughput improvements. In this paper, we first analyze the coding procedure of ANC, and elaborate the fact that the maximum possible number of coding nodes in ANC is two for common wireless transceivers. Then, we propose a rate adaptation scheme for ANC. We show that the transmission powers of nodes affect the data rate and transmitting at the maximum power is not always optimal. Based on this observation, we propose a method to find the optimal transmission power that maximizes the data rate. We also discuss some issues that need to be considered when implementing the proposed scheme in practical wireless systems. Afterwards, performance of the proposed scheme is evaluated through extensive simulations. The simulation results show that the proposed joint rate and power adaptation scheme outperforms ANC without power adaptation, and it is beneficial over other relaying schemes in a broad range of scenarios of mobile and vehicular applications.**

***Index Terms*—Communication, analog network coding, physical layer network coding, rate and power adaptation, wireless networks.**

I. INTRODUCTION

WIRELESS network coding is a promising approach of packet forwarding in wireless networks [1]. With wireless network coding, packets are encoded at relay nodes before they are forwarded to the respective destinations. The broadcast nature of wireless channels is exploited by this method, and the required number of timeslots for packet exchange can be reduced. Compared with conventional packet forwarding schemes, wireless network coding brings improved performance in terms of throughput, delay and network reliability.

In *Conventional Network Coding* (C-NC), packets are sent to the relay separately. Physical layer network coding [8]–[10] extends the idea of C-NC by mixing the packets at the physical layer, through superposing the electromagnetic waves transmitted by different source nodes. By this means, the source nodes transmit the packets simultaneously to the relay, and the network throughput can be further increased. Two general schemes of physical layer network coding have been investigated in the literature. The *DeNoise-and-Forward* (DNF) [11]–[13] scheme decodes the superposed signal at the relay node before forwarding. Although this scheme avoids noise amplification and signal attenuation (which will be further discussed in Section III-C), it requires synchronization among source nodes, which is difficult to realize in practice [14], [15]. The *Amplify-and-Forward* (AF) scheme [16]–[20] simply amplifies the superposed signal at the relay node, without any decoding operation. The AF scheme of physical layer network coding is often referred as *Analog Network Coding* (ANC) [16]. ANC is easier to implement since it only requires coarse synchronization at the packet level.

In the example network topology shown in Fig. 1, node s_1 intends to send a packet to node d_1, and node s_2 intends to send a packet to node d_2. The node pairs $s_1 \rightarrow d_1$ and $s_2 \rightarrow d_2$ are called *sessions*, x_1 and x_2 are the signals transmitted by s_1 and s_2 respectively. For this topology, conventional 4-phase relaying requires 4 timeslots to allow each session to send one data packet to its intended destination. With C-NC, three timeslots are required, since the relay can forward an encoded packet. The advantage of ANC becomes obvious in this case because only two timeslots are necessary, as illustrated in Fig. 1.

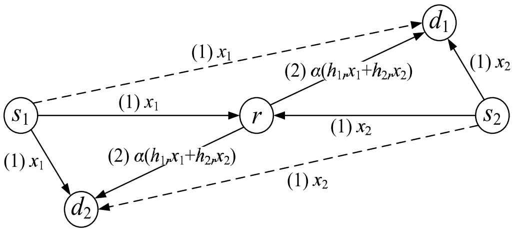

Fig. 1. Analog network coding with opportunistic listening. The number in the brackets indicates the timeslot. Dashed lines indicate the interference signal.

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The variables x_1 and x_2 denote the transmitted signals from s_1 and s_2 respectively, h_{1r} and h_{2r} are channel gains, and α is an amplification factor.

Since the quality of wireless channels varies with the location of nodes and the wireless environment, it would be beneficial if the data rate can be adapted based on the channel quality, as in many existing wireless systems [21]. When the Medium Access Control (MAC) policy ideally schedules the transmission of nodes so that the destructive interference between nodes is ignorable, the data rate in conventional wireless networks can generally be maximized by setting the average transmission power of nodes to its maximum value. The reason is that in the case of zero interference, maximizing the transmission power implies maximizing the Signal-to-Noise Ratio (SNR) at the receiver and subsequently maximizing the achievable data rate [21]. But this is not the case when using ANC for the following reasons:

1) When opportunistic listening is present, the two simultaneously transmitting source nodes interfere with each other at the opportunistic listeners. As long as ANC is used, this interference cannot be cancelled with any MAC scheme. Hence, it is intuitively beneficial to reduce the transmission power of strong interferers.
2) Suppose in Fig. 1, the distance between s_1 and r is significantly smaller than the distance between s_2 and r, then x_1 will dominate the amplified and forwarded signal broadcasted by the relay, if both source nodes transmit at the maximum transmission power. The reason is that the transmission power of the relay is also constrained to a maximum value. If x_1 is very large it will "suppress" x_2. Consequently, x_2 will have a small SNR at the destination node, which will lead to a data rate reduction of both sessions since only one packet from each session can be coded together (which is further explained in Section III-A).

The above discussions imply that the data rate is affected by the *interaction* between simultaneously coded sessions, and power adaptation is necessary for ANC. The objective of power adaptation is to maximize the data rate of both sessions.

The example network in Fig. 1 contains only two sessions, which is also the common case in the related work on ANC [16]–[20]. An interesting question is whether we can encode more than two sessions with ANC, as in C-NC [22], in order to improve the network performance. We will discuss this problem in Section III-B.

The main contributions of this paper are as follows.

1) We propose a rate adaptive ANC scheme, and discuss the maximum possible number of coding nodes. We show that the transmission power has an impact on the data rate and maximum power transmission is not always the optimal choice.
2) We provide an analysis of transmission power optimization, and propose a method to find the optimal transmission power in order to maximize the transmission rate.
3) We evaluate the performance of the proposed scheme through extensive simulations. We show that the proposed joint rate and power adaptation scheme outperforms ANC without power adaptation, and the proposed scheme is beneficial over other relaying schemes.

The remainder of this paper is organized as follows. Section II discusses the related work. Section III describes the model of rate adaptive ANC and discusses the impact of transmission power on the data rate. We also investigate the maximum possible number of coding nodes in Section III. In Section IV, we propose a transmission power optimization method. Section V discusses some practical considerations. The performance of the proposed scheme is evaluated in Section VI. Section VII draws conclusions and discusses some future work.

II. RELATED WORK

Over the past decade, much work has been focusing on improving the performance of communication networks through investigating appropriate routing strategies [1]–[6]. Network coding was initially proposed in [2]. It was shown that with network coding, the occupied bandwidth can be reduced in a wired network. A bi-directional relaying approach where the relay encodes the received data packets before forwarding was proposed in [3], which is similar to network coding. Considering the broadcast characteristic of wireless channels, a practical implementation of network coding in wireless networks (COPE) was proposed in [4]. It turns out that network coding is beneficial for unicast sessions in wireless networks.

The idea of physical layer network coding was first introduced in [7], to the best of our knowledge. And it was further elaborated in [8] and [11], DNF and AF (or ANC) schemes have been investigated. The throughput capacity of physical layer network coding in wireless networks has been studied in [10]. It was shown that physical layer network coding brings throughput improvement compared with conventional relaying or network coding schemes.

A drawback of the DNF scheme is the strict synchronization requirement. Recently, an optimized constellation design method without the requirement of phase synchronization was proposed in [12]. However, the constellation design procedure is complex, especially with high-level modulation. Hence, it is difficult to work with adaptive rate modulation schemes. In [13], the authors proposed a method to encode more than two sessions. It is based on the assumption that the received superposed signals have phase synchronization at both the relay and the opportunistic listener, which is difficult to realize in practice.

ANC has received much attention for its easy-to-implement characteristic. A wireless network prototype for ANC was proposed in [16]. Considering the issue of applying ANC in distributed wireless networks, a cooperative protocol supporting ANC was recently proposed in [17]. In broadband applications [18], [19], classical rate and power adaptation schemes to avoid fading [21] were adopted. The classical adaptation schemes are different from our work since our goal is to obtain an optimal data rate for all the sessions joining a specific coding operation. Our work is closely related to the term network coding noise [20], i.e. the additional noise that is introduced when performing ANC. The power adaptation intends to reduce the network coding noise, which subsequently makes it possible to transmit at a higher data rate.

Recently, several rate adaptation schemes for C-NC have been proposed, which consider the interaction between different sessions and aim at increasing the data rate for multiple sessions. Ref. [23] discussed whether rate adaptation is beneficial for network coding, and indicates that reducing the data rate can bring more overhearing opportunity, and thereby increasing the coding opportunity. A rate adaptation scheme was proposed for C-NC. Another interesting observation in [23] is that the optimal number of coded sessions is no more than two when the channel access opportunities of nodes are optimized. In [24], a MAC scheme for C-NC which takes advantage of rate adaptation was proposed. Ref. [25] considers rate adaptation and optimal relay selection of C-NC in fading channels. And a rate adaptation scheme for C-NC where the opportunistic listeners receive data at a lower rate was proposed in [26]. However, to the best of our knowledge, rate adaptation for physical layer network coding that takes the interaction between different sessions into account has not been investigated in the literature.

III. RATE ADAPTIVE ANALOG NETWORK CODING

A. ANC for Two Sessions

We first summarize the procedure of ANC for two sessions. In Fig. 1, the sessions $s_1 \rightarrow d_1$ and $s_2 \rightarrow d_2$ share the same relay node r. Two timeslots are required for transmitting two packets (one packet in each session). In the first timeslot, the source nodes s_1 and s_2 broadcast their packets simultaneously to the relay r and their respective opportunistic listeners d_2 and d_1. The opportunistic listener d_1 (or respectively, d_2) decodes the packet originated from s_2 (or s_1), and regards the signal from s_1 (or s_2), which is generally lower than the signal from s_2 (or s_1), as noise. In the second timeslot, the relay r amplifies the superposed signal it has received in the first timeslot, and broadcasts the signal to the destinations d_1 and d_2. Afterwards, the destination node d_1 (or respectively, d_2), re-modulates the packet from s_2 (or s_1), subtracts the re-modulated signal from the received superposed signal, and finally obtains the packet intended to itself. If the source node of one session is the destination node of the other session, we have $s_1 = d_2$ or/and $s_2 = d_1$.

In this paper, we assume that all the packets are of the same length, and all sessions send at the same packet rate due to fairness issues. Within one transmission round, each session transmits one packet to its intended destination. The source nodes are assumed to be backlogged and always have packets to send.

B. Can We Encode More Than Two Sessions?

Now we consider whether it is possible to encode more than two sessions with ANC. In network coding, each destination node has to obtain the packets from the other simultaneously coded sessions, either through opportunistic listening or it is the source node of the other session itself, so that it can decode the packet intended to itself. When the number of coded sessions is $n > 2$, let $s_1, s_2, \ldots, s_n$ denote the source nodes and $d_1, d_2, \ldots, d_n$ denote the respective destination nodes. A specific destination node d_i has to overhear the packets originated from $s_1, \ldots, s_{i-1}$, $s_{i+1}, \ldots, s_n$, in order to decode the packet sent by s_i from the coded packet forwarded by the relay. The overheard packets can be uncoded or coded with the same coding operation as the coding operation performed at the relay node.

In C-NC, n timeslots are required for transmitting a packet in each session to the relay and the opportunistic listeners of the respective source nodes. And there exists the possibility of encoding more than two sessions, as shown in [22]. In physical layer network coding, there exists at least one timeslot in which no less than two source nodes transmit simultaneously. Otherwise, it is the same with C-NC and physical layer network coding is not performed. It follows that the number of required timeslots is smaller than or equal to $n - 1$ when performing physical layer network coding. When $n > 2$, at least one destination node, say d_i, has to overhear and decode a superposed signal. This superposed signal is different from the superposed signal received by the relay and contains (possibly not all, but at least two) packets whose intended destinations are different from d_i. In [13], phase level synchronization is assumed and the coding operation is the same for the overheard packets and the relayed packet when using the DNF strategy. Without the assumption of phase and symbol level synchronization, the coding operation of the overheard packets and the relayed packet are different. In this case, the opportunistic listener has to decode all the independent packets that are mixed in the superposed signal. This can only be achieved using interference cancellation strategies up to present knowledge [27], [28]. For cases where a destination node is also a source node in another session, it is still impossible for that source node to decode the simultaneously transmitting packets as long as the radio transceiver operates in half-duplex way. The above discussions can be summarized by the following fact.

Fact 1: Without symbol level synchronization and interference cancellation strategies, the maximum possible encoding number of ANC for half-duplex wireless networks is *two*.

We neglect the use of symbol level synchronization and interference cancellation strategies due to their high complexity to implement in wireless network terminals. And wireless network terminals are generally half-duplex. For these reasons and from Fact 1, we restrict the number of coded sessions to two in this paper. And it was also found in [23] that encoding two sessions is the optimal case.

C. Channel Status and Data Rate

In this subsection, we consider the relationship between the channel status and the data rate, and show that the transmission power has an impact on the data rate. In our analysis, we consider a slow-varying flat-fading channel, so that the channel gains are the same in the first and second timeslot. We use the Shannon's formula for channel capacity over Additive White

Gaussian Noise (AWGN) channels [29] to evaluate the data rate. The reason for using the Shannon's formula lies in that 1) modern channel coding schemes approach the Shannon limit [30]; and 2) under a specific bit-error-rate constraint, the achievable data rate for Multiple Quadrature Amplitude Modulation (MQAM) with coherent receiver and maximum likelihood decoder has a similar expression, except for a power loss factor [21]. By maximizing the data rate evaluated from the Shannon's formula, we also maximize the data rate for MQAM and other relevant modulation and channel coding schemes.

In the following discussions, z_n denotes the AWGN with variance σ_n^2, h_{ij} $(i, j = 1,2)$ is the channel gain from the source node s_i to the destination node d_j, h_{ir} is the channel gain from the source node s_i to the relay r, and h_{rj} is the channel gain from the relay r to the destination node d_j. P_i denotes the transmission power of s_i, and P_r denotes the transmission power of relay r.

In the *first timeslot*, the superposed signal received at the relay

$$y_{r(1)} = h_{1r}x_1 + h_{2r}x_2 + z_{n(1)}, \tag{1}$$

where the number in the brackets in the subscript indicates the timeslot. And the signals received by the opportunistic listeners d_1 and d_2 are:

$$y_{1(1)} = h_{21}x_2 + (h_{11}x_1 + z_{n(1)}), \tag{2}$$

$$y_{2(1)} = h_{12}x_1 + (h_{22}x_2 + z_{n(1)}). \tag{3}$$

The opportunistic listener d_j intends to receive the packet sent by the source node s_i $(i \neq j)$. And the signal transmitted by s_j is generally very weak at d_j, i.e. $|h_{jj}x_j|^2$ is small. Otherwise, the session $s_j \rightarrow d_j$ can communicate without relaying. Therefore, (2) and (3) form the classical Gaussian interference channel with weak interference [31], where $h_{jj}x_j$ can be considered as a part of the noise since it is weak and receivers have no interference cancellation schemes. Then, the SNR (strictly speaking, it is the Signal-to-Interference-plus-Noise Ratio, SINR, but we use the term SNR here since we regard the interference as a part of the noise) at d_1 and d_2 can be evaluated by:

$$\gamma_{1(1)} = \frac{|h_{21}|^2 P_2}{|h_{11}|^2 P_1 + \sigma_n^2}, \tag{4}$$

$$\gamma_{2(1)} = \frac{|h_{12}|^2 P_1}{|h_{22}|^2 P_2 + \sigma_n^2}. \tag{5}$$

Eq. (2)–(5) indicate that the simultaneously transmitted packets interfere with each other, which implies that the transmission powers of both source nodes need to be carefully selected.

From the Shannon's formula, the overhearing data rates of d_1 and d_2 (respectively originating from s_2 and s_1) on unit bandwidth are respectively constrained by:

$$R_{1(1)} \le C(\gamma_{1(1)}) \text{ (bps/Hz)}, \tag{6}$$

$$R_{2(1)} \le C(\gamma_{2(1)}), \tag{7}$$

where $C(\gamma) = \log_2(1+\gamma)$. When $s_i = d_j$ $(i \neq j)$, we regard $|h_{ij}|^2 \rightarrow \infty$, and have $\gamma_{j(1)} \rightarrow \infty$ and $R_{j(1)} \le \infty$. No rate calculation for the relay is needed since the relay does not decode the packets.

In the *second timeslot*, the relay r amplifies and forwards the superposed signal received by the relay in the first timeslot. The amplification factor is defined as:

$$\alpha = \left(\frac{P_r}{|h_{1r}|^2 P_1 + |h_{2r}|^2 P_2 + \sigma_n^2} \right)^{1/2}, \tag{8}$$

which ensures that the transmission power of the relay is P_r. The signals received at the destination nodes d_1 and d_2 are:

$$y_{1(2)} = h_{r1}\alpha y_{r(1)} + z_{n(2)} = h_{r1}\alpha(h_{1r}x_1 + h_{2r}x_2 + z_{n(1)}) + z_{n(2)}, \tag{9}$$

$$y_{2(2)} = h_{r2}\alpha y_{r(1)} + z_{n(2)} = h_{r2}\alpha(h_{1r}x_1 + h_{2r}x_2 + z_{n(1)}) + z_{n(2)}, \tag{10}$$

where $z_{n(1)}$ and $z_{n(2)}$ are independent.

From (8), (9) and (10), we can observe that the total transmission power of the relay is divided to both of the superposed signals, so that each individual signal is transmitted at a power lower than the relay's transmission power. This is different from DNF or C-NC, where the packets arrived at the relay are decoded prior to relaying. After decoding, the relay can transmit a combined packet at its full transmission power. Hence, in ANC, the relay introduces signal attenuation compared with DNF and C-NC schemes. When $|h_{1r}|^2$ is different from $|h_{2r}|^2$, the signal sent through the weak channel will be suppressed after relaying if the two source nodes transmit at an identical power. This is because after relaying, the power ratio of the two signals remains the same as when they have been received by the relay. Therefore, reducing the power of the strong signal can be beneficial for the weak signal, in the sense that the weak signal will be amplified with a higher gain. However, we also have to consider the strength of the signal received by the opportunistic listeners. This implies that a more sophisticated power optimization scheme is necessary.

We assume that the destination nodes have complete knowledge of the compound channel gains $h_{ri}\alpha h_{ir}$ and $h_{ri}\alpha h_{jr}$ $(i \neq j)$. The destination d_i first re-modulates the packet x_j, which has been received in the first timeslot, to $h_{ri}\alpha h_{jr}x_j$. Then, $h_{ri}\alpha h_{jr}x_j$ is subtracted from $y_{i(2)}$, yielding

$$y'_{1(2)} = h_{r1}\alpha h_{1r}x_1 + (h_{r1}\alpha z_{n(1)} + z_{n(2)}), \tag{11}$$

$$y'_{2(2)} = h_{r2}\alpha h_{2r}x_2 + (h_{r2}\alpha z_{n(1)} + z_{n(2)}), \tag{12}$$

from which we can decode the intended packets x_1 and x_2. The SNR of these two signals are:

$$\gamma_{1(2)} = \frac{|h_{r1}\alpha h_{1r}|^2 P_1}{(|h_{r1}\alpha|^2 + 1)\sigma_n^2} = \frac{|h_{1r}h_{r1}|^2 P_r}{\left[\frac{|h_{2r}|^2 P_2 + |h_{r1}|^2 P_r + \sigma_n^2}{P_1} + |h_{1r}|^2 \right] \sigma_n^2}, \tag{13}$$

and similarly,

$$\gamma_{2(2)} = \frac{|h_{2r}h_{r2}|^2 P_r}{\left[\frac{|h_{1r}|^2 P_1 + |h_{r2}|^2 P_r + \sigma_n^2}{P_2} + |h_{2r}|^2 \right] \sigma_n^2}. \tag{14}$$

From the Shannon's formula, the data rates are constrained by:

$$R_{1(2)} \le C(\gamma_{1(2)}), \tag{15}$$

$$R_{2(2)} \le C(\gamma_{2(2)}). \tag{16}$$

Since the data rates are selected by the source nodes and the relay only amplifies the signal, we have $R_{2(1)} = R_{1(2)}$ and $R_{1(1)} = R_{2(2)}$. As discussed in Section III-A, we consider packets of the same length and only one packet from each session can be encoded within one round of packet exchange. Hence, we set $R_{1(2)} = R_{2(2)}$, because one source node transmitting at a higher data rate does not provide throughput improvement and is unnecessary. Therefore, the actual data rate has to satisfy:

$$\begin{aligned} R &= R_{1(1)} = R_{1(2)} = R_{2(1)} = R_{2(2)} \\ &\le \min\{C(\gamma_{1(1)}), C(\gamma_{1(2)}), C(\gamma_{2(1)}), C(\gamma_{2(2)})\} \\ &= \log_2(\min\{\gamma_{1(1)}, \gamma_{1(2)}, \gamma_{2(1)}, \gamma_{2(2)}\}). \end{aligned} \tag{17}$$

And the optimal data rate is

$$R^* = \log_2(\min\{\gamma_{1(1)}, \gamma_{1(2)}, \gamma_{2(1)}, \gamma_{2(2)}\}). \tag{18}$$

IV. TRANSMISSION POWER OPTIMIZATION

A. Problem Formation

The above discussions indicate that the transmission powers of the relay and source nodes P_r, P_1 and P_2 affect the SNR values and thereby affect the data rate. Hence, they have to be adjusted in order to maximize the data rate R. This optimization problem can be formulated as follows:

$$\begin{aligned} &\max_{P_r, P_1, P_2} \; R \\ &\;s.t. \quad 0 \le P_r, P_1, P_2 \le P_{\max}, \end{aligned} \tag{19}$$

where $R = \log_2(\min\{\gamma_{1(1)}, \gamma_{1(2)}, \gamma_{2(1)}, \gamma_{2(2)}\})$ and $P_{\max}$ is the maximum power constraint. Since the base-2 logarithmic function is an increasing function, the optimization problem (19) can be simplified as

$$\begin{aligned} &\max_{P_r, P_1, P_2} \; \min\{\gamma_{1(1)}, \gamma_{1(2)}, \gamma_{2(1)}, \gamma_{2(2)}\} \\ &\;s.t. \quad 0 \le P_r, P_1, P_2 \le P_{\max}, \end{aligned} \tag{20}$$

i.e. we would like to maximize the minimal SNR value. In the following discussions, let $\gamma_{\min} = \min\{\gamma_{1(1)}, \gamma_{1(2)}, \gamma_{2(1)}, \gamma_{2(2)}\}$.

It can be observed from (4), (5), (13) and (14) that γ_l, where l stands for the subscripts "1(1)", "1(2)", "2(1)" and "2(2)", are monotonic functions of the transmission powers. The first order derivatives of γ_l with respect to P_r, P_1 and P_2 are listed in Table I.

TABLE I
FIRST ORDER DERIVATIVES OF SNR VALUES

l	$\partial\gamma_l/\partial P_r$	$\partial\gamma_l/\partial P_1$	$\partial\gamma_l/\partial P_2$
1(1)	0	<0	>0
1(2)	>0	>0	<0
2(1)	0	>0	<0
2(2)	>0	<0	>0

Since γ_l is a non-decreasing function of P_r, the optimal solution of the transmission power of the relay is $P_r^* = P_{\max}$. The derivatives of γ_l with respect to P_1 and P_2 are smaller than zero for some values of l. Hence, it is generally not optimal when both source nodes transmit at the maximum power. And we have to find methods to optimize the transmission powers P_1 and P_2.

B. Derivation of the Optimization Method

In this subsection, we search for some specific values of P_1 and P_2 that are possible optimal values.

Proposition 1: At least one of the optimal values P_1^* and P_2^* of P_1 and P_2 has to be equal to $P_{\max}$.

Proof: Suppose $P_1^* \ne P_{\max}$ and $P_2^* \ne P_{\max}$, since $0 \le P_1^*, P_2^* < P_{\max}$, there exists $k > 1$, such that $0 \le kP_1^*, kP_2^* \le P_{\max}$. We have

$$\begin{aligned} \gamma_{1(1)}(kP_1^*, kP_2^*) &= \frac{|h_{21}|^2 kP_2^*}{|h_{11}|^2 kP_1^* + \sigma_n^2} = \frac{|h_{21}|^2 P_2^*}{|h_{11}|^2 P_1^* + \dfrac{\sigma_n^2}{k}} \\ &> \frac{|h_{21}|^2 P_2^*}{|h_{11}|^2 P_1^* + \sigma_n^2} = \gamma_{1(1)}(P_1^*, P_2^*), \end{aligned}$$

and similarly, $\gamma_{2(1)}(kP_1^*, kP_2^*) > \gamma_{2(1)}(P_1^*, P_2^*)$;

$$\begin{aligned} \gamma_{1(2)}(kP_1^*, kP_2^*) &= \frac{|h_{1r}h_{r1}|^2 P_r}{\left[\dfrac{|h_{2r}|^2 kP_2^* + |h_{r1}|^2 P_r + \sigma_n^2}{kP_1^*} + |h_{1r}|^2\right]\sigma_n^2} \\ &= \frac{|h_{1r}h_{r1}|^2 P_r}{\left[\left(|h_{2r}|^2 P_2^* + \dfrac{|h_{r1}|^2 P_r + \sigma_n^2}{k}\right)\Big/ P_1^* + |h_{1r}|^2\right]\sigma_n^2} \\ &> \frac{|h_{1r}h_{r1}|^2 P_r}{\left[\dfrac{|h_{2r}|^2 P_2^* + |h_{r1}|^2 P_r + \sigma_n^2}{P_1^*} + |h_{1r}|^2\right]\sigma_n^2} \\ &= \gamma_{1(2)}(P_1^*, P_2^*), \end{aligned}$$

and similarly, $\gamma_{2(2)}(kP_1^*, kP_2^*) > \gamma_{2(2)}(P_1^*, P_2^*)$.

This implies that no matter which γ_l is equal to $\gamma_{\min}$, kP_1^*, kP_2^* is always a better solution for (20) than P_1^*, P_2^*. This proves a contradiction. ■

Note that Proposition 1 does *not* say that P_1^* and P_2^* are *both* equal to $P_{\max}$.

Proposition 2: If P_1^* and P_2^* are optimal values of P_1 and P_2, at least one of the following conditions has to be satisfied: 1) $\gamma_{1(1)} = \gamma_{2(1)}$, 2) $\gamma_{1(1)} = \gamma_{1(2)}$, 3) $\gamma_{2(1)} = \gamma_{2(2)}$, 4) $\gamma_{1(2)} = \gamma_{2(2)}$.

Proof: Suppose P_1^* and P_2^* are optimal values and none of the conditions 1) – 4) are satisfied. When $\gamma_{1(1)} = \gamma_{\min}$, the solution can be optimized by decreasing P_1^* or increasing P_2^* while remaining $\gamma_{1(1)} = \gamma_{\min}$ (unless $P_1^* = 0$ and $P_2^* = P_{\max}$, but $P_1^* = 0$ is obviously not an optimal solution). The case is similar when $\gamma_l = \gamma_{\min}$ and $l \neq$ "1(1)". This is contradictory with the statement that P_1^* and P_2^* are optimal values. Hence, at least one of the conditions 1) – 4) has to be satisfied. ■

It can be observed from Table I that when at least one of the conditions 1) – 4) is satisfied, $\gamma_{\min}$ reaches its local maxima.

C. The Optimization Process

According to Proposition 1 and 2, we can find the optimal solution by enumerating the possible optimal values of P_1^* and P_2^* and selecting those values that result in the highest $\gamma_{\min}$. The transmission power optimization process can be formulated as Algorithm 1. The solutions for P_1 (or P_2) when knowing P_2 (or P_1) in the conditions 1) – 4) in Proposition 2 can be found in the Appendix. The conditions 1) – 4) are all quadratic equations in terms of P_1 (or P_2), which generally have two solutions. And we test 16 possible values of P_1^* and P_2^* to find the optimal solution. We neglect those solutions that are outside the interval $[0, P_{\max}]$.

Algorithm 1 Transmission Power Optimization

```
$P_r^* = P_1^* = P_2^* = P_{\max}$
Obtain $|h_{ij}|^2$ $(i, j = 1,2)$, $|h_{ir}|^2$ and $|h_{rj}|^2$
for $q = 1$ to 4
    $P_1 = P_{\max}$,
    Solve $P_2$ from condition $q$) in Proposition 2, the
    solutions are $P_{21}$ and $P_{22}$
    $P_{1,2q-1} = P_{1,2q} = P_1$, $P_{2,2q-1} = P_{21}$, $P_{2,2q} = P_{22}$
end for
for $q = 1$ to 4
    $P_2 = P_{\max}$,
    Solve $P_1$ from condition $q$) in Proposition 2, the
    solutions are $P_{11}$ and $P_{12}$
    $P_{2,2q+7} = P_{2,2q+8} = P_2$, $P_{1,2q+7} = P_{11}$, $P_{1,2q+8} = P_{12}$
end for
for $q = 1$ to 16
    if $P_{1,q}, P_{2,q} \in [0, P_{\max}]$ and $\gamma_{\min}(P_{1,q}, P_{2,q}) >$
    $\gamma_{\min}(P_1^*, P_2^*)$
        $P_1^* = P_{1,q}$, $P_2^* = P_{2,q}$
    end if
end for
```

V. PRACTICAL CONSIDERATIONS

In this section, we discuss some practical issues closely related to the proposed adaptive rate and power ANC scheme. Some other practical considerations, such as how to solve the problem of frequency offset, phase jitter, sampling offset etc., were discussed in [16] and [32]. Note that the phase tracking procedure of ZigZag decoding [32] can also be used for ANC.

A. Which Node Performs the Calculations?

Since the two source nodes have to transmit packets with the same data rate and approximately at the same time, a straightforward idea is that the relay performs the transmission power optimization and sends the optimized powers and transmission rate to the source nodes. In addition, the relay has to act as a coordination node that provides coarse packet level synchronization between the two source nodes. The powers and transmission rate information can be sent together with the control packets used for synchronization.

B. Obtaining the Channel Gains

In order to allow the relay to perform rate and power adaptation, the relay has to obtain the channel gains occurred in (4), (5), (13) and (14) and the noise power. The general process is to store a series of historical channel gains, and predict the channel gains at the next transmission time.

The historical channel gains can be estimated from the received data packets. Nodes can also send periodic beacons when the channel has been idle for a long time, in order to allow their neighboring nodes to estimate the channel status. When a superposed signal is being received, the channel gains can be obtained by correlating the known preamble with the preamble of packets as in [32]. If the head-and-tail packet structure as in [16] is used, we can also estimate the gains from the respective non-overlapped data blocks at the head and the tail [33]. With the same method, the compound channel gains $h_{ri}\alpha h_{ir}$ and $h_{ri}\alpha h_{jr}$ $(i \neq j)$ can be estimated, which are assumed to be known in Section III-C. The design of a specific packet format depends on the MAC layer protocol, which is left as our future work and is not considered in this paper.

Regarding the methods to predict the channel gains at the next transmission time, we first notice that only the amplitude values of the channel gains are used. And (4), (5), (13) and (14) can also be easily expressed in terms of SNR on the specific channels. Hence, we can also perform the rate and power adaptation with the SNR values. From a sequence of known SNR values, the SNR at a future time can be predicted, for instance [34]. The prediction is relatively accurate when the prediction time is short, and the prediction error increases with the prediction time. Hence, we can start with a low data rate session when the channel has been idle for a long time, and increase the data rate after we have adequate samples to make a more precise prediction. When the predictions are not made by the relay, nodes have to send the prediction results to the relay. For a prediction with low accuracy, a pessimistic approach is to choose the channel gain amplitude (or SNR on the respective channels) that results in the lowest possible γ_l for rate computation. This results in a low data rate but also ensures a low outage probability. Better results can be obtained by performing calculations based on the outage probability requirement and the prediction method, which also adds the complexity.

Since each receiver has its lowest sensible SNR value, which we denote as SNR_{sens} in our further discussions, in some scenarios we cannot obtain the gain of every channel. For those channels that we cannot obtain the channel gain because the received signal is too weak, we set the channel gain amplitude to those values that result in the lowest γ_l while keeping the respective channels to be non-sensible.

C. Frequency-selective Fading Channels

It is assumed in Section III and IV that the channels are all flat-fading. However, in practice, wireless channels exhibit frequency-selective fading to some extent, especially when the bandwidth is high. Equalizers are typically used in practical systems to eliminate the Inter-Symbol Interference (ISI) caused by frequency-selective fading channels. The equalizers are trained to match the signal that has to be received. When using ANC, however, the superposed signal amplified by the relay has to be received by the destinations in the second timeslot. Conventional equalizers are generally unable to match both (independent) signals since they traversed different channels and are not synchronous at the symbol level.

Fortunately, in ANC, the overheard signal is subtracted from the superposed signal prior to decoding. Hence, similar to the case of flat-fading channels, we first re-modulate the overheard signal, and then subtract the re-modulated signal from the superposed signal. The remaining signal can then be passed through an equalizer and decoded by the decoder. The difference is that in the case of frequency-selective fading channels, the channel gain h has more than one tap, which can also be estimated using techniques described in [33]. Meanwhile, multiplication of a channel gain and a signal or of two channel gains should be replaced with convolution, and $|h|^2$ should be replaced with $\sum_{m=-M}^{M} |h[m]|^2$, where the number of taps is $2M+1$.

D. The Procedure

According to the above discussions, the procedure of exchanging packets for a pair of sessions with the proposed rate and power adaptation scheme can be summarized as follows:

1) Each node collects a specific number of the previous squared channel gain amplitudes (or the SNR values) of the channel directed to itself, i.e. the relay r collects $|h_{ir}|^2$, the destination d_j collects $|h_{ij}|^2$ and $|h_{rj}|^2$.
2) The values of $|h_{ir}|^2$, $|h_{ij}|^2$ and $|h_{rj}|^2$ at the next transmission time are predicted by the respective nodes that have collected the previous values. The destination d_j sends the predicted $|h_{ij}|^2$ and $|h_{rj}|^2$ to the relay.
3) The relay performs the power optimization algorithm and selects the data rate according to (18). Afterwards, it sends the resulting transmission powers and the data rate to the source nodes.
4) Data transmission, as discussed in Section III.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed rate and power adaptation scheme. The network setting is the same as Fig. 1, which contains two sessions and one relay node. We compare the throughput of the proposed method with several other rate adaptive transmission schemes, which include rate adaptive ANC without power optimization, Conventional Network Coding (C-NC) with rate adaptation, rate adaptive 4-phase relaying without network coding and rate adaptive transmission without relaying. When no power adaptation is used, we set $P_1 = P_2 = P_{\max}$. The throughput is defined as the number of bits transmitted in unit bandwidth and unit time. We first evaluate the throughput analytically, given the knowledge of the squared amplitude of the channel gains. Afterwards, the throughput with more general node placements and in fading channels is investigated through simulations.

A. Throughput Analysis

1) The Ideal Case

When the control overhead is ignored, the throughput of rate adaptive ANC is $S_{\text{ANC}}=R$, because two packets are exchanged in two timeslots with each packet transmitting at rate R.

C-NC consists of three timeslots when encoding two sessions. The maximum data rates in these three timeslots are respectively:

$$R_{\text{C-NC}(1)} = \log_2(P_{\max} \cdot \min\{|h_{1r}|^2, |h_{12}|^2\} / \sigma_n^2), \tag{21}$$

$$R_{\text{C-NC}(2)} = \log_2(P_{\max} \cdot \min\{|h_{2r}|^2, |h_{21}|^2\} / \sigma_n^2), \tag{22}$$

$$R_{\text{C-NC}(3)} = \log_2(P_{\max} \cdot \min\{|h_{r1}|^2, |h_{r2}|^2\} / \sigma_n^2). \tag{23}$$

Assume the packet consists of L bits on unit bandwidth, the transmission time for C-NC is

$$T_{\text{C-NC}} = \frac{L}{R_{\text{C-NC}(1)}} + \frac{L}{R_{\text{C-NC}(2)}} + \frac{L}{R_{\text{C-NC}(3)}}. \tag{24}$$

Since two packets are sent during this time, the throughput of C-NC is $S_{\text{C-NC}} = 2L / T_{\text{C-NC}}$.

Similarly, for 4-phase relaying, the transmission time is

$$\begin{aligned} T_{\text{4-phase}} = & \frac{L}{\log_2(P_{\max}|h_{1r}|^2/\sigma_n^2)} + \frac{L}{\log_2(P_{\max}|h_{r1}|^2/\sigma_n^2)} \\ & + \frac{L}{\log_2(P_{\max}|h_{2r}|^2/\sigma_n^2)} + \frac{L}{\log_2(P_{\max}|h_{r2}|^2/\sigma_n^2)}, \end{aligned} \tag{25}$$

and the throughput is $S_{\text{4-phase}} = 2L / T_{\text{4-phase}}$.

In the case of transmitting without relaying, the transmission time is

$$T_{\text{No relay}} = \frac{L}{\log_2(P_{\max}|h_{11}|^2/\sigma_n^2)} + \frac{L}{\log_2(P_{\max}|h_{22}|^2/\sigma_n^2)}, \tag{26}$$

and the throughput is $S_{\text{No relay}} = 2L / T_{\text{No relay}}$.

2) Considering the Control Overhead

In rate adaptive transmission schemes, each node transmits data at a dynamic data rate, which can be evaluated from the respective channel gains as discussed in Section III-C and Section VI-A-1). Control packets need to be sent in order to or notify the channel gains or the optimized data rate to the relay and the source nodes.

In the proposed ANC scheme, the relay computes the data rate, as discussed in Section V-D. When rate adaptive ANC is performed without power adaptation, the steps are similar. The only difference is that the relay does not perform power optimization. But the number of required control packets is the same, since the channel gains estimated at the destinations are also used for rate selection and the resulting data rate has to be sent to the source nodes. It follows from the discussions in Section V-D that three control packets need to be sent for rate adaptive ANC (both with and without power adaptation) to update the data rate.

For C-NC, the necessary number of control packets for data rate update is also three. The reason is that the destination d_j has to send $|h_{ij}|^2$ and $|h_{rj}|^2$ to the relay, in order to allow the relay to select the data rate according to (21), (22) and (23). Afterwards, the relay has to broadcast the resulting data rate to both source nodes. The same number of control packets is required for 4-phase relaying, where the two destinations have to respectively feedback $|h_{r1}|^2$ and $|h_{r2}|^2$ to the relay, and the relay has to feedback $|h_{1r}|^2$ and $|h_{2r}|^2$ to the source nodes. In this case, the data rates of the source nodes can be selected by the source nodes themselves. When no relaying is used, only two control packets are used to feedback the channel status from the destinations to the respective source nodes, so that the source nodes can choose the appropriate date rates.

We assume that all the control packets are of the same length L_c. As can be inferred from the above discussions, only a few bytes need to be sent in every control packet. Hence, the length of the effective control information is small compared to the length of the physical layer and MAC headers [35], and the above assumption is appropriate. Let R_c denote the date rate of control packets, which is generally the lowest supported data rate [35], then the time required for sending one control packet is $T_c = L_c / R_c$. Assume the data rate is updated for N_c times during time T_{total}, the throughput considering the impact of control overhead can be evaluated as

$$S_{\text{actual}} = \max\left\{0, S_{\text{ideal}} \cdot \left(1 - \frac{M_c N_c T_c}{T_{\text{total}}}\right)\right\}, \tag{27}$$

where S_{ideal} represents the ideal throughput evaluated in Section VI-A-1), and M_c denotes number of control packets required to update the data rate. As discussed above, M_c is equal to three for rate adaptive ANC, C-NC and 4-phase relaying; and it is equal to two when no relaying is performed. The ratio N_c / T_{total} describes how often the data rate is updated, which is dependent on channel variation and is the same for all the transmission schemes. A practical estimation of the time the channel gains are highly correlated can be performed using the conservative definition of coherence time [36]. Let T_{coh} denote the coherence time of channels, then the data rates need to be updated at an interval of T_{coh}, in order to adapt to the most recent channel condition. It follows that N_c / T_{total} = 1 / T_{coh}, yielding

$$S_{\text{actual}} = \max\left\{0, S_{\text{ideal}} \cdot \left(1 - \frac{M_c T_c}{T_{\text{coh}}}\right)\right\}. \tag{28}$$

B. Simulation Setup

In our simulations, we consider a Rician flat-fading channel with a specific Rician factor K, where K = 5 dB except in Section VI-C-3). The case of frequency-selective fading channels is similar with flat-fading channels as discussed in Section V-C, but the performance is affected by the equalizer design which is an independent topic. Therefore, we focus on flat-fading channels in our discussion.

The expected squared amplitude of the gain of the channel from node a to node b is evaluated by $E|h_{ab}|^2 = 1/D_{ab}^4$, where E is the expectation operator and D_{ab} is the distance in meters from node a to node b. We assume that all the channel gains at the next transmission time are known except in Section VI-C-5). In our simulations, the noise power density is −174 dBm/Hz, the receiver bandwidth is 1 MHz and the noise figure is 6 dB. We set the lowest sensible SNR value SNR_{sens} = 3 dB for the proposed scheme. For those schemes used for comparison, we assume that any value of SNR is sensible. The maximum transmission power $P_{\max}$ is set to 10 dBm except in Section VI-C-4). The control packet length is set to 400 bits. The normalized transmission rate of control packets is fixed as 1 bps/Hz, which corresponds to the spectrum efficiency of Binary Phase Shift Keying (BPSK) shaped with a raised cosine pulse with roll-off factor β = 1 [21]. This transmission rate can be regarded as the basic data rate of all transmissions. We set the channel coherence time to 1s in order to consider a slow-varying channel, except for Section VI-C-6). Each simulation is run with 10000 different random seeds to obtain the overall performance.

We assume that the transmission range of nodes is D_{TX}, which means that relaying is assumed to be required when the distance between the source and the destination is greater than D_{TX}. In our simulations, we set D_{TX} = 250 m. The scenario in the simulations is set up so that relaying is necessary, otherwise the upper layer protocols will not choose relaying in practical systems and network coding is unnecessary. We place the two source nodes s_1, s_2 and the relay node r in the same line, with the relay in the middle, as shown in Fig. 2. The destination nodes d_2 and d_1 are respectively placed within a specific range of s_1 and s_2, in order to allow opportunistic listening. Unless specifically stated, the distance between the source node and the relay is randomly chosen within the interval [125, 250] m, and the location of d_2 (or d_1) is randomly chosen within the area bounded by a circle with radius D_{OP}=100 m centered at s_1 (or s_2). Meanwhile, the location of nodes must ensure that the distance between s_1 and d_1 and the distance between s_2 and d_2 are both greater than D_{TX}. But the distance between s_1 (or s_2, d_1, d_2) and the relay must be smaller than or equal to D_{TX}, as shown in Fig. 2. This setting ensures that the relay is a suitable relay for both sessions and the destination nodes have overhearing opportunity.

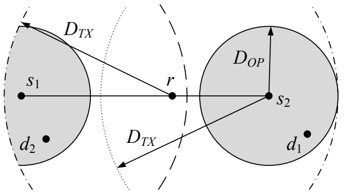

Fig. 2. Example of node placement. The variable D_{TX} denotes the transmission range of nodes s_1, s_2 and r. The dashed, dotted and dash-dotted lines respectively indicate the border of the transmission area covered by s_1, s_2 and r. And the shaded area indicates the possible location of opportunistic listeners which is bounded by a circle with radius D_{OP} and should lie inside the transmission range of the relay but outside the transmission range of the distant source node.

C. Simulation Results

1) Impact of the Node Location

We first consider the impact of node location on the throughput. We place the source node s_1 at a distance of 250 m from the relay. The distance between the other source node s_2

and the relay is set to different values ranging from 0 to 250 m. With this scenario, we evaluate the performance when the node placements are imbalanced. The radius of the circle centered at s_2, which bounds the area of d_1, is the minimum value of 100 m and the distance between s_2 and the relay. Fig. 3 shows the resulting throughput and Fig. 4 shows the throughput gain.

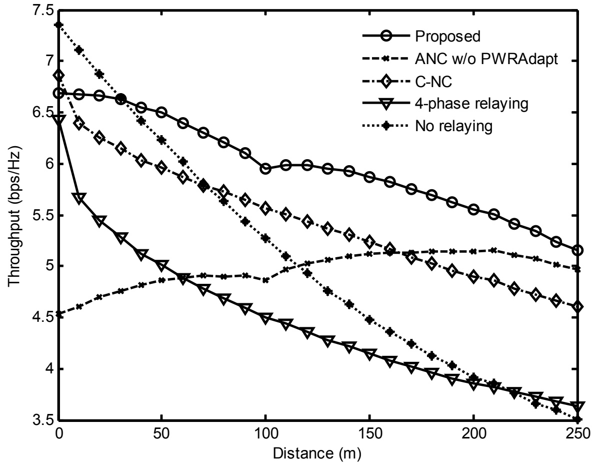

Fig. 3. Throughput vs. the distance between source node s_2 and relay r.

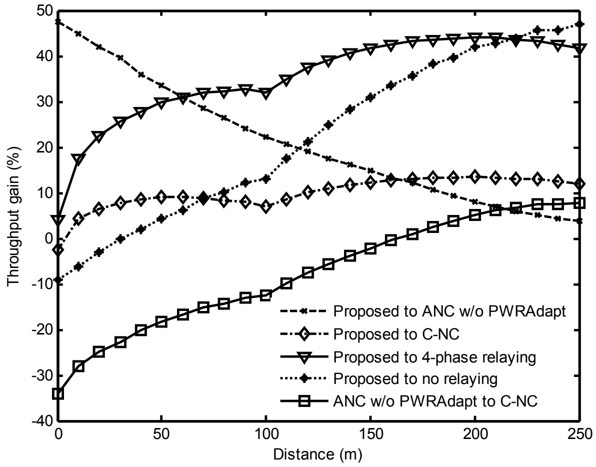

Fig. 4. Throughput gain vs. the distance between source node s_2 and relay r.

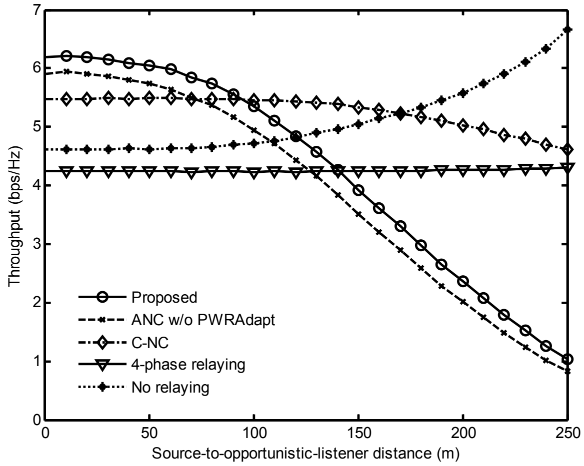

Fig. 5. Throughput vs. source-to-opportunistic-listener distance.

It can be observed that when the distance is greater than 30 m, the proposed scheme outperforms the other schemes. For a very small distance, no relaying is actually needed. Compared with ANC without power adaptation, the proposed scheme has a substantial throughput gain (up to approx. 50%) when the distance is small. When the distance between s_2 and r is the same with the distance between s_1 and r, we have 4% throughput gain. This means that power adaptation is beneficial for ANC, especially when the distances between the source nodes and the relay have significant difference. The proposed scheme is beneficial over C-NC except when the distance is 0, and the average throughput gain is approx. 10%. The throughput gain is lower than 50%, which is the theoretical throughput gain of physical layer network coding to C-NC without rate adaption [16]. One reason is that in rate adaptive C-NC, we can adapt the rate independently for the two source nodes and the relay, since they transmit in independent timeslots and the relay decodes the packet. But in ANC, we have to consider the joint effects of the channels. Another reason is that ANC exhibits signal attenuation and noise amplification at the relay and interference at the opportunistic listeners, which further reduces the possible data rate. We can also observe that without power adaptation, ANC performs worse than C-NC when the distance is smaller than 170 m.

2) *Impact of the Source-to-opportunistic-listener Distance*

Now we consider how the locations of opportunistic listeners affect the throughput. In this subsection, the location of d_2 (or d_1) is only chosen on the boundary of the shaded area in Fig. 2. With this setting, the distance between the source s_1 (or s_2) and its opportunistic listener d_2 (or d_1) is fixed as D_{OP}, which we refer as the source-to-opportunistic-listener distance. Fig. 5 shows the results.

It can be observed that in all cases, the proposed method is beneficial over ANC without power adaptation, with a throughput gain ranging from 5% to 26%. The throughput of ANC decreases rapidly when D_{OP} increases. The reason is that with ANC, the interference signal at the opportunistic listeners is high compared with the intended signal when D_{OP} is large. The throughput of C-NC slightly decreases when D_{OP} increases. When using 4-phase relaying or no relaying, opportunistic listening is not performed. The throughput of no relaying slightly increases with D_{OP}, since when D_{OP} is large it is more likely that the source and destination node is nearer. The results imply that ANC is beneficial when the opportunistic listeners are near to the source nodes. When D_{OP} is 0, the proposed method has 13% throughput gain to C-NC.

3) *Impact of the Rician Factor K*

The impact of the Rician factor K on the throughput is studied in this subsection. From the results in Fig. 6, we can observe that for all relaying methods, the throughput increases with K. When K is small, the throughput of the proposed method is slightly lower than the throughput of C-NC. And the proposed method outperforms all the other methods when $K > 0$ dB. When K = 20 dB, the throughput gain of the proposed method is 15% to C-NC. The proposed method also outperforms ANC without power adaptation with a throughput gain ranging from 4% to 10%. The results indicate that ANC is beneficial for channels with a strong non-fading component. The reason is that the data rate in ANC is affected by the status of many independent channels. And the data rate will be low when a specific channel is in deep fading.

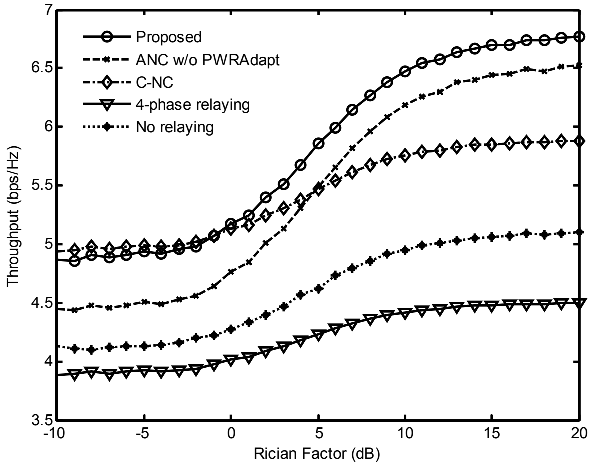

Fig. 6. Throughput vs. the rician factor K.

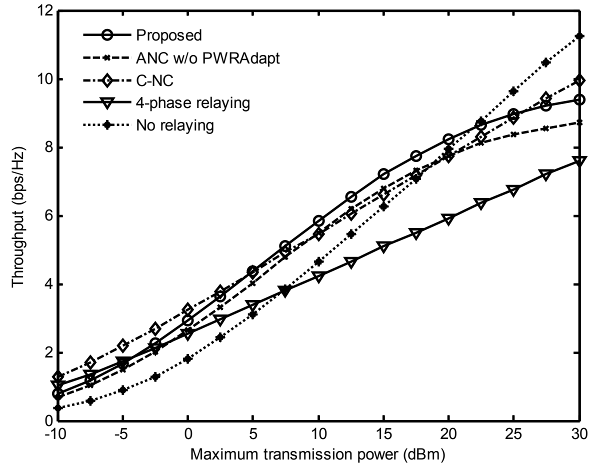

Fig. 7. Throughput vs. maximum transmission power.

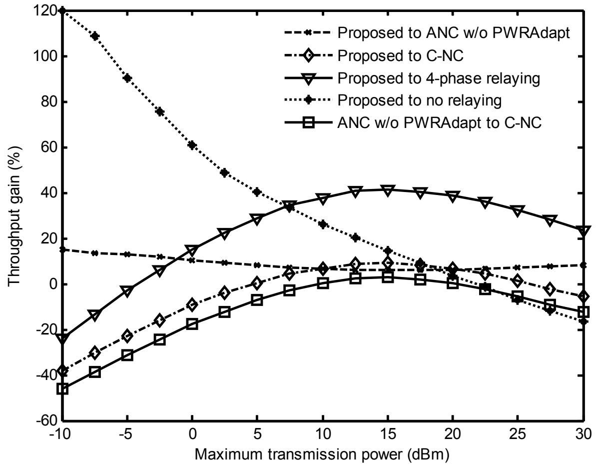

Fig. 8. Throughput gain vs. maximum transmission power.

4) *Impact of the Maximum Transmission Power*

The maximum transmission power $P_{\max}$ has also an impact on the data rate. We can observe from the results shown in Fig. 7 and Fig. 8 that the proposed method is beneficial when $P_{\max}$ ranges between 5 and 20 dBm, with a maximum throughput gain of 9% to C-NC. When $P_{\max} < 5$ dBm, C-NC outperforms the other relaying schemes. This is because the characteristics of ANC will cause low SNR for certain channel realizations. And the SNR has a greater impact on the data rate in the power-limited regime. When $P_{\max} > 20$ dBm, the throughputs of the ANC schemes tend to saturate, which is due to the increased power of the interference signal. And in this region, better performance can be obtained by transmitting without relaying, since no relaying is actually required in this region.

5) *Impact of the Prediction Error*

In this subsection, we consider the case where prediction errors of $|h|^2$ exist. When errors exist, we use the pessimistic approach described in Section V-B, i.e. we use the worst possible value for computation. The results are shown in Fig. 9. We can observe that when the prediction error is less than 2.5 dB, which can be obtained through several prediction methods at present knowledge [34], the proposed method outperforms the other methods. From the discussions in Section VI-C-3), it can also be inferred that higher prediction errors can be tolerated when $K > 5$ dB.

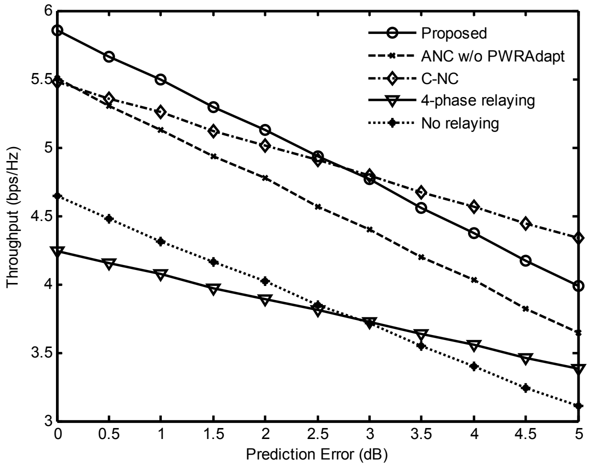

Fig. 9. Throughput vs. prediction error.

6) *Impact of the Channel Coherence Time*

The throughput at different channel coherence time, which measures the level of channel variation, is studied in this subsection. It can be observed from Fig. 10 that when the coherence time is above 100 ms, the throughput remains approximately constant. This is because the channels vary slowly in this region and the control overhead is low enough to be neglected. The proposed method is beneficial over the other relaying methods when the coherence time is greater than 3 ms. This value corresponds to the coherence time of the channel between vehicles with a relative speed of 72 km/h using a 900 MHz carrier or with a relative speed of 27 km/h using a 2.4 GHz carrier [36]. When the coherence time is below 3 ms, it is more advantageous to transmit without relaying, which requires fewer control packets for data rate adaptation, as discussed in Section VI-A-2). It can also be observed that when regarding the basic data rate as 1 bps/Hz, rate adaptive transmission is not beneficial for any scheme if the coherence time is smaller than 1 ms. The results indicate that the proposed scheme is beneficial for low to medium speed applications.

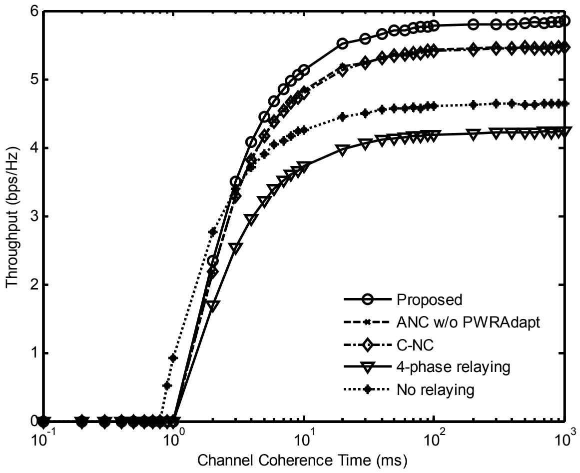

Fig. 10. Throughput vs. channel coherence time.

VII. CONCLUSIONS

In this paper, we proposed a rate and power adaptation scheme for analog network coding. We showed that power adaptation is necessary for ANC in order to obtain the optimal transmission rate, and introduced a joint rate and power adaptation method. The simulation results show that the proposed method outperforms rate adaptive ANC without power adaptation. Comparing with several other transmission schemes, we found that the proposed scheme is beneficial over other schemes in various scenarios. However, for some scenarios or network parameters, other transmission schemes are more advantageous. Hence, it is possible to develop a scheme that always selects the transmission scheme that has the best performance.

We considered a network that consists of only two sessions in this paper. When containing multiple sessions, an optimization scheme can be developed to determine which two sessions should be encoded. The MAC mechanism in this paper was assumed to be contention-free and pre-scheduled, as in most related work on ANC in the literature. This simplifies the analysis in the sense that it does not need to concern the random access issues which arise in contention-based MAC schemes. But considering the difficulties of network-wide scheduling, a contention-based MAC protocol that is compatible with ANC can be designed in the future. Meanwhile, especially with contention-based MAC protocols, the destructive interference between nodes in a general multi-hop network needs to be considered. This requires the investigation of a network-wide power optimization scheme, which is also a part of our future work.

APPENDIX

According to (4), (5), (13) and (14), the conditions 1) – 4) in Proposition 2 can be arranged into the following form of a quadratic equation:

$$c_1 P_1^2 + c_2 P_2^2 + c_3 P_1 P_2 + c_4 P_1 + c_5 P_2 = 0\,. \qquad (29)$$

And it can be solved as

$$P_1 = \frac{-(c_3 P_2 + c_4) \pm \sqrt{(c_3 P_2 + c_4)^2 - 4c_1(c_2 P_2^2 + c_5 P_2)}}{2c_1}, \qquad (30)$$

$$P_2 = \frac{-(c_3 P_1 + c_5) \pm \sqrt{(c_3 P_1 + c_5)^2 - 4c_2(c_1 P_1^2 + c_4 P_1)}}{2c_2}. \qquad (31)$$

The coefficients $c_1 - c_5$ for different conditions are shown in Table II.

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TABLE II
COEFFICIENTS IN (29) – (31)

Condition	c_1	c_2	c_3	c_4	c_5
1) $\gamma_{1(1)} = \gamma_{2(1)}$	$\lvert h_{12}\rvert^2 \lvert h_{11}\rvert^2$	$-\lvert h_{21}\rvert^2 \lvert h_{22}\rvert^2$	0	$\lvert h_{12}\rvert^2 \sigma_n^2$	$-\lvert h_{21}\rvert^2 \sigma_n^2$
2) $\gamma_{1(1)} = \gamma_{1(2)}$	$\frac{\lvert h_{1r} h_{r1}\rvert^2 \lvert h_{11}\rvert^2 P_r}{\sigma_n^2}$	$-\lvert h_{21}\rvert^2 \lvert h_{2r}\rvert^2$	$-\lvert h_{21}\rvert^2 \lvert h_{1r}\rvert^2$	$\lvert h_{1r} h_{r1}\rvert^2 P_r$	$-(\lvert h_{r1}\rvert^2 P_r + \sigma_n^2)\lvert h_{21}\rvert^2$
3) $\gamma_{2(1)} = \gamma_{2(2)}$	$\lvert h_{12}\rvert^2 \lvert h_{1r}\rvert^2$	$-\frac{\lvert h_{2r} h_{r2}\rvert^2 \lvert h_{22}\rvert^2 P_r}{\sigma_n^2}$	$\lvert h_{12}\rvert^2 \lvert h_{2r}\rvert^2$	$(\lvert h_{r2}\rvert^2 P_r + \sigma_n^2)\lvert h_{12}\rvert^2$	$-\lvert h_{2r} h_{r2}\rvert^2 P_r$
4) $\gamma_{1(2)} = \gamma_{2(2)}$	$\lvert h_{1r} h_{r1}\rvert^2 \lvert h_{1r}\rvert^2$	$-\lvert h_{2r} h_{r2}\rvert^2 \lvert h_{2r}\rvert^2$	$\lvert h_{1r} h_{r1}\rvert^2 \lvert h_{2r}\rvert^2 - \lvert h_{2r} h_{r2}\rvert^2 \lvert h_{1r}\rvert^2$	$(\lvert h_{r2}\rvert^2 P_r + \sigma_n^2)\lvert h_{1r} h_{r1}\rvert^2$	$-(\lvert h_{r1}\rvert^2 P_r + \sigma_n^2)\lvert h_{2r} h_{r2}\rvert^2$

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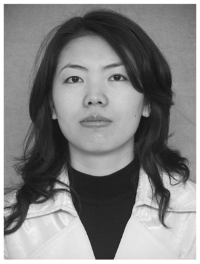

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