EXACT LOCALIZATION OF ACOUSTIC REFLECTORS FROM QUADRATIC CONSTRAINTS

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ABSTRACT

In this paper we discuss a method for localizing acoustic reflectors in space based on acoustic measurements on source-to-microphone reflective paths. The method converts Time of Arrival (TOA) and Time Difference of Arrival (TDOA) into quadratic constraints on the line corresponding to the reflector. In order to be robust against measurement errors we derive an exact solution for the minimization of a cost function that combines an arbitrary number of quadratic constraints. Moreover we propose a new method for the analytic prediction of reflector localization accuracy. This method is sufficiently general to be applicable to a wide range of estimation problems.

Index Terms— Microphone arrays, space-time audio processing, environment reconstruction, acoustic reflector localization

1. INTRODUCTION

Knowing the geometry of the acoustic environment can be very useful for numerous space-time processing applications. For example, in [1] source localization is approached using a maximum likelihood estimator whose data model incorporates the prediction of the early reflections. In [2] an environment-aware acoustic rendering system is proposed, in which early reflections are included in the propagation model from the loudspeakers to the listening area. This has the result of making their rendering system robust to mild reverberation. Consequently, the problem of estimating the geometry of the environment through acoustic acquisitions is an area of increasing interest. In [3] a method is proposed for estimating the reflectors based on the inverse mapping of the acoustic multi-path propagation problem. In [4] the parameters of a constrained room model are estimated through ℓ_1-regularized least-squares. In [5] the problem of the estimation of the room geometry is approached through the measurement of the Times of Arrival (TOAs) of the reflective path from the source to the microphone. Here TOAs are converted into geometric constraints that locate the line that the reflector lies upon. For a single source-microphone pair such constraints express that this line should be tangential to an ellipse that is parameterized by the locations of the source and the microphone and by the TOA. Using multiple observations with a microphone array, the reflector is found as the common tangent to all such ellipses, which is estimated through the iterative minimization of a fourth-order polynomial cost function. In [6] the authors generalize this approach with a two-step process based on a single source and multiple microphones. The source is first localized through the estimation of the Time Differences Of Arrival (TDOAs) on microphone pairs. The source location is then used for converting TDOAs of reflective paths into TOAs. The localization of the reflector is then approached in a similar fashion as in [5].

The cost functions defined in [5, 6] are inherently nonlinear, therefore they exhibit numerous local minima in which adaptive optimization algorithms could easily become trapped, particularly in the presence of relevant measurement errors. In this paper we propose an exact minimization procedure that determines the correct global minimum of the cost function while circumventing the problem of local minima. The problem is reformulated as the constrained minimization of a second-order polynomial, which admits an exact solution. This reformulation is inspired by [7], where a source localization problem is approached with an exact minimization of a constrained least-squares cost function. This algorithm is particularly useful when TOAs are estimated from TDOAs using information on the source location, as TOAs could be affected by a relevant error. In this paper we also propose a methodology for error propagation analysis, which aims to characterize the error that the reflector localization is affected by using some prior information on the error on TOA measurements. The ideas behind this analysis are partially borrowed from catastrophe theory [8], which allows us to derive an approximate linear relationship between the error in the distance of the acoustic path and the error on the localization of reflectors. This technique turns out to be general enough to be applicable whenever the estimation of the variable of interest is accomplished through the minimization of a cost function, under the hypothesis of a small bias in the estimated variable. Moreover, this method is more general than the well-known Cramer-Rao Lower Bound (CRLB). In fact, while CRLB provides a bound for an estimation problem, the proposed approach gives the theoretical limit for a specific cost function applied to that problem. Moreover, it can be shown that CRLB corresponds to the error propagation analysis applied to a maximum-likelihood cost function, and therefore it can be seen as a particular case of the method based on catastrophe theory. A Matlab toolbox is available [9], which can be used for assessing the accuracy of this class of estimation procedures.

The paper is organized as follows: in Section 2 we introduce the relevant notation and summarize the procedure used in [5, 6] to derive the cost function. In Section 3 we reformulate the cost function in order to be able to find an exact solution. Section 4 concerns the error propagation analysis. In Section 5 we show some simulation results that prove the validity of the error propagation analysis

The authors acknowledge the financial support of the Future and Emerging Technologies (FET) programme within the 7th Framework Programme for Research of the European Commission, under FET-Open grant number: 226007 SCENIC

as well as the improvement brought by the exact formulation over the iterative one. Conclusions are drawn in Section 6.

2. REFLECTOR LOCALIZATION

2.1. Notation

In this Section we describe the data model and the notation used throughout the paper. The microphones of the array are assumed to be placed in $\boldsymbol{x}_1, \dots \boldsymbol{x}_N$. The acoustic source is in $\boldsymbol{s}$ and, with no loss of generality, we can assume the origin of the reference frame to be placed in that location. Let us consider a single planar reflector lying on the line of equation $l_1 x + l_2 y + l_3 = 0$, described by the parameters $\mathbf{l} = [l_1\ l_2\ l_3]^T$. With reference to Fig.1, the image source $\boldsymbol{s}'$ is obtained by mirroring $\boldsymbol{s}$ over $\mathbf{l}$. The vector $\boldsymbol{\tau} = [\tau_1, \dots \tau_N]^T$

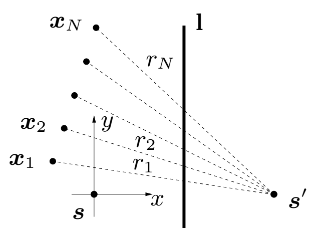


Figure 1: An acoustic source located at $\boldsymbol{s}$ is reflected over the line $\mathbf{l}$ to its image position $\boldsymbol{s}'$. The microphones at $\boldsymbol{x}_1, \dots \boldsymbol{x}_N$ estimates their distances $r_1, \dots r_N$ from the image source in $\boldsymbol{s}'$.

contains the TOAs of the reflective paths between the image source and the sensors, which are either measured [5] or estimated from the TDOAs [6]. The length of the reflective paths can be estimated as $r_i = \tau_i c$ and arranged in the vector $\mathbf{r} = \boldsymbol{\tau} c = [r_1, \dots r_N]^T$, where c is the speed of sound.

2.2. Cost function

As shown in [5, 6], the TOA measures corresponding to the reflective paths can be converted into quadratic constraints (in the homogeneous space) describing an ellipse. More specifically, as shown in Fig. 2, the ellipse has foci in $\boldsymbol{x}_i = [x_i\ y_i]^T$ and $\boldsymbol{s}$, and its major axis is r_i. This ellipse is tangential to the reflector line $\mathbf{l}$ at the reflection point $\boldsymbol{p}_i$. In order to find the equation of this ellipse we

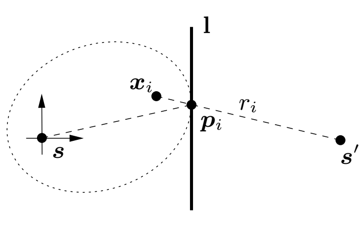


Figure 2: The length of the reflected path r_i from the image source in $\boldsymbol{s}'$ to the microphone at $\boldsymbol{x}_i$ constrains the reflector line $\mathbf{l}$ to be tangent to an ellipse whose major axis is r_i and whose foci are $\boldsymbol{s}$ and $\boldsymbol{x}_i$. $\boldsymbol{p}_i$ is the reflection point on $\mathbf{l}$.

start from constraint $\|\boldsymbol{x} - \boldsymbol{x_i}\| + \|\boldsymbol{x}\| = r_i$, which can be written as

$$\sqrt{(x - x_i)^2 + (y - y_i)^2} = r_i - \sqrt{x^2 + y^2} \ . \tag{1}$$

After taking the square power of both sides of eq. (1) we derive

$$\sqrt{x^2 + y^2} - 2x x_i - 2y y_i = r_i^2 - x_i^2 - y_i^2 \ . \tag{2}$$

Squaring again both the sides of eq. (2), we derive the implicit form of the ellipse described by the homogeneous parameter vector $[a_i\ b_i\ c_i\ d_i\ e_i\ f_i]^T$, which is given by

$$a_i x^2 + b_i x + c_i y^2 + d_i x + e_i y + f_i = 0 \ , \tag{3}$$

where

$$\begin{aligned} a_i &= -4(r_i^2 - x_i^2) \ , & d_i &= 4[x_i r_i^2 - x_i(x_i^2 + y_i^2)] \ , \\ b_i &= 8 x_i y_i \ , & e_i &= 4[y_i r_i^2 - y_i(x_i^2 + y_i^2)] \ , \\ c_i &= -4(r_i^2 - y_i^2) \ , & f_i &= r_i^4 - 2r_i^2(x_i^2 + y_i^2) + (x_i^2 + y_i^2)^2 \ . \end{aligned}$$

Eq. (3) can be expressed in matrix form as

$$\mathbf{x}^T \mathbf{C}_i \mathbf{x} = 0 \ , \ \mathbf{C}_i = \begin{bmatrix} a_i & b_i/2 & d_i/2 \\ b_i/2 & c_i & e_i/2 \\ d_i/2 & e_i/2 & f_i \end{bmatrix} \tag{4}$$

where $\mathbf{x} = [\boldsymbol{x}\ 1]^T$ is the homogeneous representation of a point $\boldsymbol{x}$ lying on the ellipse; and $\mathbf{C}_i$ is the *point-conic* matrix. The dual form of the conic expresses the conic as the set of lines $\mathbf{l}$ tangent to it, i.e. $\mathbf{l}^T \mathbf{C}_i^* \mathbf{l} = 0$, where $\mathbf{l} = [l_1\ l_2\ l_3]^T$ is the homogeneous representation of a line tangent to the ellipse; and $\mathbf{C}_i^* = \det(\mathbf{C}_i)\mathbf{C}_i^{-1}$ represents the *line-conic* matrix. Considering the set of N TOA measurements, a cost function collecting the corresponding N constraints can be defined as

$$J(\mathbf{l}, \mathbf{r}) = \sum_{i=1}^{N} \left(\mathbf{l}^T \mathbf{C}_i^* \mathbf{l} \right)^2 \ . \tag{5}$$

The reflector line is then estimated as the common tangent to all the ellipses by minimizing $J(\mathbf{l}, \mathbf{r})$. As all the vectors $k\mathbf{l}$, $k \neq 0$, form a class of equivalence, an infinite number of solutions turns out to minimize the cost function. In order to find an unique solution and avoid the trivial solution $\mathbf{l} = [0\ 0\ 0]^T$, some additional constraint needs to be used. For example, in [5], the minimization problem is formulated on the sub-space $\mathbf{l}_\alpha = [l_1 = \cos\alpha,\ l_2 = \sin\alpha,\ l_3]^T$, and the reflector is estimated as

$$\hat{\mathbf{l}}_\alpha = \underset{\mathbf{l}_\alpha}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\mathbf{l}_\alpha^T \mathbf{C}_i^* \mathbf{l}_\alpha \right)^2 \ . \tag{6}$$

3. EXACT SOLUTION

We now need to reformulate the cost function of Section 2.2 in order to turn the optimization problem into a linear Least-Squares (LS) one. As noted in [7], these problems are referred to as *generalized trust region subproblems* (GTRS), whose exact solution can be derived quite efficiently.

We first analyze the structure of the dual-conic, whose matrix

$$\mathbf{C}_i^* = \begin{bmatrix} a_i^* & b_i^*/2 & d_i^*/2 \\ b_i^*/2 & c_i^* & e_i^*/2 \\ d_i^*/2 & e_i^*/2 & f_i^* \end{bmatrix} \tag{7}$$

is symmetric, and its parameters can be written as

$$\begin{aligned} a_i^* &= 4r_i^2(r_i^2 - x_i^2 - y_i^2)^2 \ , & d_i^* &= 16 r_i^2 x_i (r_i^2 - x_i^2 - y_i^2) \ , \\ b_i^* &= 0 \ , & e_i^* &= 16 r_i^2 y_i (r_i^2 - x_i^2 - y_i^2) \ , \\ c_i^* &= a_i^* \ , & f_i^* &= 16 r_i^2 (r_i^2 - x_i^2 - y_i^2) \ . \end{aligned} \tag{8}$$

By replacing eq. (8) into the cost function (6), after some manipulation we obtain

$$J(\mathbf{l}, \mathbf{r}) = \sum_{i=1}^{N} \left[a_i^*(l_1^2 + l_2^2) + d_i^* l_1 l_3 + e_i^* l_2 l_3 + f_i^* l_3^2 \right]^2 \ . \tag{9}$$

In order to find a unique minimum for $J(\mathbf{l},\mathbf{r})$, we focus on the subspace defined by $\mathbf{l}' = [l_1\ l_2\ 1]^T$, and look for minima of the cost function lying on $l_3 = 1$. This leads to

$$\hat{\mathbf{l}}' = \underset{\mathbf{l}'}{\operatorname{argmin}} \sum_{i=1}^{N} \left[a_i^*(l_1^2+l_2^2) + d_i^* l_1 + e_i^* l_2 + f_i^*\right]^2 . \quad (10)$$

Notice that the condition $l_3 = 1$ rules out the potential reflectors passing through the origin. As the origin is the location of the source, this does not constitute a serious limitation. The simple substitution $w = l_1^2 + l_2^2$ allows us to rewrite the vector of the unknowns as $\mathbf{w} = [w\ l_1^2\ l_2^2]^T$, therefore the optimization problem can be written as

$$\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}} \left\{\|\mathbf{A}\mathbf{w}-\mathbf{b}\|^2 : \mathbf{w}^T\mathbf{D}\mathbf{w} + 2\mathbf{f}^T\mathbf{w} = 0\right\} \quad (11)$$

where

$$\mathbf{A} = \begin{bmatrix} a_1^* & d_1^* & e_1^* \\ \vdots & \vdots & \vdots \\ a_N^* & d_N^* & e_N^* \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} -f_1^* \\ \vdots \\ -f_N^* \end{bmatrix}$$

and

$$\mathbf{D} = \mathrm{diag}(0,1,1)\ ,\ \mathbf{f} = \begin{bmatrix}-0.5 & 0 & 0\end{bmatrix}^T .$$

Assuming that $\mathbf{A}$ has full column rank, the problem can be solved quite efficiently, and the exact solution is readily found using the approach described in [7]. In particular, the minimum is found as

$$\hat{\mathbf{w}}(\lambda) = (\mathbf{A}^T\mathbf{A} + \lambda\mathbf{D})^{-1}(\mathbf{A}^T\mathbf{b} - \lambda\mathbf{f})\ ,$$

where λ is the unique solution of $\hat{\mathbf{w}}(\lambda)^T\mathbf{D}\hat{\mathbf{w}}(\lambda) + 2\mathbf{f}^T\hat{\mathbf{w}}(\lambda) = 0$ on the interval for which $\mathbf{A}^T\mathbf{A} + \lambda\mathbf{D}$ is positive definite [7]. From the solution $\hat{\mathbf{w}} = [\hat{w}\ \hat{l}_1\ \hat{l}_2]^T$, the estimated reflector line is finally given by $\hat{\mathbf{l}}' = [\hat{l}_1\ \hat{l}_2\ 1]^T$.

4. ERROR PROPAGATION ANALYSIS

In this Section we propose a method for predicting the impact of the error on TOAs on the localization of reflectors using a formulation based on Catastrophe Theory [8].

Let $\mathbf{l}_0$ be the true reflector and $\mathbf{r}_0$ be the theoretical propagation distances of the reflective paths. In a real scenario the measurement of $\mathbf{r}_0$ is affected by noise $\delta\mathbf{r}$, and noisy measurements are denoted by $\bar{\mathbf{r}} = \mathbf{r}_0 + \delta\mathbf{r}$. Consequently, the new position of the minimum of $J(\mathbf{l};\bar{\mathbf{r}})$ becomes $\bar{\mathbf{l}} = \mathbf{l}_0 + \delta\mathbf{l}$. Assuming the error $\delta\mathbf{r}$ to be sufficiently small, we want to find a relationship between $\delta\mathbf{r}$ and $\delta\mathbf{l}$. We do so by computing the second-order Taylor expansion of $J(\mathbf{l};\bar{\mathbf{r}})$ centered about $(\mathbf{l}_0;\mathbf{r}_0)$. The term $(\nabla_\mathbf{l} J)^T|_{\mathbf{l}_0,\mathbf{r}_0}$ is zero, as the function with the true TOAs $\mathbf{r}_0$ has a minimum in $\mathbf{l}_0$. We can thus take the first-order derivative of the Taylor expansion and set it to zero to obtain

$$\mathbf{H}_{\mathbf{l},\mathbf{l}}(J)|_{\mathbf{l}_0,\mathbf{r}_0}\delta\mathbf{l} + \mathbf{H}_{\mathbf{l},\mathbf{r}}(J)|_{\mathbf{l}_0,\mathbf{r}_0}\delta\mathbf{r} = 0\ , \quad (12)$$

where

$$\mathbf{H}_{\mathbf{l},\mathbf{l}}(J) = \begin{bmatrix} J_{l_1l_1} & J_{l_1l_2} & J_{l_1l_3} \\ J_{l_2l_1} & J_{l_2l_2} & J_{l_2l_3} \\ J_{l_3l_1} & J_{l_3l_2} & J_{l_3l_3} \end{bmatrix},\ \mathbf{H}_{\mathbf{l},\mathbf{r}}(J) = \begin{bmatrix} J_{l_1r_1} & \dots & J_{l_1r_N} \\ J_{l_2r_1} & \dots & J_{l_2r_N} \\ J_{l_3r_1} & \dots & J_{l_3r_N} \end{bmatrix}$$

and

$$J_{l_il_j} = \frac{\partial^2 J}{\partial l_i \partial l_j},\quad J_{l_ir_j} = \frac{\partial^2 J}{\partial l_i \partial r_j} .$$

From (12) we finally obtain

$$\delta\mathbf{l} = \mathbf{G}\delta\mathbf{r}\ , \quad (13)$$

where $\mathbf{G} = -\mathbf{H}_{\mathbf{l},\mathbf{l}}(J)|_{\mathbf{l}_0,\mathbf{r}_0}^{-1} \cdot \mathbf{H}_{\mathbf{l},\mathbf{r}}(J)|_{\mathbf{l}_0,\mathbf{r}_0}$ In a real scenario we cannot assume $\delta\mathbf{r}$ to be known. However, some statistical information could be available in advance or could be estimated from the data. It is therefore important to find a relation between statistical descriptors of the noise $\delta\mathbf{r}$ and of $\delta\mathbf{l}$. The relationship between the covariance matrix $\mathbf{M}_\mathbf{l}$ of the estimation, and the covariance matrix $\mathbf{M}_\mathbf{r}$ of $\delta\mathbf{r}$ is

$$\mathbf{M}_\mathbf{l} = \mathbf{G}\mathbf{M}_\mathbf{r}\mathbf{G}^T, \quad (14)$$

where

$$\mathbf{M}_\mathbf{l} = \begin{bmatrix} \sigma_{l_1}^2 & \sigma_{l_1}\sigma_{l_2} & \sigma_{l_1}\sigma_{l_3} \\ \sigma_{l_1}\sigma_{l_2} & \sigma_{l_2}^2 & \sigma_{l_2}\sigma_{l_3} \\ \sigma_{l_1}\sigma_{l_3} & \sigma_{l_2}\sigma_{l_3} & \sigma_{l_3}^2 \end{bmatrix}$$

and

$$\mathbf{M}_\mathbf{r} = \begin{bmatrix} \sigma_{r_1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{r_2}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{r_N}^2 \end{bmatrix}$$

under the assumption of statistical independence of the measurement errors.

5. EVALUATION AND DISCUSSION

In order to test the solutions proposed in this paper, we first compared the accuracies of the exact and iterative techniques; and then we validated the error propagation analysis by comparing the RMSE of the exact solution and that predicted by eq. (14).

5.1. Setup

All the simulations were conducted with reference to the setup of Fig. 3. The microphone array was made of 5 sensors uniformly

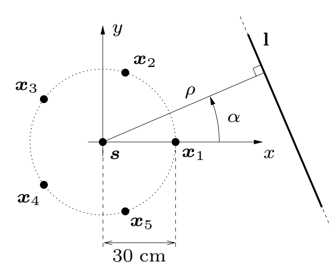
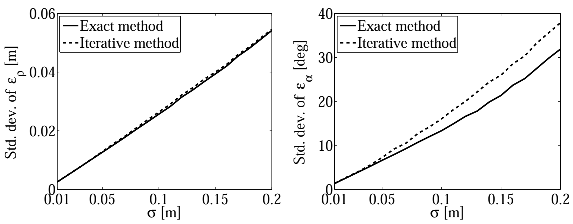


Figure 3: Simulation setup.

spaced on a circle of radius 30 cm centered in the origin of the reference frame (corresponding to the acoustic source). TOAs between microphones and source were calculated. The simulations were performed on a set of 9000 test reflector lines $\mathbf{l} = [\cos\alpha,\ \sin\alpha,\ -\rho]^T$ defined by their distance ρ and angle α with respect to the origin, as shown in Figure 3. The test reflectors were defined by distances in the range $[1\ \mathrm{m} \sim 4\ \mathrm{m}]$ and angles in the range $[0\ \sim 2\pi]$.

5.2. Comparison between exact and iterative methods

Using the above setup we compared the performance of the exact and iterative methods for minimizing the cost function of Section 2.2. The iterative method considered for the comparison is that proposed in [5], with a cost function of the form (6). For each reflector position, the distance measurements $\mathbf{r}$ were corrupted by 1000 realizations of independent identically distributed zero-mean Gaussian noise with standard deviation σ. The performance was evaluated by considering the distance error $\epsilon_\rho = \rho - \hat{\rho}$ and the angular error $\epsilon_\alpha = \alpha - \hat{\alpha}$ of the estimated reflector represented by the pair $(\hat{\rho}, \hat{\alpha})$ with respect to the true reflector position (ρ, α). Figs. 4-(a) and 4-(b) show the standard deviation of the distance error and of the angular error as a function of σ, respectively, averaged over all the tested locations and repetitions. As far as the distance error is concerned, the iterative and the exact solutions turned out to exhibit almost identical errors, which were proportional to the standard deviation σ of the measurement error. As for the angular error, for values of σ below 0.05 m, the two approaches had the same results, but for higher values of σ, the iterative method was affected by larger errors. This was due to the presence of multiple local minima in the cost function. For large measurement errors, the risk of encountering local minima increases as the cost function becomes less smooth. Although this phenomenon occurs occasionally, its impact on the standard deviation of the angular error is quite noticeable. The exact solution is therefore preferable over the iterative one, especially for large measurement errors.

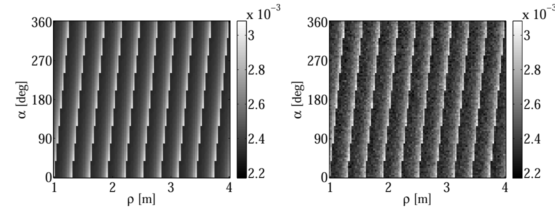


(a) Averaged std. dev. of ϵ_ρ (b) Averaged std. dev. of ϵ_α

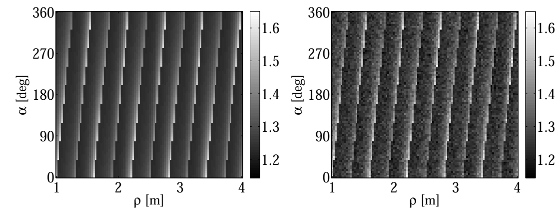
Figure 4: Comparison between the iterative and the exact solutions.

5.3. Validation of the error propagation analysis

We now validate the method for the error propagation analysis proposed in Section 4. In this case the standard deviation of the measurement noise is kept to $\sigma = 0.01$ m. The standard deviation of the error predicted with the analytic method is compared with the results of the simulations conducted on the same testing reflector positions. The results shown in Fig. 5 show the distance error for theoretical (a) and simulated (b) analysis, respectively. Similarly, Fig. 5 shows the theoretical (c) and simulated (d) results relative to the angular error. The results of the simulations accurately match the theoretical ones: they present the same mean error of the expected values (2.5 mm for the distance and $1.3°$ for the angle). The patterns of local maxima (i.e. diagonal white lines) correspond to configurations where two or more reflective paths are collinear, thus producing similar ellipses. In this situation, therefore, two measurements yield the same information, thus reducing the robustness of the estimation.



(a) Theoretical std. dev. of ϵ_ρ (b) Estimated std. dev. of ϵ_ρ



(c) Theoretical std. dev. of ϵ_α (d) Estimated std. dev. of ϵ_α

Figure 5: Comparison between the theoretical standard deviation of the error (predicted with the error propagation analysis) and the simulation results.

6. CONCLUSIONS

In this paper we proposed an exact technique for the localization of acoustic reflectors and a new method for the prediction of the related accuracy, which is valid also for other estimation problems. Simulations showed that the exact solution brings performance improvements over the iterative one, especially in the presence of large error on TOA measures. We also proved the accuracy and the effectiveness of the error propagation analysis.

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