PATH UNCERTAINTY ROBUST BEAMFORMING
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ABSTRACT
• Phase differences between received microphone signals are not a deterministic function of the array geometry.
• They are subject to random variations arising from position uncertainties and fluctuations in the sound velocity in the propagation channel.
• They degrade beamformer performance [1].
• We present a framework for modelling these uncertainties and show that improved beamformers are obtained when they are taken into account.

BEAMFORMER WEIGHTS DESIGN
• Beamformer weights are based on the covariance of the array data, $\mathbf{x}$.
• Array data is modelled as propagation coefficients, $\mathbf{D}$, multiplied by the sources, $\mathbf{s}$, with some additive noise.
• We redefine these propagation coefficients with an additional random propagation time, $\mathbf{T}$:
$$\langle \mathbf{xx}^H \rangle = \mathbf{x} = \mathbf{D}s + \mathbf{v} \quad \mathbf{D} = \mathbf{D} \odot \exp(j\omega_k \mathbf{T})$$
• Which gives us a new model for array covariance:
$$\mathbf{D} \odot \exp(j\omega_k \mathbf{T}) \mathbf{S} \left( \mathbf{D} \odot \exp(j\omega_k \mathbf{T}) \right)^H + \langle \mathbf{vv}^H \rangle$$
• We can expand the propagation term as follows:
$$a_{i,j} = \sum_{p=1}^{P} d_{ip} \bar{d}_{jp} \langle s_p^2 \rangle \times \exp \left( -\frac{\omega^2}{2} \left( \langle t_{ip}^2 \rangle + \langle t_{jp}^2 \rangle - 2 \langle t_{ip} t_{jp} \rangle \right) \right)$$
• We need the covariance in the additional random propagation times, $\mathbf{T}$:
$$\langle t_{ip} t_{jp} \rangle = \langle t_{ip} t_{jp} \rangle_S + \langle t_{ip} t_{jp} \rangle_C$$

ELEMENT POSITION UNCERTAINTIES
• Assume the positions of all elements are not exact.
• Source and microphone positions are given as $\mathbf{p}_i$ and $\mathbf{m}_j$.
• They deviate from their nominal positions through a zero-mean Gaussian distribution ($\mathbf{p}_i$ and $\mathbf{m}_j$).
• The covariance in path length changes between microphones $i$ and $j$ is:
$$\langle \delta(i,p) \delta(j,p) \rangle = \frac{1}{c^2} \langle \delta(i,p) \delta(j,p) \rangle$$
• The resulting time covariance is:
$$\langle t_{ip} t_{jp} \rangle_S = \frac{1}{c^2} \langle \delta(i,p) \delta(j,p) \rangle$$

CHANNEL UNCERTAINTIES
• It is commonly assumed that the channel propagation speed between a source and two different microphones is identical.
• This does not apply when the arrays are far apart [2].
• Variations can become large enough to cause phase differences that degrade the performance of the beamformer [3].
• These channel uncertainty cause correlated errors terms in $\langle \mathbf{xx}^H \rangle$.
• Propagation is modelled using the inverse velocity as a function of position $z$, where $g(z)$ is a zero-mean Gaussian variable:
$$\frac{1}{c(z)} = \frac{1}{c_0} + g(z)$$
• The total path delay from $z_a$ to $z_b$ is given by:
$$q \left( x_a, x_b \right) = \frac{\left| z_a - z_b \right|}{c_0} + \int_0^1 g \left( x_a + \left( x_b - x_a \right) t \right) dt$$
• The time difference covariance matrix is given by:
$$\langle t_{ip} t_{jp} \rangle_C = \left( \langle q(0, m_i - p_j) - \frac{m_i - p_j}{c_0} \rangle \left( \langle q(0, m_i - p_j) - \frac{m_i - p_j}{c_0} \rangle \right)^* \right)$$

REFERENCES

SIMULATIONS
SINGLE BEAMFORMER
• We applied the above model to the array geometry shown on the left.
• The correlation, $c_{ij}$, of M3 with the other microphones is on the right.
• Neighbouring propagation paths have a higher correlation in propagation time delay.

GENERAL TESTS
• Model applied to 300 random beamformer geometries.
• We compared SNR optimal beamformer weights [4] with (1) and without (2) considering random deviations:
$$\mathbf{D} = \left( \mathbf{D} \odot \exp(j\omega_k \mathbf{T}) \right)$$
• In the presence of Gaussian sensor noise and spatially diffuse noise.
• Measured resulting SNR in each case.
• Results are averaged over all 300 beamformers.
• Considering only channel uncertainties we observe the following difference in SNR between (1) and (2).

• As the correlation in the channels decays, the SNR of (1) degrades.
• Whereas (2) is able to utilise the most reliable microphones to maintain higher SNR.
• The more microphones in the array, the larger the SNR gains.
• Considering channel uncertainties and positional uncertainties with standard deviations of 5cm in each element, we obtain the following:

• This further improved the performance of the robust beamformer compared to the conventional.