Estimating the Topology of Neural Networks from Distributed Observations

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Connectivity within the Brain

• **Structural connectivity** describes the physical connections between different neurons.
  - Diffusion tensor imaging
  - Tractography from magnetic resonance imaging

• **Functional connectivity** refers to statistical dependencies between different units in the brain.
  - Functional MRI (fMRI)
  - Electroencephalography (EEG)
  - Magnetoencephalography (MEG)
  - Multielectrode array (MEA)

• **Effective connectivity** describes the causal relationships between neurons.
  - Structural equation modelling
  - Dynamics causal modelling
  - Granger causality
Calcium imaging: functional imaging of neural activity

• Can monitor activity of 100s-1000s of neurons simultaneously, at single cell resolution.

• Can image *in vivo* in behaving animals.

• Can image same cell populations over multiple months.

NetRate Algorithm for Network Topology Inference

• NetRate is an algorithm proposed by Gomez-Rodriguez, used to infer the edges of a static, directed network [2].

• The spreading model is the susceptible-infected one.

• Each edge from node $j$ to $i$ is assigned the conditional likelihood $f(t_i|t_j, \alpha_{j,i})$, of node $i$ to get infected at time $t_i$, given node $j$ was infected at time $t_j$, and the edge weight $\alpha_{j,i}$.

• The parameters $\alpha_{j,i}$ represent the transmission rates associated with edges.

NetRate Algorithm for Network Topology Inference

• The algorithm assumes access to multiple independent cascades of information.

• Each cascade is generated by randomly selecting a source node, and allowing information to spread according to the likelihoods $f(t_i|t_j, \alpha_{j,i})$.

• Each cascade contains the infection times of all the network nodes.

• NetRate aims to infer the transmission edges $\alpha_{j,i}$, by maximizing the likelihood of the observed cascades.
NetRate Algorithm for Network Topology Inference

Likelihood of a cascade

• The probability node $i$ to be infected at time $t_i$ given node $j$ was infected at time $t_j$ is $f(t_i|t_j, \alpha_{i,j})$.

• The probability that node $i$ is not infected by node $j$ by time $t_i$ is given by the survival function:

$$S(t_i|t_j, \alpha_{i,j}) = 1 - F(t_i|t_j, \alpha_{i,j})$$

• The hazard function is defined as the instantaneous infection rate, and given by:

$$H(t_i|t_j, \alpha_{i,j}) = \frac{f(t_i|t_j, \alpha_{i,j})}{S(t_i|t_j, \alpha_{i,j})}$$
NetRate Algorithm for Network Topology Inference

Likelihood of a cascade

• The likelihood of a cascade is the probability of observing the state of the susceptible and infected nodes:

\[
 f(t^c; A) = \prod_{t_i < T} \prod_{t_m > T} S(T | t_i, \alpha_{i,m}) \times \prod_{k: t_k < t_i} S(t_i | t_k, \alpha_{k,i}) \sum_{j: t_j < t_i} H(t_i | t_j, \alpha_{j,i})
\]

• Assuming independent cascades, the NetRate algorithm aims to solve the network inference problem given by:

\[
 \min_A - \sum_{c \in C} \log f(t^c; A)
\]

where:

\[
 A := \{\alpha_{j,i} > 0 | i, j = 1, ..., N, i \neq j\},
\]

\( C \) is the set of cascades,
\( c \) is a cascade in this set,
\( t^c \) are the observed infection times in cascade \( c \),
\( T \) is the length of the observation window.
NetRate Algorithm for Network Topology Inference

Likelihood of a cascade

- NetRate aims to solve the network inference problem given by:
\[
\min_A - \sum_{c \in C} \log f(t^c; A)
\]

where:
- \( A := \{\alpha_{j,i} > 0 | i, j = 1, ..., N, i \neq j\} \),
- \( C \) is the set of cascades,
- \( c \) is a cascade,
- \( t^c \) are the observed infection times in cascade \( c \),
- \( T \) is the length of the observation window.

- This problem is convex if the transmission likelihood has log-concave survival function and concave hazard function.

- The network inference problem is convex for the exponential, power-law and Rayleigh models.
NetRate Algorithm for Brain Topology Inference

• We have access to multiple independent cascades of information.
  • Cascades generated using constant input to Izhikevich’s neuron model.

• The spreading of information within the brain follows the susceptible-infected model.
  • During a cascade, each neuron spikes at most once.

• The diffusion of information between neurons can be modelled probabilistically.
  • Proved through stability analysis of Izhikevich’s dynamical system.

• The network inference problem is convex if the underlying distribution \( f(t_i|t_j, \alpha_{i,j}) \) follows the exponential, power-law or Rayleigh models.
  • The shape of this likelihood is derived empirically, using stability analysis of Izhikevich’s dynamical system.
Temporal Dynamics of Neural Networks

Spiking Neuron Model

• Izhikevich’s spiking neuron model accurately replicates the spiking behaviour of biological neurons [3]:

\[
\begin{align*}
\frac{dv(t)}{dt} &= 0.04v^2(t) + 5v(t) + 140 - u(t) + I \\
\frac{du(t)}{dt} &= a(bv(t) - u(t))
\end{align*}
\]

• If \( v(t) > 30mV \), then:

\[
\begin{align*}
\begin{cases}
    v(t) &\leftarrow c, \\
    u(t) &\leftarrow u + d
\end{cases}
\end{align*}
\]

• Regular spiking behaviour is obtained by setting: \( a = 0.02, b = 0.2, c = -65, d = 8 \).

Temporal Dynamics of Neural Networks

Transmission Likelihood

• We identify the causes of neuron spikes through stability analysis of Izhikevich’s system.
• If a neuron’s initial state is unstable, its potential will diverge to infinity, equivalent to a spike.
• Initial values of membrane potential and recovery determine the time a neuron takes to spike.

![Graphs showing membrane potential and simulation time]
Temporal Dynamics of Neural Networks

Transmission Likelihood

• Initial values of membrane potential and recovery determine the time a neuron takes to spike.
• If \((v_{\text{init}}, u_{\text{init}}) = (-80, -20)\), the time to spike is \(t \approx 7s\).
• If \((v_{\text{init}}, u_{\text{init}}) = (-40, -30)\), the time to spike is \(t \approx 1s\).
• A pre-synaptic node \(j\) can drive neuron \(i\) into a different unstable state, compared to pre-synaptic neuron \(k\), if \(\alpha_{k,i} \neq \alpha_{j,i}\).
• This shows that the diffusion of information between neurons can be modelled probabilistically, according to the rates \(\alpha\).
Temporal Dynamics of Neural Networks

Transmission Likelihood

• For each neuron $i$ that spikes, we identify the pre-synaptic neuron that spikes when $i$ became unstable.

• For example, neuron 5 fires at time $t = 257$. It enters the unstable region at time $t = 254$, the exact time when neuron 6 fires.

• The time delay between the spikes is $t_{6,5} = 3$, and the transmission rate $\alpha_{6,5} = 21$. 

![Graph showing neurons and time delays](image)
Temporal Dynamics of Neural Networks

Transmission Likelihood

• For small transmission rates, the shape of the likelihood is approximately exponential.
• For large transmission rates, the shape is approximately Rayleigh.
• We choose Rayleigh in order to accurately detect larger transmission rates.
• This proves the optimisation problem imposed by NetRate is convex.
Simulations

• Generate independent cascades of information.
  • Supply excitatory spiking neurons with a constant input:
    \[
    \frac{dv(t)}{dt} = 0.04v^2(t) + 5v(t) + 140 - u(t) + I
    \]
    \[
    \frac{du(t)}{dt} = a(bv(t) - u(t))
    \]
  • This makes them spike periodically, generating independent cascades of information.

• Run NetRate on small-world networks and random geometric graphs.
Simulations

![Bar chart showing Accuracy, Precision, and Recall for Small-world and Random graph models.](image-url)
Conclusion

• We proposed a novel method to infer the topologies of biological neural networks, using the NetRate algorithm.

• The spike propagation has a probabilistic nature. The shape of the pairwise transmission likelihood is found empirically.

• We showed that the optimisation problem NetRate solves for neural connectivity inference, is convex.

• Results indicate that NetRate is a suitable algorithm for neural network inference.

Future Work

• Define a weighted transmission likelihood, such that NetRate accurately infers both small and larger weights.
Thank you for listening!

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Any questions?