EE1 and ISE1 Communications I

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Lecture twelve
Lecture Aims

• Angle Modulation
  – Phase and Frequency modulation
  – Concept of instantaneous frequency
  – Examples of phase and frequency modulation
  – Power of angle-modulated signals
Consider a modulating signal $m(t)$ and a carrier $v_c(t) = A \cos(\omega_c t + \theta_c)$.

The carrier has three parameters that could be modulated: the amplitude $A$ (AM) the frequency $\omega_c$ (FM) and the phase $\theta_c$ (PM).

The latter two methods are closely related since both modulate the argument of the cosine.
Instantaneous Frequency

• By definition a sinusoidal signal has a constant frequency and phase: $A \cos(\omega_c t + \theta_c)$

• Consider a generalized sinusoid with phase $\theta(t)$: $\phi(t) = A \cos \theta(t)$

• We define the instantaneous frequency $\omega_i$ as:

$$\omega_i(t) = \frac{d\theta}{dt}$$

• Hence, the phase is

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha) d\alpha.$$
Phase modulation

We can transmit the information of $m(t)$ by varying the angle $\theta$ of the carrier. In phase modulation (PM) the angle $\theta(t)$ is varied linearly with $m(t)$:

$$\theta(t) = \omega_c t + k_p m(t)$$

where $k_p$ is a constant and $\omega_c$ is the carrier frequency. Therefore, the resulting PM wave is

$$\phi_{PM}(t) = A \cos[\omega_c t + k_p m(t)]$$

The instantaneous frequency in this case is given by

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$
Frequency modulation

In PM the instantaneous frequency $\omega_i$ varies linearly with the derivative of $m(t)$. In frequency modulation (FM), $\omega_i$ is varied linearly with $m(t)$. Thus

$$\omega_i(t) = \omega_c + k_f m(t).$$

where $k_f$ is a constant. The angle $\theta(t)$ is now

$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)]d\alpha = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha.$$

The resulting FM wave is

$$\phi_{FM}(t) = A \cos \left[ \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha \right]$$
Sketch FM and PM signals if the modulating signal is the one above (on the left). The constants $k_f$ and $k_p$ are $2\pi \times 10^5$ and $10\pi$, respectively, and the carrier frequency $f_c = 100\, MHz$. 
• Instantaneous angular frequency $\omega_i = \omega_c + kf m(t)$

• Instantaneous frequency $f_i = f_c + \frac{kf}{2\pi} m(t) = 10^8 + 10^5 m(t)$

$$
(f_i)_{min} = 10^8 + 10^5[m(t)]_{min} = 99.9\text{MHz}
$$

$$
(f_i)_{max} = 10^8 + 10^5[m(t)]_{max} = 100.1\text{MHz}
$$
PM example

- Instantaneous frequency 
  \[ f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t) = 10^8 + 5\dot{m}(t) \]

  \[ (f_i)_{\text{min}} = 10^8 + 5[\dot{m}(t)]_{\text{min}} = 10^8 - 10^5 = 99.9\text{MHz} \]

  \[ (f_i)_{\text{max}} = 10^8 + 5[\dot{m}(t)]_{\text{max}} = 10^8 + 10^5 = 100.1\text{MHz} \]
Power of an Angle-Modulated wave

- General angle modulated waveform

\[ \phi(t) = A \cos \theta(t) \]

- Instantaneous phase and frequency vary with the time, but amplitude \( A \) remains constant.

- Thus, the power of angle–modulated waves is always \( \frac{A^2}{2} \).
Conclusions

Examined

- Instantaneous frequency
- PM and FM modulations
- Examples of PM and FM signals