1. INTRODUCTION AND MOTIVATION

A generic sampling setup in 2-D

Input signal
\( g(x,y) \)

Acquisition device

\( \phi_{j}(x,y) \)

Sampling

\( S_{j,k} = \{ \delta(x-jT_1,y-kT_2) \} \)

Set of samples in 2-D

\( \sum_{j,k} \delta_{j}(x-jT_{1},y-kT_{2}) \)

Motivation

Image resolution enhancement and super-resolution photogrammetry.

What signals? Non-bandlimited signals but with a finite number of degrees of freedom (rate of innovation), and thus known as signals with Finite Rate of Innovation (FRI) [Vetterli et al.] E.g., Streams of Diracs, non-uniform splines, and piecewise polynomials. Present Focus: Sets of 2-D Diracs, bilevel and planar polygons.


Sampling kernel and properties? Any kernel \( \phi(x,y) \) that is of compact support and can reproduce polynomials of degrees \( \gamma \), \( \gamma \in \mathbb{Z} \), \( \gamma \in \mathbb{R} \) such that

\[ \sum_{j,k} \delta_{j}(x-jT_{1},y-kT_{2}) \]

2. SAMPLING OF SIGNALS WITH FINITE RATE OF INNOVATION (FRI) IN 2-D

• SETS OF 2-D DIRACS (LOCAL RECONSTRUCTION)

Consider \( g(x,y) = \sum_{j,k} \phi_{j}(x,y) \delta_{j}(x-jT_{1},y-kT_{2}) \) and \( \phi_{j}(x,y) \) with support \( L_{x}xL_{y} \) such that there is at most one Dirac \( \delta_{j}(x-jT_{1},y-kT_{2}) \) in an area of size \( L_{x}T_{1}xL_{y}T_{2} \).

From the partition of unity the amplitude is determined as:

\[ a_{p,q} = \sum_{j,k} S_{j,k} \]

And using polynomial reproduction properties along \( x \)-axis and \( y \)-axis, the coordinate position \( (x_{p},y_{q}) \) is determined as:

\[ x_{p} = \sum_{j,k} C_{p,j} S_{j,k} \]

• BILEVEL POLYGONS & DIRACS USING COMPLEX-MOMENTS (GLOBAL APPROACH)

Moments are used to characterize unspecified objects. [Shohat et al. 1943, Eldad et al. 2004]. For a convex, bilevel polygon \( g(x,y) \) with \( N \) corner points, and an analytic function \( \phi \) in closure \( \alpha \), the complex-moments of the polygon follow [Milanfar et al.][3].

\[ \gamma_{j,k}(x,y) = \int_{\alpha} \phi(x,y) \gamma_{j,k}(x,y) dxdy \]

The \( \gamma_{j,k} \) can be retrieved from \( \gamma_{j,k} \) using annihilating filter \( \gamma_{j,k} \) (Prony’s like method) such that \( d_{j,k}+\gamma_{j,k} = 0 \).

Now for both bilevel polygon and set of Diracs, using complex-moments and annihilating filter method, it is straightforward to see that

\[ C_{p,j} = \sum_{j,k} \gamma_{j,k}(x,y) \]

where \( \gamma_{j,k} \) are complex weights and \( \gamma_{j,k} \) are corner point coordinates of the bilevel polygon \( g(x,y) \), \( n = 2,3,\ldots,2N+1 \).

• PLANAR POLYGONS based on DIRECTIONAL DERIVATIVES (LOCAL APPROACH)

Intuitively, for a planar polygon, two successive directional derivatives along two adjacent sides of the polygon results into a 2-D Dirac at the corner point formed by the respective sides.

A planar triangle needs three pairs of directional differences to get decomposed in three 2-D Diracs

3. CONCLUSION AND ONGOING WORK

Conclusion: Local and global sampling choices for the classes of 2-D FRI signals with varying degrees of complexity.

Current investigations: Sampling of more general shapes such as circles, ellipses, and polygons containing polygonal holes inside.

Future plans: Extension of our 2-D sampling results in higher dimensions and effect of noise. Integration of sampling results with wavelet footprints for image resolution enhancement and super-resolution photogrammetry.

4. KEY REFERENCES


