A Balancing Voltage Transformation for Robust Frequency Estimation in Unbalanced Power Systems

Yili Xia*, Kai Wang*, Wenjiang Pei* and Danilo P. Mandic†
*School of Information Science and Engineering, Southeast University, 2 Sipailou, Nanjing 210096, P. R. China.
E-mail: yili.xia06@gmail.com, kaiwang@seu.edu.cn, wjpei@seu.edu.cn
†Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ London, UK.
E-mail: d.mandic@imperial.ac.uk

Abstract—This paper addresses the detection of the fundamental frequency of power systems under unbalanced and distorted conditions. By using the second order information, both the autocorrelation and pseudo-autocorrelation, within the Clarke’s transformed voltage, a novel balancing voltage transformation (BVT) is proposed to accurately detect the underlying phase angle evolution of the positive sequence component. This removes the biggest obstacle in current power systems and makes possible to use any frequency estimator for single-tone exponential on unbalanced power systems. The robustness of the proposed phase angle detection technique is illustrated for two well-known and efficient frequency estimators, that is, a discrete Fourier transform (DFT) coefficient interpolation method [1] and the weighted linear predictor (WLP) [2]. A window technique is used to cater for the fast and computationally affordable frequency estimation purposes. Simulations over a range of unbalanced conditions, including voltage dips and swells, frequency deviations and the presence of higher order harmonics support the analysis.

I. INTRODUCTION

Frequency is an important power quality parameter which is only allowed to vary within a small predefined range, a consequence of the dynamic unbalance between the generation and the load [3]. Its accurate estimation is essential, as maintaining the nominal frequency value is a prerequisite for both the stability of the grid and for normal operation of electrical devices.

A variety of architectures and algorithms have been proposed for this purpose, including phase-locked loops (PLL) [4], least squares techniques [2], [5–7], Kalman filters [8], adaptive notch filters [9] and recursive DFT methods [1], [10], [11]. In three-phase systems, standard frequency estimators employ Clarke’s transformation [8], a framework that simultaneously considers all the available information among the three-phase reference channels, to produce the complex-valued system voltage, from which the instantaneous phase information of the positive sequence component within the noncircular system voltage is proposed. This is achieved by using a novel balancing voltage transformation (BVT) which considers all the available second order information. The proposed transformation maps the noncircular system voltage onto a circular positive sequence component without loss of information, where the phase angle information is detected directly, hence facilitating the implementation of the existing frequency estimators for a single tone exponential. To illustrate the robustness of the proposed BVT technique, two well-known and efficient frequency estimators, that is, the weighted linear predictor (WLP) [2] and a coefficient interpolated discrete Fourier transform (DFT) method [1], are been implemented using a sliding window, for on-line frequency estimation based on the transformed voltage. Simulations over a range of unbalanced conditions support the analysis.

This work was partially supported by the National Natural Science Foundation of China (Grant No. 61201173, 61271058, 61401094), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20110092110008), the Natural Science Foundation of Jiangsu Province (Grant No. SBK201140040, BK2011060, BK20140645).

978-616-361-823-8 © 2014 APSIPA

APSIPA 2014
II. UNBALANCED THREE-PHASE POWER SYSTEMS

The three-phase voltages of a power system in a noise-free environment can be represented in a discrete time form as

\[ v_a(k) = V_a \cos(2\pi f k \Delta T + \phi) \]
\[ v_b(k) = V_b \cos(2\pi f k \Delta T + \phi - \frac{2\pi}{3}) \]
\[ v_c(k) = V_c \cos(2\pi f k \Delta T + \phi + \frac{2\pi}{3}) \]

(1)

where \( V_a, V_b, V_c \) are the peak values of each fundamental voltage component at time instant \( k \), \( \Delta T = \frac{T}{f} \) is the sampling interval, \( f_s \) is the sampling frequency, \( \phi \) is the initial phase, and \( f \) is the system frequency. In the analysis of power systems, three-phase voltages are typically transformed into a zero-sequence \( v_0 \) and the direct and quadrature-axis components, \( v_a \) and \( v_b \), by means of the orthogonal \((\alpha\beta0)\) Clarke's) transformation matrix, given by [8]

\[
\begin{bmatrix}
\sqrt{3} & \sqrt{3} & \sqrt{3} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & -\frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
v_0(k) \\
v_a(k) \\
v_b(k)
\end{bmatrix} = \begin{bmatrix}
\sqrt{3} & \sqrt{3} & \sqrt{3} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & -\frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
v_0(k) \\
v_a(k) \\
v_b(k)
\end{bmatrix}
\]

(2)

The factor \( \sqrt{2/3} \) is used to ensure that the system power is invariant under this transformation. In balanced system conditions, \( v_a, v_b, v_c \) are identical, giving \( v_0(k) = 0, v_a(k) = A \cos(2\pi f k \Delta T + \phi) \) and \( v_b(k) = A \sin(2\pi f k \Delta T + \phi) \). Under the transform in (2), it can be shown that the amplitude \( A = \sqrt{3} \frac{v_a(k)}{v_b(k)} \) is constant, while \( v_a(k) \) and \( v_b(k) \) represent the orthogonal coordinates of a point whose position is time variant at a rate proportional to the system frequency. Practical applications typically employ only \( v_a \) and \( v_b \) parts in the form of the complex voltage \( v(k) \), given by

\[ v(k) = v_a(k) + j v_b(k) = A e^{j(\omega k + \phi)} \]

(3)

where \( \omega = 2\pi f \Delta T \) is the normalised angular frequency. Notice that the amplitude-phase distribution of \( v(k) \) on the complex plane is rotation invariant, since for any real \( \theta \) both \( v \) and \( ve^{j\theta} \) rotate around a circle with radius \( A \). In terms of augmented statistics, this means that \( v(k) \) is second order circular (proper) [15], for which the powers in the \( v_a \) and \( v_b \) are equal, and standard autocorrelation matrix \( C \) has enough degrees of freedom to fully describe the second order statistics, while the pseudo-autocorrelation matrix \( P = E[\ddot{v}v^T] = 0 \) vanishes [16-18]. When the three-phase power system operates in abnormal conditions, for example, under different levels of dips or transients (typically \( V_a \neq V_b \neq V_c \)), then the complex Clarke's voltage becomes [12-14]

\[ v(k) = Ae^{j\omega k} + Be^{-j\omega k} \]

(4)

where

\[ A = \frac{\sqrt{3}(V_a + V_b + V_c)}{6} e^{j\phi} \]
\[ B = \left( \frac{\sqrt{3}(2V_a - V_b - V_c)}{12} - \frac{j}{4} \sqrt{2}(V_b - V_c) \right) e^{-j\phi} \]

(5)

Note that this expression is theoretically accurate for both the balanced and unbalanced system conditions. For balanced system condition, \( V_a = V_b = V_c \) and \( B = 0 \), whereas for unbalanced conditions, \( B \neq 0 \), and the so introduced negative sequence \( Be^{-j\omega k} \) causes the samples of \( V(k) \) generated from (5) to deviate from the circle with constant radius, making the distribution of \( v(k) \) rotation dependent (noncircular) [12-14]. This, in turn, results in the failure of standard phase angle calculation algorithms designed for balanced power systems, represented by the unavoidable estimation bias and oscillations at double the system frequency [6, 7, 12-14].

III. THE PROPOSED BALANCING VOLTAGE TRANSFORM (BVT) FOR UNBALANCED POWER SYSTEMS

To cancel out the negative sequence component within (4) which biases the frequency estimation of standard strictly linear algorithms, for unbalanced systems, our aim is to find a suitable data transformation for \( v(k) \). To this end, we consider the following balancing voltage transform (BVT)

\[ \ddot{v}(k) = v(k) - av^* \]
\[ \ddot{v}(k) = v(k) - \frac{B}{A^*} v^*(k) \]
\[ \ddot{v}(k) = Ae^{j\omega k} + Be^{-j\omega k} \]
\[ \ddot{v}(k) = \left( A - \frac{|B|^2}{A^*} \right) e^{j\omega k} \]
\[ \ddot{v}(k) = (1 - |a|^2)Ae^{j\omega k} \]

(6)

where the ratio \( a = \frac{B}{A^*} \) indicates the degree of system imbalance. In this way, for both the balanced and unbalanced system conditions, the transformed voltage \( \ddot{v}(k) \) contains only the positive sequence component \( e^{j\omega k} \) with a modified voltage magnitude; the transformed system voltage has therefore a rotation invariant amplitude-phase distribution, and is hence second order circular (proper) [15].

Indeed, for balanced conditions, the term \( |a| = 0 \), and we have a standard model, whereas for unbalanced conditions, a comparison of (4) and (6) shows the circular nature of (6). The problem then boils down to the estimation on the system imbalance ratio \( a \). This is not feasible based solely on \( v(k) \) or its complex conjugate \( v^*(k) \), and we need to resort to the full autocorrelation structure of \( v(k) \), where e.g. based on (4) the autocorrelation coefficient with lag \( m \), is given by

\[ r(m) = E[v(k)v^*(k - m)] \]
\[ = E[|A|^2 e^{j\omega m}] + E[|B|^2 e^{-j\omega m}] + E[A^*Be^{-j\omega(2k - m)}] + E[AB^*e^{j\omega(2k - m)}] \]
\[ \approx |A|^2 e^{j\omega m} + |B|^2 e^{-j\omega m} \]

(7)

In a similar way, the pseudo-autocorrelation coefficient \( p(m) \) of \( v(k) \) at lag \( m \) can be expanded as

\[ p(m) = E[v(k)v^*(k - m)] \]
\[ = E[|A|^2 e^{j\omega(2k - m)}] + E[|B|^2 e^{-j\omega(2k - m)}] + E[AB^*e^{j\omega m}] + E[ABe^{-j\omega m}] \]
\[ \approx AB^*e^{j\omega m} + AB e^{-j\omega m} \]

(8)
Combining (7) and (8), we arrive at
\[ r(m) - a^* p(m) = |A|^2 e^{j\omega_m} + |B|^2 e^{-j\omega_m} - \frac{B^*}{A} (A B e^{j\omega_m} + A B e^{-j\omega_m}) \]
\[ = (|A|^2 - |B|^2) e^{j\omega_m} \]  
and similarly,
\[ r(m) - p(m) = (|B|^2 - |A|^2) e^{-j\omega_m} \]  
Taking the complex conjugate of both sides of (10), we obtain
\[ r^*(m) - p^*(m) = (|B|^2 - |A|^2) e^{j\omega_m} \]  
Suppose that for a specific time lag \( m \), \( r(m) \) and \( p(m) \) can be estimated from sufficiently long observations of \( v(k) \). In (9) and (11), there are two unknown terms, \( e^{j\omega_m} \) and \( a \), therefore, it is possible to construct a quadratic polynomial of \( a \), in the form
\[ p^*(m)a^2 - (r(m) + r^*(m))a + p(m) = 0 \]  
for which the two roots are given by
\[ a = \frac{r(m) + r^*(m) \pm \sqrt{(r(m) + r^*(m))^2 - 4|p(m)|^2}}{2p^*(m)} \]  
Since, from (5), \( |A| > |B| \), and thus \( |a| < 1 \), we finally arrive at the solution for the degree of system imbalance in the form
\[ a = \frac{r(m) + r^*(m) - \sqrt{(r(m) + r^*(m))^2 - 4|p(m)|^2}}{2p^*(m)} \]  
Substituting (14) into (6), we obtain the transformed system voltage \( \tilde{v}(k) \), which admits application of any frequency estimator designed for single tone exponential.

IV. FAST AND ACCURATE FREQUENCY ESTIMATORS FOR POWER SYSTEMS

It is well known that the maximum likelihood (ML) estimator of the frequency under the model in (6) is given by the argument of the periodogram maximiser, that is
\[ \hat{f}_{ML} = \arg \max f \left( \sum_{k=0}^{N-1} \tilde{v}(k) e^{-j2\pi f k} \right)^2 \]  
However, the numerical maximisation of (15) is not a computationally simple task and may suffer from convergence and resolution problems. A variety of architectures and algorithms have been proposed to cater for fast and accurate unbiased frequency estimation, required by current power grids. However, this means that statistical accuracy has to be traded for computational efficiency. A review of recent advances in the design of rapid and computationally affordable frequency estimators can be found in [19]. We here verify the validity of the proposed voltage transformation on two classic block based frequency estimators: the discrete Fourier transformation (DFT) coefficient interpolation method [1] and the weighted linear predictor (WLP) [2].

True ML estimation of the frequency of a single complex sinusoid requires locating the peak in a periodogram. In [20], Palmer suggested using the DFT for locating this maximum, resulting in a coarse frequency estimator. Subsequently, Rife and Boorstyn [21] refined Palmer’s approach with an iterative search algorithm to perform a fine search. Motivated by their two-step structure, different reduced-complexity periodogram-based estimators have been proposed. Typically, these estimators first use \( N \)-point DFT to perform a coarse search for the periodogram peak, that is, to find the sample index \( p_N \) with the largest magnitude within the \( N \) bins. With the assumption that SNR is relatively high, it is very likely that the true frequency lies in the interval \([p_N - 0.5, p_N + 0.5]\). Then, a fine search for the true frequency is performed within this interval by using different coefficient interpolation procedures [1], [22]. We here use the approach proposed by Aboutanios and Mulgrew [1], which employs a simple but efficient iterative search strategy, and can be summarised as
\[ \tilde{V}(n) = \sum_{k=0}^{N-1} \tilde{v}(k)e^{-j\frac{2\pi kn}{N}} \]
\[ p_N = \arg \max_n \tilde{V}(n) \]
\[ \tilde{V}_{p_N \pm 0.5} = \sum_{k=0}^{N-1} \tilde{v}(k)e^{-j\frac{2\pi kn}{N} \pm 0.5} \]
\[ \delta = \delta + \frac{1}{2} \left( |\tilde{V}_{p_N \pm 0.5}^\ast| - |\tilde{V}_{p_N \pm 0.5}| \right) \]
\[ \hat{f} = \frac{p_N + \delta}{N} \]  
In [2], Kay developed a weighted linear predictor (WLP) from a different statistical model, where the additive noise is approximated by coloured Gaussian phase noise. This approximation is valid at a high SNR, where the phase measurements are samples in a weighted linear regression model. For a linear model with coloured noise, weighted least squares estimation is equivalent to ML estimation, and closed-form expression for the estimated frequency can be summarised as
\[ w(k) = \frac{3}{N} \sum_{k=0}^{N-1} \tilde{v}(k) e^{-j2\pi f k} \]
\[ \hat{f} = \frac{f_s}{2\pi} \sum_{k=0}^{N-2} w(k) \tilde{v}^*(k) \tilde{v}(k+1) \]  
Therefore, as long as the proposed voltage transformation can detect precisely the phase angle of the positive sequence component, the high estimation accuracy of the frequency estimators considered here can be maintained.

V. SIMULATIONS

To illustrate the suitability of the proposed BVT technique for the generality of system conditions, performance analysis of the considered WLP and DFT based frequency estimators
was investigated for various power system unbalanced conditions. Simulations were conducted in the Matlab programming environment at a sampling frequency of 1000 Hz, while the nominal power system frequency was 50 Hz. To cater for on-line frequency estimation, a sliding window was used for both the frequency estimation methods and for the calculation of the autocorrelation and pseudo-autocorrelation coefficients $r(m)$ and $p(m)$ in (7) and (8); the window was shifted by one step forward as the time evolved. Ten samples were used to update the estimates of both the second order statistics, $r(m)$ and $p(m)$, and also to update the system imbalance coefficient $a$ within the proposed voltage transformation. In principle, the time lag $m$ can be any integer, however, for practical purpose, the lag $m = 0$ should be ruled out, as the estimate $r(0) = \sigma_v^2 + \sigma_n^2$ can be heavily deteriorated by the noise. Without loss of generality, we set $m = 1$ in the simulations, while the window size $N$ varied in different scenarios. In principle, a large $N$ improves the performance of the considered frequency estimators, reducing the estimation bias and variance, however, this also delays the response to the system frequency variation. To quantify the degree of system unbalance in different conditions, the noncircularity index $\eta = \tau_2^2/\sigma_v^2$ was used, where $\tau_2^2 = E[v(k)v(k)]$ is the pseudo-correlation of $v(k)$ [17]. The values of $\eta$ lie in the interval $[0, 1]$, with the value of $\eta = 0$ indicating that $v(k)$ is perfectly circular, otherwise indicating a second order noncircular $v(k)$, containing a negative sequence component.

In the first set of simulations, the simulated power system experienced a single-phase fault, where a 40% voltage dip occurred in phase $v_a$. The noncircularity index of the complex-valued system voltage $v(k)$ obtained by the $\alpha\beta$ transformation of the three-phase voltage was $\eta = 0.11$. A geometric view of its noncircular distribution via the ‘real-imaginary’ scatter plot is shown in Fig. 1(a). This, in turn, indicates that the negative sequence component $B(k)$ is not negligible, and therefore, the proposed voltage transformation should be performed on $v(k)$ before applying the standard frequency estimators. The distribution of the transformed system voltage $\tilde{v}(k)$ is shown in Fig. 1(a), where its circular nature (single-tone exponential) can be clearly observed, thus facilitating unbiased and minimum variance estimation even for the standard frequency estimators, as shown in Fig. 1(b), otherwise, a direct implementation of standard frequency estimators would result in unavoidable estimation oscillations.

In the next stage, the robustness of the proposed voltage transformation in noisy environments was investigated based on the analysis of estimation bias of the considered frequency estimators. As shown in Fig. 2, in the high SNR region, the unbiased property of both frequency estimators for unbalanced power systems was maintained when using the proposed
that is, the sliding window length. The DFT based method tracked frequency dynamics more accurately. In Fig. 3(b), at \( t = 1 \text{ s} \), the unbalanced power system was contaminated by noise to give \( \text{SNR} = 50 \text{ dB} \), meanwhile, the system frequency underwent a rise at a rate of 4 \( \text{Hz/s} \), staying at 51 Hz for 0.5 s, followed by a 4 \( \text{Hz/s} \) decay from 1.8 s to 2 s. In all cases, based on the transformed system voltage, both estimators tracked the frequency variations fast and accurately.

In the next set of simulations, to access the impact of higher order harmonics on the performance of the proposed voltage transformation, an unbalanced system was further distorted by 10% of 3rd, 5% of 5th and 1% of 7th harmonic of the fundamental frequency, to give a noncircularity index \( \eta = 0.1318 \). The estimation results of both frequency estimators with different sliding window lengths \( N \) are shown in Fig. 4. Due to the breakdown of the single tone voltage modelling, both estimators exhibited unavoidable estimation bias and oscillations for this highly distorted system voltage. A relatively long sliding window of \( N = 80 \), lasting for 0.08 s, can effectively attenuate the impact of higher order harmonics with the maximum estimation error of \(< 0.02 \text{ Hz} \).

In the last set of simulations, the robustness of the proposed voltage balancing transformation was investigated in the context of a real-world power system. The three-phase voltage was recorded at a 110/20/10 kV transformer station. The REL 531 numerical line distant protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor changes in the three phase ground voltages. The measured three phase-ground voltages with a system frequency around 50 Hz were sampled at 1 kHz and were normalised with respect to their normal peak voltage values. As shown in Fig. 5(a), at around \( t = 0.12 \text{ s} \), the system experienced a voltage unbalance problem, where the voltage of phase \( v_b \) swelled to 139.8% of its normal value and meanwhile, both phases \( v_a \) and \( v_c \) experienced 19.7% and 9.7% voltage drops respectively.
Simulations over a range of unbalanced conditions, (balanced power system) directly on the transformed system within the system voltage. This has made it possible to apply phase angle evolution of the positive sequence component second order statistics of the system voltage, yielding accurate systems. This has been achieved by accounting for the full system frequency of both balanced and unbalanced power systems. This has enabled both DFT and WLP frequency estimators for real-world unbalanced three-phase power system. (a) The waveforms of the three-phase original system voltage $v_a$, $v_b$, $v_c$ for the real-world unbalanced three-phase power system. (b) Frequency estimation results.

As shown in Fig. 5(b), the proposed voltage balancing transformation enabled both DFT and WLP frequency estimators provided accurate frequency estimates in both balanced and unbalanced voltage conditions.

VI. CONCLUSION

We have introduced a novel balancing voltage transformation (BVT) for unified online estimation of the fundamental system frequency of both balanced and unbalanced power systems. This has been achieved by accounting for the full second order statistics of the system voltage, yielding accurate phase angle evolution of the positive sequence component within the system voltage. This has made it possible to apply any frequency estimator designed for single tone exponential (balanced power system) directly on the transformed system voltage. Simulations over a range of unbalanced conditions, including voltage dips and swells, phase deviation, the presence of higher order harmonics, and for real-world unbalanced power systems support the analysis.

REFERENCES