Multivariate Extensions of EMD
Applications in Data Fusion and BCI

Danilo P. Mandic

Imperial College London, UK

Ackn: Naveed Ur Rehman, David Looney, Cheolsoo Park

d.mandic@imperial.ac.uk, URL: www.commsp.ee.ic.ac.uk/~mandic
Outline

- Concept of Data Fusion Via Fission
- Empirical Mode Decomposition (EMD)
- Hilbert-Huang reconstruction and examples
- Complex EMD
- Applications: Image Restoration and Image Fusion
- Trivariate EMD: Methodology and Applications
- Multivariate EMD (MEMD)
- Filter bank property of MEMD
- Noise-assisted MEMD
- Conclusions
Problem Statement

Classic spectrum estimation: From a **finite** record of stationary data sequence, **estimate** how the total power is distributed over frequency. Has found a tremendous number of applications:


Modern view: From a **finite or infinite** record of non-stationary and non-linear data sequence, **estimate** how the total power is distributed over time-frequency.

**What are the basis for this analysis**

- **Parametric:** stochastic models (AR, MA, ARMA), ...
- **Non-parametric:** Fourier analysis, wavelets, Vigner-Ville, ...
- **Data driven:** adaptivity in the derivation of the bases
Bases for Signal Decomposition

"Linear methods" - signal analysed based on inner products with a predefined family of basis functions.

- Fourier: bases are \( \sin \) and \( \cos \) which are orthonormal and in general require an infinite number of terms in the expansion

\[
\mathcal{F}\{x[n]\} = \sum_{n=0}^{N-1} x[n] e^{-j\omega n}
\]

\[
= < x, e > \quad x = [x_0, \ldots, x_{N-1}]^T, \quad e = [1, e^{j2\omega}, \ldots, e^{j(N-1)\omega}]^T
\]

- Wavelet: assumes projecting on a pre-defined "mother" wavelet, which shrinks and expands, thus bypassing some of the problems of Fourier analysis, but not entirely

The bases (template functions) determine the properties of representation and influences the physical reading

\textit{e.g.} "frequency bins" instead of "instantaneous frequency"
Real World Signals – ‘Nonlinearity’ and ‘Stochasticity’

- Periodic oscillations
- Small nonlinearity
- Route to chaos
- Route to chaos
- Small noise
- HMM and others

Hence: we need to look into nonparametric representations of nonlinear and nonstationary signals
Speech Example – saying “Matlab” – ‘specgramdemo’

Frequency

For every time instant “t”, the PSD is plotted along the vertical axis. Darker areas: higher magnitude of PSD.
STFFT of a speech signal

(wide band spectrogram)  (narrow band spectrogram)

Data=[4001x1], Fs=7.418 kHz

-50  -40  -30  -20  -10  0  10  20  30

Frequency, kHz

-5  0  5

Time, ms

Ampl

515.5028 ms 0.0000 Hz 29.2416 dB

(win-len=256, overlap=200, ftt-len=32)

241.5745 ms 1.8545 kHz 3.2925 dB

(win-len=512, overlap=200, ftt-len=256)

(c) D. P. Mandic

The Third HHT Conference, Qingdao, China
What is the Right Basis for Real World Data?

Consider
- Amplitude modulated signal \( x(t) = m(t) \cos(\omega_0 t) \rightarrow m(t) \) - envelope
- Phase modulated signal \( x(t) = a \cos(\Phi(t)) \rightarrow \Phi(t) \) - phase

**Problem:** there is an infinite number of pairs \([a(t), \Phi(t)]\) s.t. \( m(t) \cos(\omega_0 t) = a \cos(\Phi(t)) \)

**Solution:** an analytic transform \( z(t) = x(t) + j\mathcal{H}(x(t)) \)

**Remark#1:** \( z(t) \) **cannot** be real, as \( \mathcal{F}(z(t)) = 0 \) for \( \omega < 0 \)

**Remark#2:** Hilbert transform (analytic signal) makes it possible to associate a **unique** pair \([a(t), \Phi(t)]\) to any real \( x(t) = \Re\{a(t)e^{j\Phi(t)}\} \)

**Remark#3:** For \( x(t) = a(t) \cos \Phi(t) \) \( \Rightarrow \mathcal{H}\{x(t)\} = a(t) \sin \Phi(t) \)

**Remark#4:** From instantaneous phase \( \Phi(t) \) \( \rightarrow \) instantaneous frequency

\[ f(t) = \frac{d\Phi(t)}{dt} \]

so we have an excellent resolution and do not depend on stationarity
MEMD vs STFFT for a speech signal

(STFFT spectrogram) (Hilbert-Huang spectra)

Data=[4001x1], Fs=7.418 kHz

Frequency, kHz

Time, ms

Ampl

(515.5028 ms 0.0000 Hz 29.2416 dB)

(win-len=256, overlap=200, ftt-len=256)

(no. of directions=64)

© D. P. Mandic

The Third HHT Conference, Qingdao, China
Empirical Mode Decomposition (EMD): Introduction

- Empirical Mode Decomposition has been recently introduced for the time-frequency analysis of nonstationary and nonlinear signals.

- Projection-based techniques assume stationarity and/or linearity in the input signal (Fourier and Wavelet approach), and hence, are not suitable for non-linear, non-stationary data.

- The Empirical mode decomposition (EMD) algorithm is a fully data-driven method which extracts the basis functions adaptively from the input data, through the so called sifting process.

- The adaptive nature of EMD also facilitates accurate time-frequency representation of the signal at the level of instantaneous frequency (through the Hilbert-Huang spectrum).

- Standard EMD is limited to the analysis of single data channels - modern applications require its multichannel extensions.

- For data fusion ⇔ same number of IMFs for all data channels.
Hilbert-Huang Spectrum: Results

Spectrogram and Hilbert–Huang Spectrum for a sum of two frequency modulated signals and a tone. The Hilbert-Huang spectrum shown in (right) clearly shows better resolution than the spectrogram shown on (left).
Vector sensors - Data Fusion - Synchrony

Renewable Energy
2D and 3D anemometers
control of wind turbine

Body motion sensor
3D - position, gyroscope, speed
gait, biometrics

Wearable technologies
Biomechanics
virtual reality
Vector sensors - 3D anemometer
Applications: How do we Decompose - Fuse RGB

**Computer Games**
Rotation of polygons
to form 3D graphics.

**Medical Applications**
3D time-space.
Data are naturally recorded in
2D and 3D electromagnetic field.

**Avionics**
Trajectory tracking of
angular properties (eg. velocity).
A General Data Fusion Problem

How do we make a mental "image" of a meal

- **taste** bitter, rotten
- **smell** pleasant, spices
- **vision** colour, presentation, rare, medium, well done
- **touch** bread, toast
- **hearing, temperature, toughness**
Data Fusion via Fission

- **Fission**: Decomposition into “particles”
- **Fusion**: Recombination of particles into the desired signal
Automated Data Fusion via Fission (MLEMD)

**Standard EMD**

![Diagram of Standard EMD]

**EMD for Data Fusion via Fission**

![Diagram of EMD for Data Fusion via Fission]

Incorporating the scale and temporal information

Most Machine Learning or Adaptive Filtering algorithms can be incorporated (Looney and Mandic, ICASSP 2009).
Benefits of the Data Fusion Approach

The synergy of information fragments offers some advantages over standard algorithms, such as:-

- Improved confidence due to complementary and redundant information;
- Robustness and reliability in adverse conditions (smoke, noise, occlusion);
- Increased coverage in space and time; dimensionality of the data space;
- Better discrimination between hypotheses due to more complete information;
- **System being operational even if one or several sensors are malfunctioning**;
- Possible solution to the vast amount available information.
Image Enhancement and Fusion Using EMD

- Image features (object texture or unwanted noise) can be attributed to local variations in spatial frequencies

- Therefore, the behaviour of the extracted image modes can reflect these features

- Correct fusion of the “relevant” IMFs can be used to highlight (or remove) specific image attributes

We consider the fusion capabilities of EMD under the following headings:

- Image Denoising

- Image Restoration (Illumination Removal)

- Image Fusion (of Images From Multiple Image Modalities)
Image Denoising

Noise contamination is a common problem when acquiring real world images and consequently image denoising is an important element of image processing. Many existing methods, however, are sub-optimal for the following reasons:

- They make unrealistic assumptions about the data (ICA - unrealistic independence conditions, PCA - noise and original image can be separated by linear projection)

- They are not optimised for enhancing higher order (nonlinear) statistics, that are commonly associated with the perceptual quality of an image, and do not cater for other real world data characteristics such as nonstationarity (block based Weiner filtering)

- They are computationally complex (Bayesian and particle models)
Image Denoising

Consider an original image corrupted by white Gaussian noise.

Original

Contaminated (SR 12.3 dB)
Image Denoising

Decomposing the contaminated image by EMD, we obtain the following:

Note how each of ‘Image Modes’ represents the frequency scales within the image. The higher index IMFs contain high frequency detail such as the image edges while slowly oscillating effects such as illumination are contained within the low index IMFs.
Image Denoising

Noisy image, SNR = 13 dB

EMD of the image

Denoising using PREMD, SNR=17.5 dB

Denoising using MLEMD, SNR=21 dB
The Method Naturally Deals With Texture

Cracked varnish on wood: *Left* – original; *Midle* – cracks; *Right* – wood pattern

![Cracked varnish on wood images]

Carpet: *Left* – original; *Midle* – texture; *Right* – carpet pattern

The texture is separated naturally as higher frequency T-F components
Image Restoration (Illumination Removal)

- A key problem for a machine vision system is image changes that occur due to scene illumination.

- Incident light on a surface produces complex artifacts, making it difficult for the system to separate changes caused by local variations in illumination intensity and colour.

- It can be assumed that shade in images creates low valued regions with large extrema that change slowly.

- It is therefore likely that the effects of the shade will be isolated in the lower index IMFs and a shade free image can be achieved by combining the relevant IMFs.
### Illumination Removal – Real World Objects

<table>
<thead>
<tr>
<th>Image with shade</th>
<th>Shade only</th>
<th>Original image</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Image with shade" /></td>
<td><img src="image2" alt="Shade only" /></td>
<td><img src="image3" alt="Original image" /></td>
</tr>
<tr>
<td>Shade removal: the shading is now uniform across the image surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Image with shade" /></td>
<td><img src="image5" alt="Shade only" /></td>
<td><img src="image6" alt="Original image" /></td>
</tr>
</tbody>
</table>

© D. P. Mandic

The Third HHT Conference, Qingdao, China 28
Surrogate images. *Top:* Original images $I_1$ and $I_2$; *Bottom:* Images $\hat{I}_1$ and $\hat{I}_2$ generated by exchanging the amplitude and phase spectra of the original images.
Image Fusion (From Multiple Image Modalities)

- Image fusion is becoming an important area of research, particularly as different methods of image acquisition become available.

- The fused image retains all "relevant information" from the different sources while disregarding unwanted artifacts.

- Given the unique "fission" properties of EMD, it has a strong potential for fusion.

- We propose the use of complex EMD with the input images as real and imaginary components respectively.

- The instantaneous amplitude of the extracted IMFs indicates, for each frequency level at each pixel, which of the components contains the salient information. Fusion can be achieved by combining only IMF components with the largest instantaneous amplitudes.
Obstacles to Automatic Heterogeneous EMD Fusion

- The fully *adaptive* and *empirical* nature of the algorithm compromises the uniqueness of the decomposition.

- Signals with similar statistics often yield different IMFs (in both number and frequency) - difficult to compare sources in T-F

Consider sinusoid corrupted by different realisations of AWGN. Note the difference in the IMFs.

![Graph showing seven IMFs and eight IMFs](image-url)
Obstacles to Automatic Heterogeneous EMD Fusion

Automatic fusion algorithms are necessary for widespread use!
But this is not often possible using standard EMD because

- Uniqueness of the scales cannot be guaranteed;
- Comparison of IMFs from different sources is meaningless!

Thus, automatic fusion of heterogeneous sources using EMD is only possible if their IMFs are

- equal in number;
- matched in properties (frequency).

[Rotation Invariant Complex EMD, Altaf, Mandic et al., 2007, Complex EMD, Tanaka and Mandic, 2007, Bivariate EMD, Rilling, Flandrin and Goncalves, 2007]
Empirical Mode Decomposition: Underlying Idea

- The basic idea behind EMD is to consider an input signal as fast oscillations superimposed on slow oscillations.

- The fast oscillations are repeatedly sifted from the input signal until a monotonic signal (residue) is obtained.
Complex EMD - Local Mean Estimation

(e) RIEMD

(f) BEMD

(g) A complex wind signal

(h) IMF6 + IMF7
Heterogeneous EMD Fusion

◊ It was proposed [Looney and Mandic] to use the complex extensions of the algorithm to decompose heterogeneous sources simultaneously.

◊ The approach may be used to find “common scales” within different data sets, thus addressing the problem of uniqueness.

Observe how common frequency scales are found in different signals (U1 and U2) by applying complex extensions of EMD to \((U1 + jU2)\).
Fusion Results [Looney and Mandic ICDSC’08]

Visual

Thermal

Pixel Average Fusion

PCA Fusion

Wavelet Fusion

Complex EMD Fusion
Out of focus image fusion using complex EMD

Complex EMD vs Wavelets

EMD fusion

Wavelet fusion

The wavelets produce artifacts - around the text visible as shaded “boxes”
Fusion of Exposure Images (gray scale) using Complex EMD: Methodology

- Two input gray scale images are converted into vectors by concatenating their rows, to form a complex signal.

- The complex signal is processed using the bivariate EMD algorithm, resulting in multiple complex IMFs.

- Scale images corresponding to each source are then combined locally using the local fusion algorithm to give a fused image.

(Gray-scale image fusion methodology using Bivariate EMD.)
Fusion of Exposure Images (gray scale) using Complex EMD: Results

(Input image 1)  (Input image 2)

(Wavelet based image fusion)  (EMD based image fusion)
Fusion of Exposure Images (colored RGB) using Complex EMD: Methodology

- The three channels (red, green, and blue) of a color image are processed by separate applications of bivariate EMD algorithm.

- Three sets of complex-valued IMFs are obtained which correspond to the red, green, and blue channels of the input images.

- Each set is processed separately by the fusion algorithm to yield the fused red, green and blue channel, which are combined to yield a fused RGB image.

(RGB color image fusion methodology using Bivariate EMD.)
Complex EMD-based Colored (RGB) Image Fusion: Results

(Input image 1) (Input image 2)

(‘Local’ Image Fusion) (EMD based image fusion)
Other Possibilities – “Environmental Dimension”


Here, the fusion was performed manually, without using any machine learning or extensions of EMD.
Trivariate EMD (TEMD): Underlying Idea

- Designed to extend EMD to process trivariate signals.
- Basic Idea: A trivariate signal is considered as a combination of a signal with fast 3D rotating component superimposed on a slowly rotating component.
- Local mean signal is considered as a slowly rotating component.
- Huangs sifting algorithm is used to extract 3D rotating modes in a trivariate signal.

D. Looney and D. P. Mandic, IEEE Transactions on Signal Processing, 2009
TEMD: Local Mean Estimation

To calculate the local mean, multiple projections of the input signal are taken, with each corresponding to a particular direction in 3D space.

The extrema of the projected signal are interpolated to yield quaternion-valued envelopes, which are then averaged to yield an estimate of the local mean:

\[ m(t) = \frac{1}{KN} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{\theta_k}^{\phi_n} \]  

where \( p_{\theta_k}^{\phi_n} \) denotes the projections along the direction, represented by \( \{ \theta_k, \phi_n \} \), in 3-dimensional space; \( K \) and \( N \) represent the total number of directions taken along directions \( \theta \) and \( \phi \) respectively.

(Multiple envelopes along a multivariate signal.)
The choice of the set of direction vectors for generating multiple envelopes is a crucial step.

The direction vectors are chosen along multiple longitudinal lines on a sphere, to encompass the whole 3D space; a direction vector is represented by a point on the sphere.

(a) A direction vector in 3D space

(b) Set of direction vectors in 3D space used in TEMD
TEMDD: Tai-Chi data analysis

(a) 3D plots of decomposed components
(b) Decomposition of individual components of the input trivariate signal
Fusion of light exposure images (colored RGB) using TEMD: Methodology

- The three channels (red, green, and blue) of a color image are processed by separate applications of TEMD algorithm.
- Three sets of quaternion-valued IMFs are obtained which correspond to the red, green, and blue channels of the input images.
- Each set is processed separately by the fusion algorithm to yield the fused red, green and blue channel, which are combined to yield a fused RGB image.

(RGB color image fusion methodology using TEMD.)
Multiple Color Image fusion using TEMD: Example

(Input Image 1)  (Input Image 2)  (Input Image 3)

(Fused Image)
Multivariate EMD: Direction vectors on a 3D sphere

- To generate a more uniform pointset in multidimensional spaces, the set of direction vectors generated via Hammersley sequence is used.
- Direction vectors for taking projections of trivariate signals on a sphere are shown for (a) spherical coordinate system; (b) a low-discrepancy Hammersley sequence are shown.
Multivariate EMD: Direction vectors on a 4D hypersphere

Direction vectors on a hypersphere WXYZ generated by using low discrepancy Hammersley sequence (a, b and c), and equiangular coordinate system (d, e and f), are shown.
Multivariate EMD: Illustration on a Hexavariate Signal

<table>
<thead>
<tr>
<th>Original signal</th>
<th>IMF1</th>
<th>IMF2</th>
<th>IMF3</th>
<th>IMF4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Original signal" /></td>
<td><img src="image2" alt="IMF1" /></td>
<td><img src="image3" alt="IMF2" /></td>
<td><img src="image4" alt="IMF3" /></td>
<td><img src="image5" alt="IMF4" /></td>
</tr>
</tbody>
</table>

© D. P. Mandic
The Third HHT Conference, Qingdao, China 52
MEMD: Decomposition of a Hexavariate Tai Chi signal

The proposed algorithm is applied to the body motion data recorded in a Tai Chi sequence. The data was captured using two inertial 3D MTx sensors attached to the left hand and the left ankle of an athlete; these were combined to form a single hexavariate signal.
Rotational modes for 3D+3D bodysensor Tai-Chi data

Original (left hand)

Original (left ankle)

IMF3

IMF3

IMF4

IMF4

IMF5

IMF5

Residue

Residue
MEMD as a Filter Bank

Spectra of IMFs (IMF1-IMF9) obtained for a single realization of an 8-channel white Gaussian noise via MEMD (top) and the standard EMD (bottom). Overlapping of the frequency bands corresponding to the same-index IMFs is more prominent in the case of MEMD based filters.
Multivariate EMD: Noise-assisted MEMD to reduce mode-mixing (a single channel case)

- Extra noise channels are added as extra dimensions to the original signal.
- The resulting multidimensional IMFs are aligned according to the filter bank structure of MEMD reducing mode mixing within the signal IMFs.

(Standard EMD) \hspace{2cm} (NA-MEMD: Signal channel with two extra noise channels)

⇒ An alternative to EEMD without mixing signal and noise!

NA-MEMD = process your data channel and several noise channels with MEMD.
A 2-channel Case: NA-MEMD to reduce mode-mixing

- The extra noise channels help
- So with NA-EMD we have a unified framework for the decomposition of both single-channel data and M-channel data
Operation of NA-MEMD and EEMD

In NA-MEMD signal is **NOT** added to noise, but instead the signal channel and noise channels are processed in a multidimensional space using MEMD. This way we guarantee the same number of IMFs and the same frequency contents at every level of such multidimensional IMFs.

Usually 2-4 noise channels are sufficient for good decomposition (see the simulations).
**NA-MEMD vs Ensemble EMD**

**Ensemble EMD (EEMD)**
- Performs EMD over an ensemble of signal plus white noise, and averages the ensemble IMFs
- Uses the filter bank property of EMD on white noise by populating the time-frequency space
- This way it reduces mode mixing
- EMD is applied separately over an ensemble of signal plus noise
- ⇒ each realization may have different number of modes (IMFs) and output contains residual noise
- **Only valid for univariate signals**

**Noise–Assisted MEMD**
- Makes use of the filter bank property of MEMD on white noise to populate entire T–F space
- Unlike EEMD, it does not directly interfere with the original signal (as noise is added to separate channels)
- ⇒ the output contains minimal residual noise due to leakage which is negligible
- Since a single MEMD is applied to the multivariate signal → same number of modes (IMFs)
- **Is valid for both univariate and multivariate signals**
NA–MEMD vs EEMD: Some Examples

Decomposition of X via EEMD

Decomposition of X via N–A MEMD

Error Function (e(t)) for EEMD

Error function (e(t)) for N–A MEMD

EEMD for 500 realisations of WGN

NA-MEMD with two noise channels
Sensitivity to Noise Power

**Ensemble EMD**

- The decomposition error increases with an increase in noise power.
- This is because noise is added to the signal and interferes with the decomposition.

**Noise Assisted MEMD**

- The error does not increase with an increase in noise power.
- This independence of noise power is because the noise is in separate channels and does not interfere with the signal.

---

© D. P. Mandic

The Third HHT Conference, Qingdao, China 61
BCI Application: Estimating cleaned EEG using MEMD

Filter bank structure of MEMD helps to clean the EEG signal by separating the brain electrical activity from unwanted artefacts, such as the electrooculogram (EOG) and electromyogram (EMG).
Imaginary Motion BCI: Mu Rhythm (8-12Hz)

- Occupies in the alpha range (8-12Hz)
- Strongly suppressed during the performance of contralateral motor acts
- Reflects the electrical output of the synchronisation of large portions of pyramidal neurons of the motor cortex which control hand and arm movement
- Active movements/observed actions/imagined actions

(a) power of mu band on scalp during motor imagery
(b) average time-frequency representation
(c) a typical single trial showing imagery-related modulation
Motor imagery: A dynamic state during which an individual mentally simulates a given action.

Experiment: A subject imagined herself/himself moving left arm, right arm or foot.

Event-related synchronization (ERS) over ipsilateral area during the motor imagery task can be expected around 10Hz (mu rhythm).

Right hemisphere (C4+C6) during the imagination of right arm movement.

(a) STFT  (b) EMD  (c) EEMD  (d) MEMD
Classification Performances

- 200 trials of motor imagery for left hand, right hand and foot movements.
- Common spatial patterns (CSP) were used to extract features.
- Average classification rates of 100 repetitions while mixing sample order (Support vector machine was used).
- MEMD produced the best classification results using 2\(^{nd}\) and 3\(^{rd}\) IMFs for all subjects.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Algorithm</th>
<th>IMFs</th>
<th>Classification rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>DFT (8-30Hz)</td>
<td></td>
<td>81.057 ± 4.291</td>
</tr>
<tr>
<td></td>
<td>EMD 1-3</td>
<td></td>
<td>65.710 ± 5.426</td>
</tr>
<tr>
<td></td>
<td>EEMD 1-3</td>
<td></td>
<td>79.727 ± 4.492</td>
</tr>
<tr>
<td></td>
<td>MEMD 2-3</td>
<td></td>
<td><strong>84.747 ± 4.762</strong></td>
</tr>
<tr>
<td>B</td>
<td>DFT (8-30Hz)</td>
<td></td>
<td>57.370 ± 5.564</td>
</tr>
<tr>
<td></td>
<td>EMD 1-4</td>
<td></td>
<td>60.607 ± 5.882</td>
</tr>
<tr>
<td></td>
<td>EEMD 1-2</td>
<td></td>
<td>67.387 ± 5.824</td>
</tr>
<tr>
<td></td>
<td>MEMD 2-3</td>
<td></td>
<td><strong>70.167 ± 5.363</strong></td>
</tr>
<tr>
<td>C</td>
<td>DFT (8-30Hz)</td>
<td></td>
<td>64.860 ± 6.020</td>
</tr>
<tr>
<td></td>
<td>EMD 1-4</td>
<td></td>
<td>60.653 ± 6.020</td>
</tr>
<tr>
<td></td>
<td>EEMD 1-3</td>
<td></td>
<td>75.250 ± 5.117</td>
</tr>
<tr>
<td></td>
<td>MEMD 2-3</td>
<td></td>
<td><strong>78.483 ± 4.362</strong></td>
</tr>
<tr>
<td>D</td>
<td>DFT (8-30Hz)</td>
<td></td>
<td>86.700 ± 4.081</td>
</tr>
<tr>
<td></td>
<td>EMD 1-3</td>
<td></td>
<td>79.470 ± 5.447</td>
</tr>
<tr>
<td></td>
<td>EEMD 2</td>
<td></td>
<td>87.433 ± 3.917</td>
</tr>
<tr>
<td></td>
<td>MEMD 2-3</td>
<td></td>
<td><strong>89.133 ± 3.235</strong></td>
</tr>
</tbody>
</table>
Multivariate extensions of EMD

- For robust modelling, it is crucial that similar oscillatory modes from multiple channels are aligned

- Extensions of EMD for multivariate time series:
  - Complex EMD [Tanaka and Mandic, IEEE SPL, 2007]
  - Rotation Invariant EMD [Mandic et al., Proc ICASSP, 2007]
  - Bivariate EMD [Rilling, Flandrin, Goncalves, IEEE SPL, 2007]
  - Trivariate EMD [Rehman and Mandic, IEEE TSP, 2009]
  - Quadrivariate EMD [Rehman and Mandic, Proc. IJCNN, 2010]
  - Filterbank property of Multivariate EMD and Noise-Assisted MEMD [Rehman and Mandic, IEEE Tr. Sig. Proc., 2011]

- Matlab code at www.commsp.ee.ic.ac.uk/~mandic
Conclusions

- EMD is non-parametric and self adaptive which is advantageous when decomposing real world data into its natural frequency modes.

- It is a powerful tool for the purposes of “data fusion via fission”.

- Multivariate extensions of EMD have been proposed which take multiple projections of a signal by sampling hyperspheres using equi-angular coordinate system and low discrepancy quasi-Monte Carlo based Hammersley sequences.

- The proposed method extracts common rotational modes across the signal components, making it suitable for e.g. fusion of information from multiple sources, and follows a filter bank structure.

- Making use of the quasi-dyadic filter bank property of multivariate extension of EMD, noise-assisted MEMD method (NA-MEMD) has been presented which has been shown to reduce mode mixing.

- NA-EMD is a viable alternative to EEMD.
Thank you

十分感谢！