Communications II

Lecture 5: Effects of Noise on FM

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Outline

- Recap of FM
- FM system model in noise
- Derivation of output SNR
- Pre/de-emphasis
- Comparison with AM
- Reference: Lathi, Chap. 12.

Frequency Modulation

Fundamental difference between AM and FM:

AM: message information contained in the signal **amplitude** ⇒ Additive noise: corrupts directly the modulated signal.

FM: message information contained in the signal **frequency**

⇒ the effect of noise on an FM signal is determined by the extent to which it changes the frequency of the modulated signal.

Consequently, FM signals is less affected by noise than AM signals

REVISION: Frequency modulation

A carrier waveform

$$s(t) = A\cos[\theta_i(t)]$$

where

$\theta_i(t)$: the **instantaneous phase angle**.

When

$$s(t) = A\cos(2\pi f\, t) \Rightarrow \theta_i(t) = 2\pi f\, t$$

We may say that

$$\frac{d\theta}{dt} = 2\pi f \Rightarrow f = \frac{1}{2\pi}\frac{d\theta}{dt}$$

Generalisation: **instantaneous frequency**:

$$f_i(t) \equiv \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

In FM: the instantaneous frequency of the carrier varies linearly with the message:

$$f_i(t)=f_c+k_f m(t)$$

where k_f is the **frequency sensitivity** of the modulator. Hence (assuming $\theta_i(0)=0$):

$$\theta_i(t) = 2\pi\int_0^t f_i(\tau)d\tau$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau$$

Modulated signal:

$$s(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

Note:

(a) The envelope is constant

(b) Signal s(t) is a non-linear function of the message signal m(t).

Bandwidth of FM

$m_p \equiv \max |m(t)|$: peak message amplitude

$f_c - k_f m_p$ < instantaneous frequency < $f_c + k_f m_p$

Define: **frequency deviation**= the deviation of the instantaneous frequency from the carrier frequency:

$$\Delta f \equiv k_f m_p$$

Define: **deviation ratio:**

$$\beta \equiv \frac{\Delta f}{W}$$

where W: the message bandwidth.

Small β: FM bandwidth ≈ 2x message bandwidth (**narrow-band FM**)

Large β: FM bandwidth >> 2x message bandwidth (**wide-band FM**)

Carson's rule of thumb:

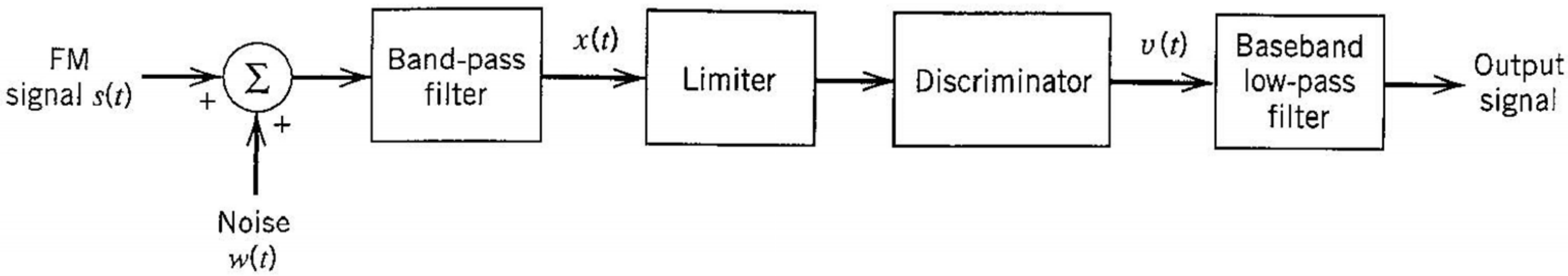
$$B_T = 2W(\beta+1) = 2(\Delta f + W)$$

$\beta << 1 \Rightarrow B_T \approx 2W$ (as in AM)

$\beta >> 1 \Rightarrow B_T \approx 2\Delta f$, independent of W

Noise in FM

Model of an FM receiver



Bandpass filter: removes any signals outside the bandwidth of $f_c \pm B_T/2$

⇒ the predetection noise at the receiver is bandpass with a bandwidth of B_T.

FM signal has a constant envelope

⇒ use a **limiter** to remove any amplitude variations

Discriminator: a device with output proportional to the deviation in the instantaneous frequency

⇒ it recovers the message signal

Final baseband low-pass filter: has a bandwidth of W

⇒ it passes the message signal and removes out-of-band noise.

Linear argument at high SNR

FM is nonlinear modulation, meaning superposition doesn’t hold.

Nonetheless, it can be shown (see Chap. 9, Lathi) that for *high SNR*, noise output and message signal are approximately independent of each other: Output ≈ Message + Noise.

Any (smooth) nonlinear systems are locally linear!

Noise does not affect power of the message signal at the output

⇒ We can **compute the signal power for the case without noise, and accept that the result holds for the case with noise too.**

Power of signal at the output **without** noise:

Instantaneous frequency of the input signal:

$$f_i = f_c + k_f m(t)$$

Output of discriminator:

$$k_f m(t)$$

So, output signal power:

$$P_S = k_f^2 P$$

where:

P : the average power of the message signal

In the presence of additive noise, the real predetection signal is

$$x(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$
$$+ n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

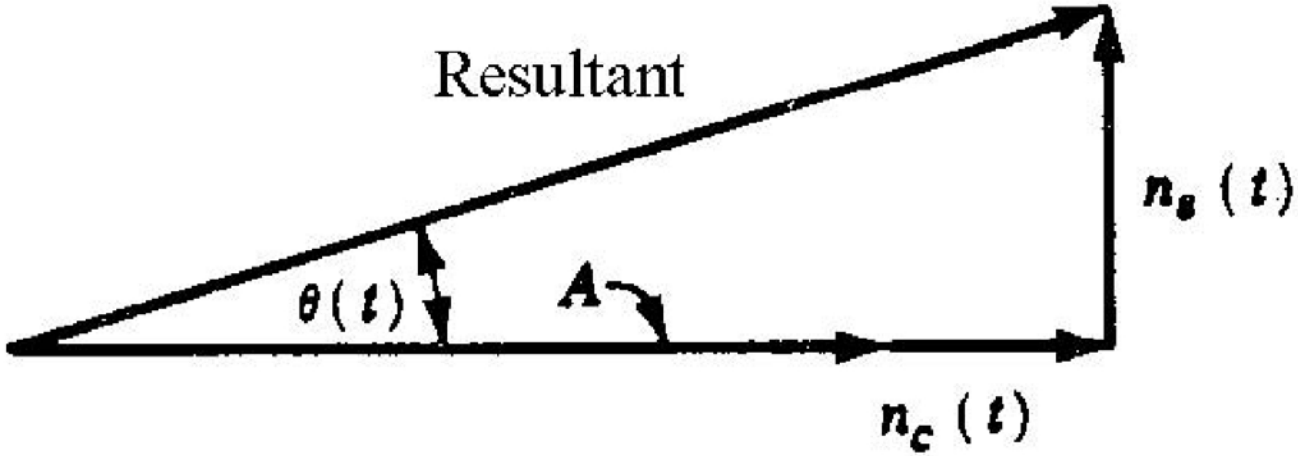
It can be shown (by linear argument): For high SNR, noise output: approximately independent of the message signal

⇒ We only have the carrier and noise signals present

⇒ In order to calculate the power of output noise, we may use:

$$\tilde{x}(t) = A\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

Phasor diagram of the FM carrier and noise signals



Instantaneous phase:

$$\theta_i(t) = \tan^{-1}\frac{n_s(t)}{A + n_c(t)}$$

For large carrier power (large A):

$$\theta_i(t) = \tan^{-1}\frac{n_s(t)}{A}$$

$$\approx \frac{n_s(t)}{A}$$

Discriminator output = instantaneous frequency:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$= \frac{1}{2\pi A}\frac{dn_s(t)}{dt}$$

The discriminator output in the presence of signal and noise:

$$k_f m(t) + \frac{1}{2\pi A}\frac{dn_s(t)}{dt}$$

What is the PSD of $\dfrac{dn_s(t)}{dt}$

Fourier theory:

$$\text{if} \quad x(t) \leftrightarrow X(f)$$

$$\text{then} \quad \frac{dx(t)}{dt} \leftrightarrow j2\pi f X(f)$$

Differentiation with respect to time = passing the signal through a system with transfer function of $H(f) = j2\pi f$

It can be shown:

$$S_o(f) = |H(f)|^2 S_i(f)$$

where:

$S_i(f)$: PSD of input signal

$S_o(f)$: PSD of output signal

$H(f)$: transfer function of the system

$$\left\{\text{PSD of } \frac{dn_s(t)}{dt}\right\} = |\, j2\pi f\,|^2 \times \left\{\text{PSD of } n_s(t)\right\}$$

$$\left\{\text{PSD of } n_s(t)\right\} = \left\{N_0 \text{ within band } \pm \frac{B_T}{2}\right\}$$

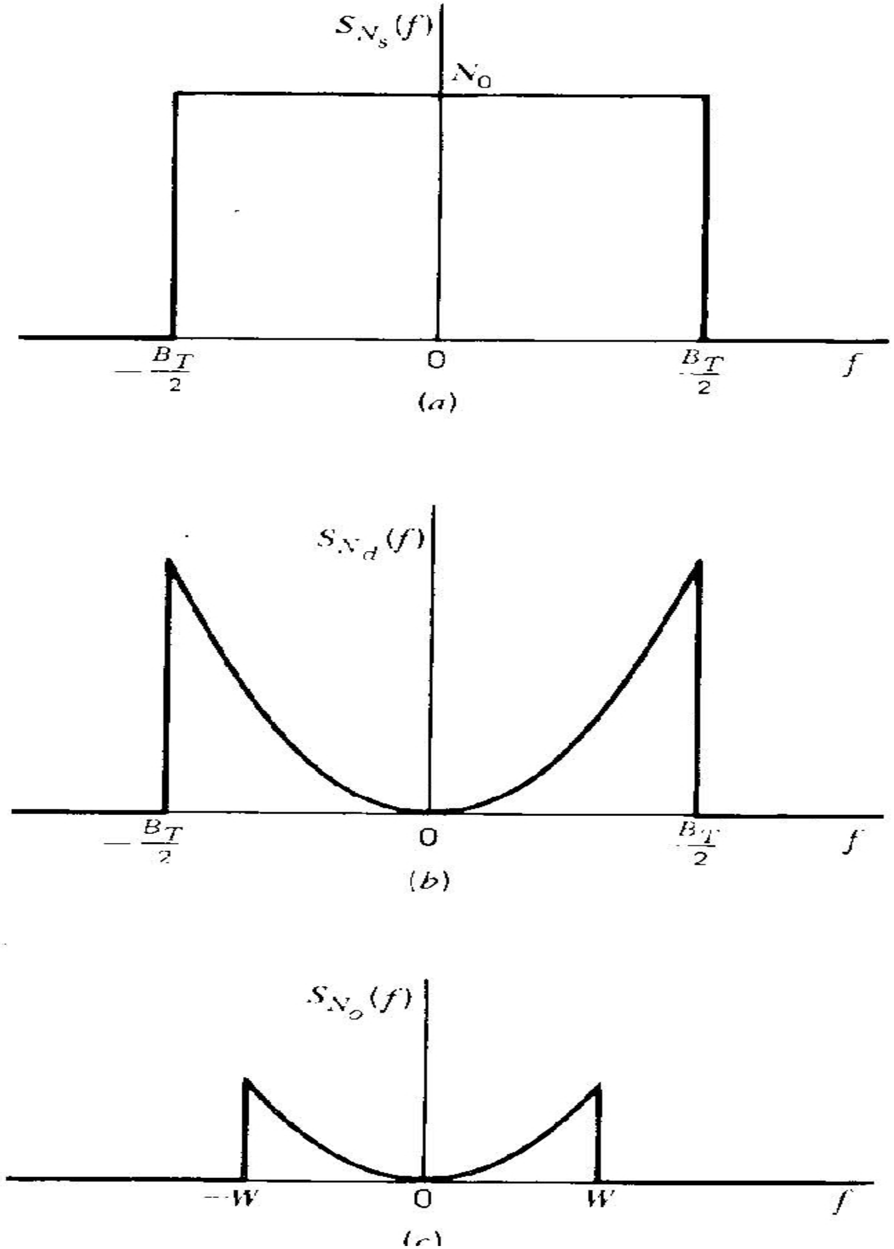
Then:

$$\left\{\text{PSD of } \frac{dn_s(t)}{dt}\right\} = |\, j2\pi f\,|^2 \times N_0$$

$$\left\{\text{PSD of } f_i(t) = \frac{1}{2\pi A}\frac{dn_s(t)}{dt}\right\} = \left(\frac{1}{2\pi A}\right)^2 |\, j2\pi f\,|^2 \times N_0 \equiv S_D(f)$$

After the baseband LPF, this is restricted in the band $\pm W$

Power spectral densities for FM noise analysis



Average noise power at the receiver output:

$$P_N = \int_{-W}^{W} S_D(f)\,df$$

Thus,

$$P_N = \int_{-W}^{W} \left(\frac{1}{2\pi A}\right)^2 |\, j2\pi f\,|^2 \, N_0\,df = \frac{2N_0W^3}{3A^2}$$

Average noise power at the output of a FM receiver $\propto \dfrac{1}{\text{carrier power } A^2}$

$A \uparrow \Rightarrow$ Noise$\downarrow$, called the *quieting effect*

$$SNR_O = \frac{3A^2 k_f^2 P}{2N_0 W^3} \equiv SNR_{FM}$$

Transmitted power of an FM waveform:

$$P_T = \frac{A^2}{2}$$

From $SNR_{baseband} = \dfrac{P_T}{N_0 W}$:

$$SNR_{FM} = \frac{3k_f^2 P}{W^2} SNR_{baseband} = 3\beta^2 \frac{P}{m_p^2} SNR_{baseband}$$

Valid when the carrier power is large compared with the noise power

The FM detector exhibits a (more pronounced) **threshold effect** like the AM envelope detector.

The threshold point occurs around when signal power is 10 time noise power:

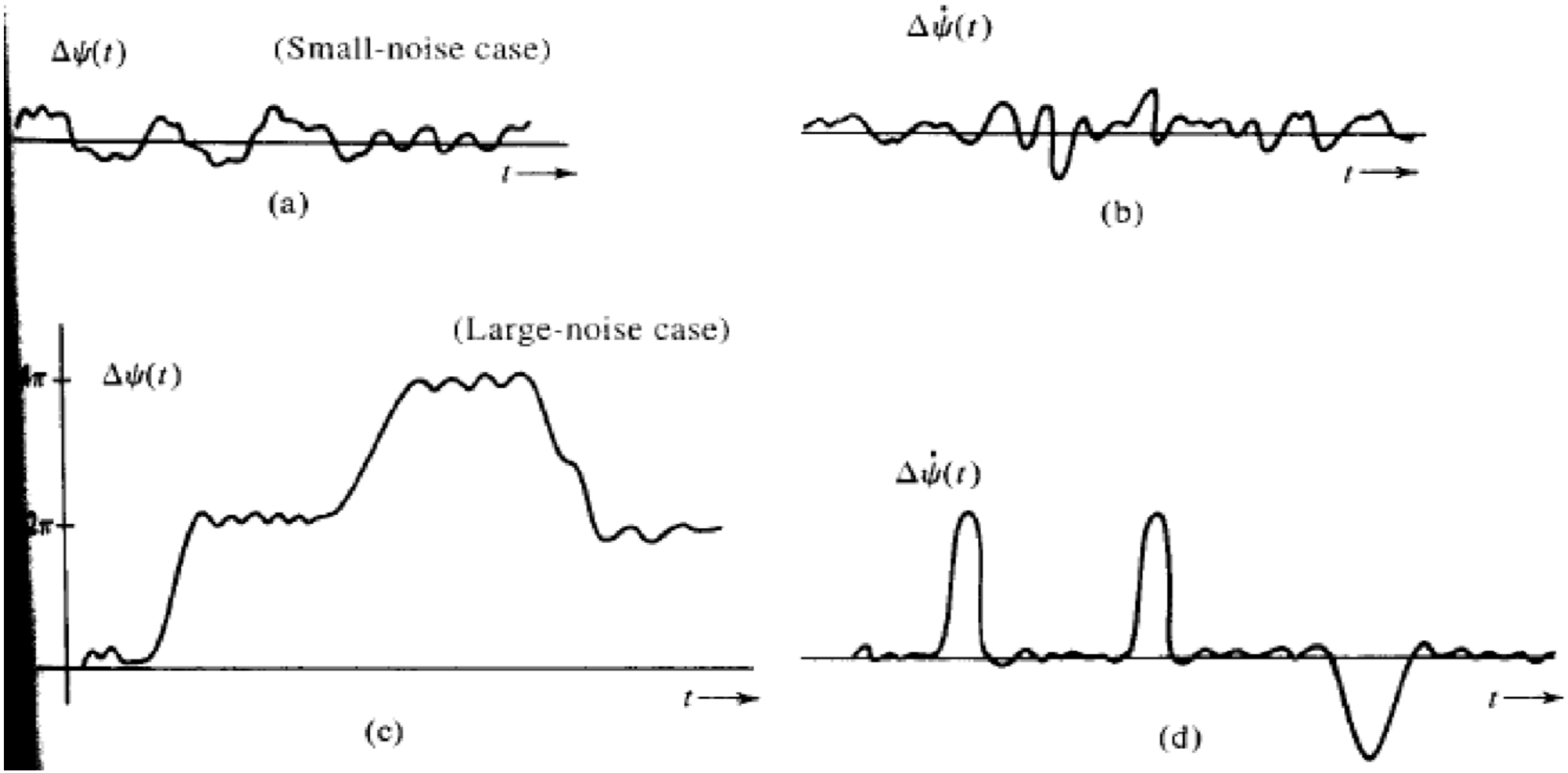
$$\frac{A^2}{2N_0B_T} = 10$$

where

$B_T = 2W(\beta + 1)$ (Carson's rule of thumb)

Qualitative discussion of threshold effect

Phase noise

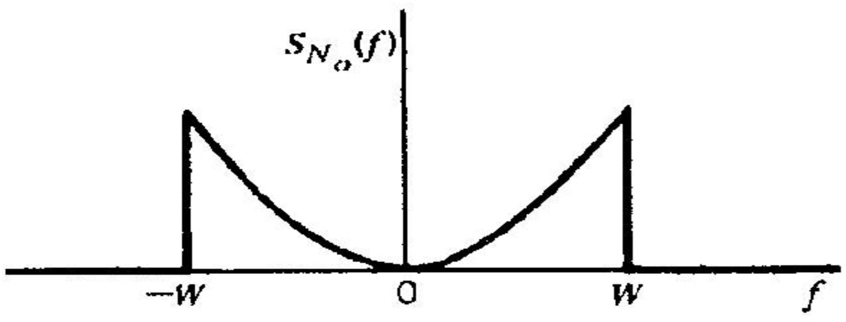


(c) phase shift 2π is caused by rotation around the origin

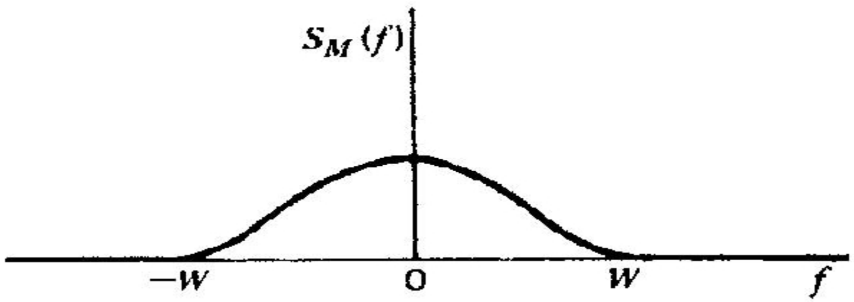
Pre-emphasis and De-emphasis: An alternative way to increase SNR_{FM}

PSD of the noise at the detector output $\propto$ square of frequency.

PSD of a typical message typically rolls off at around 6 dB per decade



(a)



(b)

To increase SNR_{FM} :

• Use a LPF to cut-off high frequencies at the output

Message is attenuated too

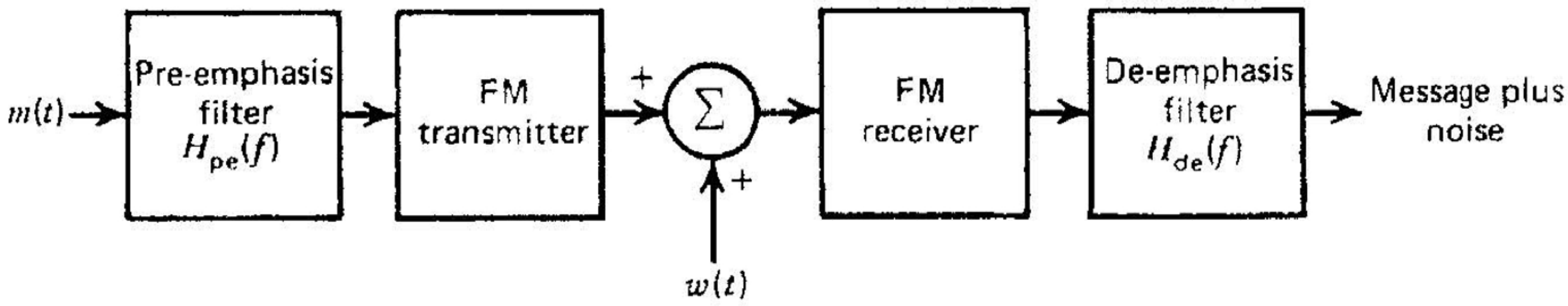
Not very satisfactory

• Use **pre-emphasis** and **de-emphasis**

Message is unchanged

High frequency components of noise are suppressed

Pre-emphasis and de-emphasis in an FM system



$H_{pe}(f)$: used to artificially emphasize the high frequency components of the message prior to modulation, and hence, before noise is introduced.

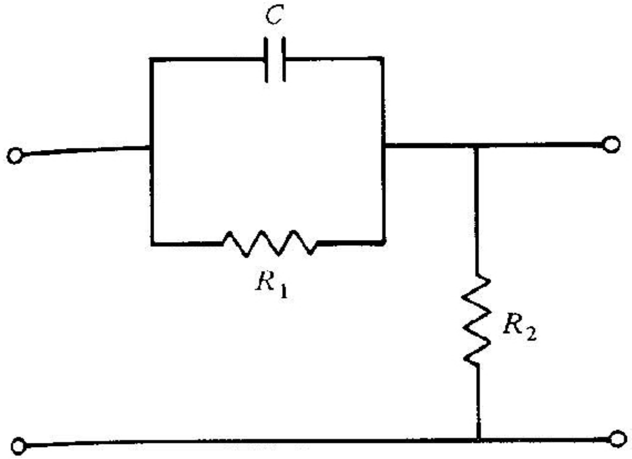
$H_{de}(f)$: used to de-emphasize the high frequency components at the receiver, and restore the original PSD of the message signal.

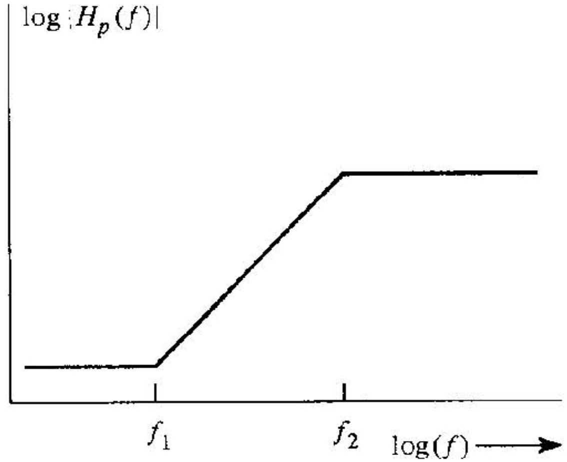
In theory, $H_{pe}(f) \propto f$, $H_{de}(f) \propto 1/f$.

This can improve the output SNR by around 13 dB.

Dolby noise reduction uses an analogous pre-emphasis technique to reduce the effects of noise (hissing noise in audiotape recording is also concentrated on high frequency).

Simple linear pre-emphasis and de-emphasis circuits

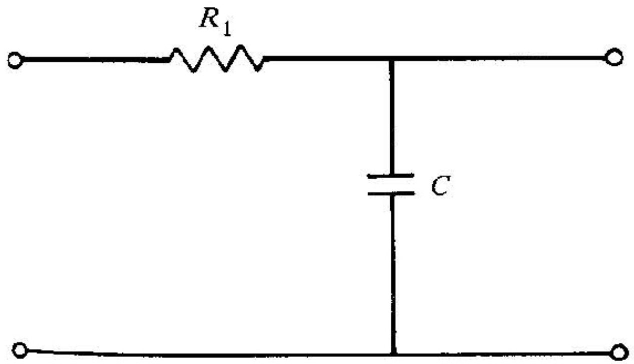


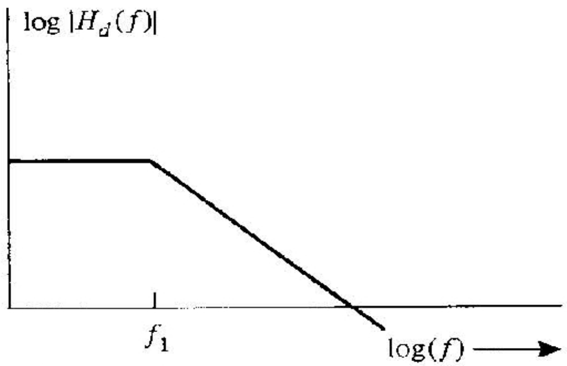


$$H_p(f) = K\,\frac{1+j(f/f_1)}{1+j(f/f_2)} \qquad \text{where } f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi R_1 C},\; f_2 = \frac{1}{2\pi\tau_2} = \frac{R_1+R_2}{2\pi R_1 R_2 C}$$

(a) Preemphasis Filter

(b) Bode Plot of Preemphasis Frequency Response





$$H_d(f) = \frac{1}{1+j(f/f_1)} \qquad \text{where } f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi R_1 C}$$

(c) Deemphasis Filter

(d) Bode Plot of Deemphasis Characteristic

Comparison of Analogue Communication Systems

Assumptions:

① single-tone modulation, ie: $m(t) = A_m \cos(2\pi f_m t)$

② the message bandwidth $W = f_m$;

③ for the AM system, $\mu = 1$;

④ for the FM system, $\beta = 5$ (which is what is used in commercial FM transmission, with $\Delta f = 75$ kHz, and $W = 15$ kHz).

With these assumptions, we find that the SNR expressions for the various modulation schemes become:

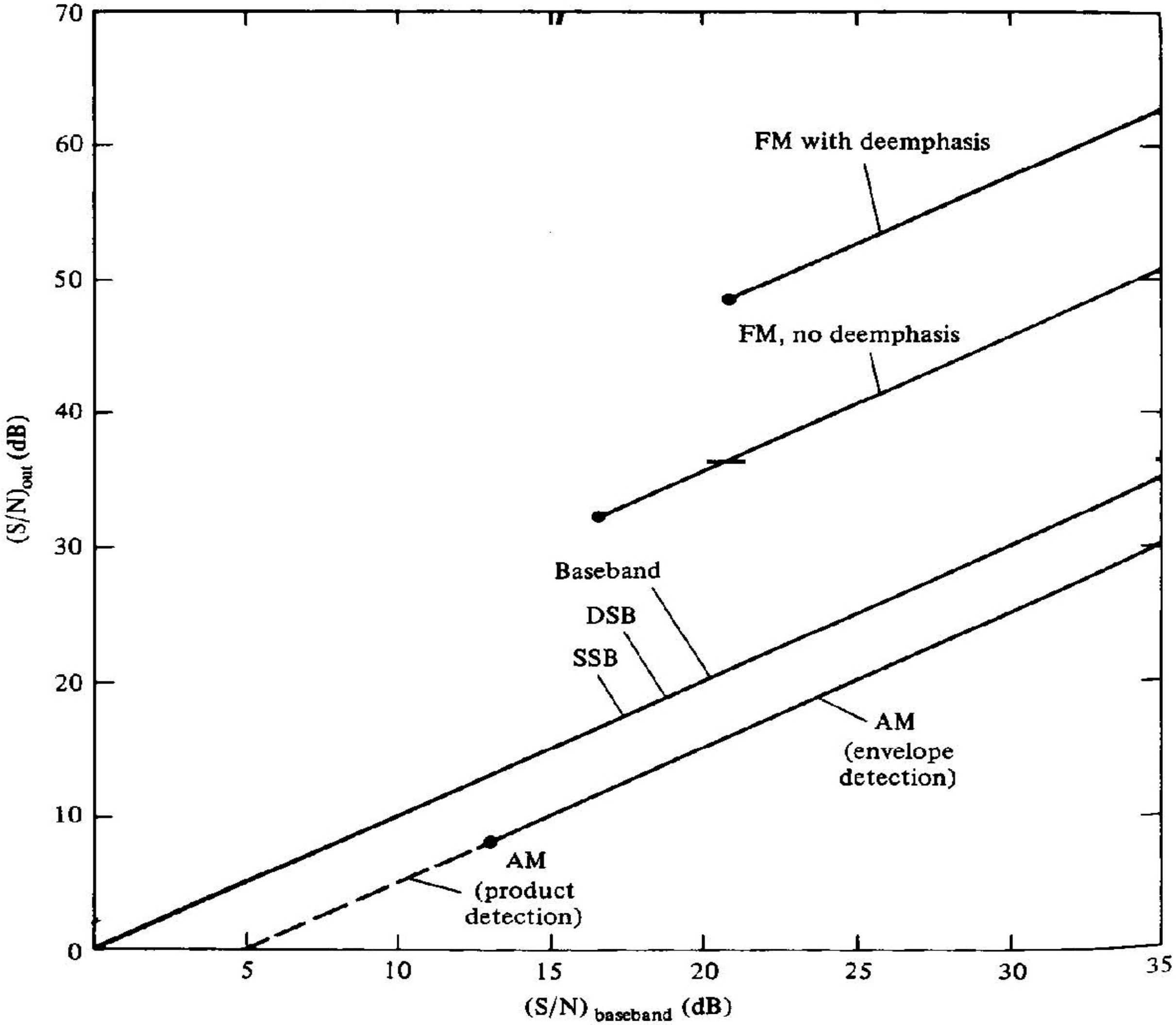
$$SNR_{DSBSC} = SNR_{baseband}$$

$$SNR_{AM} = \frac{1}{3} SNR_{baseband}$$

$$SNR_{FM} = \frac{3}{2}\beta^2 SNR_{baseband} = \frac{75}{2} SNR_{baseband}$$

where we used $\beta = 5$

Noise performance of analog communication systems



Conclusions

AM: The SNR performance is 4.8 dB worse than a baseband system, and the transmission bandwidth is $B_T = 2W$.

DSB: The SNR performance is identical to a baseband system, and the transmission bandwidth is $B_T = 2W$ (for SSB, the SNR performance is again identical, but the transmission bandwidth is only $B_T = W$).

FM: The SNR performance is 15.7 dB better than a baseband system, and the transmission bandwidth is $B_T = 2(\beta + 1)W = 12W$ (with pre- and de-emphasis the SNR performance is increased by about 13 dB with the same transmission bandwidth).