Communications II

Professor Kin K. Leung

EEE Departments

Imperial College London

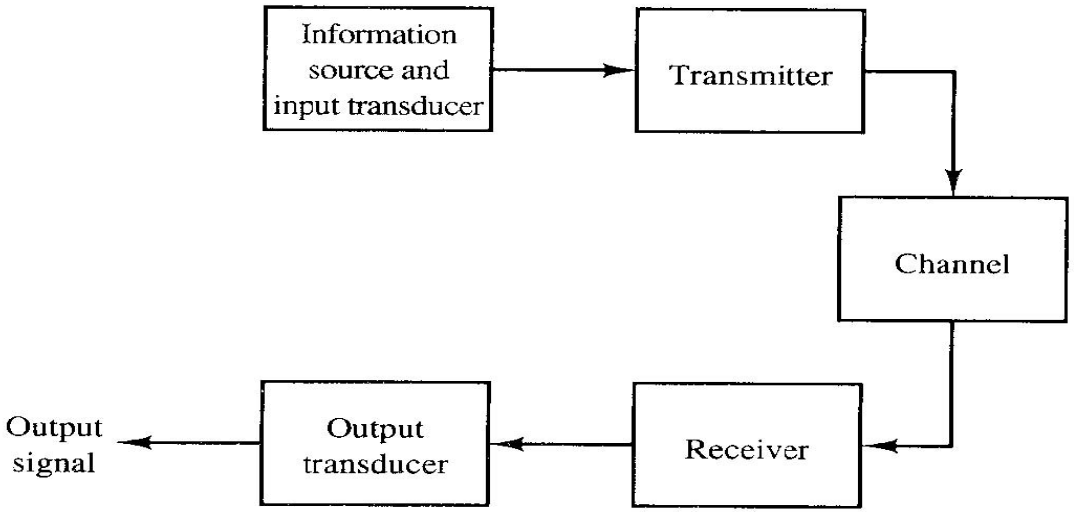
Acknowledge Contributions by Darren Ward, Maria Petrou
and Cong Ling

Lecture 1: Introduction and Review

- **What does communication mean?**

  Communication involves the transfer of information from one point to another.

- **What is a communication system?**



- **Communications I**
  - How do communication systems work?
  - About modulation, demodulation, signal analysis...
  - The main mathematical tool is the Fourier transform for deterministic signal analysis.

- **Communications II**
  - How do communication systems perform in the presence of noise?
  - The main mathematical tool is probability theory.
  - This is essential for a meaningful comparison of various communications systems.
  - About statistical aspects and noise; harder than Communications I.

Noise in Communications

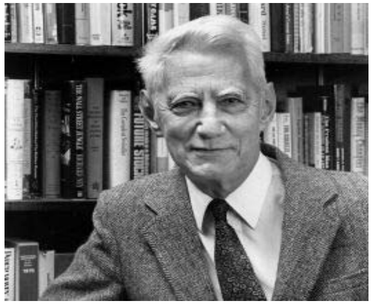
- Signal is contaminated by noise along the path.
- External noise: interference from nearby channels, human-made noise, natural noise...
- Internal noise: thermal noise, random emission... in electronic devices
- Noise is one of the basic factors that set limits on the rate of communications.
- A widely used metric is the signal-to-noise (SNR) ratio

$$\text{SNR} = \frac{\text{signal power}}{\text{noise power}}$$

Objectives of a communication system design

- The message is delivered both efficiently and reliably, subject to certain design constraints: power, bandwidth, and cost.
- Efficiency is usually measured by the number of messages sent in unit time and unit bandwidth.
- In digital communications, reliability is expressed in terms of probability of error.
- Is it possible to operate at zero error rate even though the channel is noisy?

Classic paper

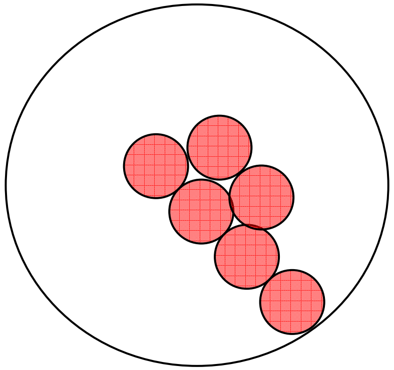


- C. E. Shannon, ”Communication in the presence of noise,” Proc. IRE, vol. 37, pp. 10-21, Jan. 1949.
  - The maximum rate of reliable transmission is calculated when the signal is perturbed by noise.
- An elaboration, from the engineering point of view, of his seminal paper
  - C. E. Shannon, ”A mathematical theory of communication,” Bell System Technical Journal, 1948.
  - Many concepts in ”Communication in the presence of noise” were fundamental and paved the way for future developments in communication theory.
  - Transmission of continuous-time sources over continuous-time channels. Using sampling theorem, he showed the famous capacity formula

$$C = W \log(1+SNR).$$

Engineering intuition via sphere packing

- A message of bandwidth W and duration T can be represented by $2TW$ samples. Signal power P, noise power N.
- Received signals (of power $P + N$) lie on the surface of a sphere with radius $\sqrt{2TW(P+N)}$
- Perturbation by noise would change a signal point into a cloud, which is a sphere with radius $\sqrt{2TWN}$
- Maximum number of signals $M = \frac{Vol_{ball}(\sqrt{2TW(P+N)})}{Vol_{ball}(\sqrt{2TWN})} = \left(\sqrt{\frac{P+N}{N}}\right)^{2TW}$
- Channel capacity $C = \frac{\log_2 M}{T} \le W \log_2 \frac{P+N}{N} = W \log_2(1+SNR)$



Comments about Shannon Capacity Formula

- Error-free transmission is possible as long as the rate R < C.
- However, Shannon didn't tell us explicitly how to construct the codes.
- Most practical modulation schemes operate far below the channel capacity.
- Powerful error-correction coding is required to approach the limit (beyond the scope of this course).
- Shannon's formula still provides a basis for the trade-off between the bandwidth and SNR, and for comparing different modulation schemes.

Learning outcomes

By the end of the course, you should be able to

- Compare the performance of various communications systems
- Describe a suitable model for noise in communications
- Determine the SNR performance of analog communications systems
- Determine the probability of error for digital communications systems
- Understand information theory and its significance in determining system performance

Syllabus

- Introduction and review (1 lecture = 2 hours)
- Noise and random processes (2 lectures)
- Effects of noise on analog communications (2 lectures)
- Effects of noise on digital communications (2 lectures)
- Information theory (1 lecture)
- …

Recommended textbooks

- S. Haykin, Communication Systems, 4th ed., Wiley, New York, 2001
- J.G. Proakis and M. Salehi, Communication Systems Engineering, Prentice-Hall, New Jersey, 1994
- B.P. Lathi, Modern Digital and Analog Communication Systems, 3rd ed., Oxford University Press, 1998
- L.W. Couch II, Digital and Analog Communication Systems, 6th ed., Prentice-Hall, New Jersey, 2001

Handouts

- Slides
- Problem sheets (solutions will be disclosed on my website later)
- All teaching materials can be found on my website at

  www.commsp.ee.ic.ac.uk/~kkleung/Communications2_2009

Signals

- What is a signal?
  - A signal is a single-valued function of time that conveys information.
- What is a deterministic Signal?
  - A deterministic signal is a completely specified function of time.
  - Example: $A\cos(2\pi f_c t + \theta)$ where A, f_c and θ are known constants.
- What is a random signal?
  - The value of a random (or stochastic) signal for each instant in time, for which the signal is defined, is the output of a random experiment.
  - Example: $A\cos(2\pi f_c t + \Theta)$ where Θ is a random variable.

Signals (Continued)

- What is an analog signal?
  - An analog signal takes continuous values.
  - It is usually a continuous function of time).
- What is a digital signal?
  - A digital signal takes discrete values.
  - It is usually a discrete function of time.

- How is the power of a signal defined?
  - Instantaneous power of a voltage or current signal: $p(t)=v(t)i(t)$
  - Ohm's law: $i(t) = v(t)/R$ where R is the resistance. Then:

$$p(t) = \frac{|v(t)|^2}{R} \quad or \quad p(t) = |i(t)|^2 R$$

- What is the normalized power?
  - Set R=1 in the above:

$$p(t) = |g(t)|^2$$

where $g(t)$ is either a voltage or current signal.

- What is the average power of a periodic signal?

$$P \equiv \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 dt$$

where T is the period of the signal.

- What is the average power of a non-periodic signal?

$$P \equiv \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 dt$$

- How is the energy of a signal defined?

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

- Question: What is the energy of a periodic signal?

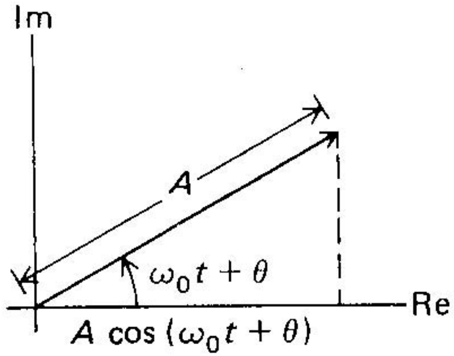
- What information is conveyed by the bandwidth of a signal?
  - The bandwidth of a signal provides a measure of the extent of significant spectral content of the signal for (positive) frequencies.

- What is the signal bandwidth for a band-limited signal?
  - It is the range of frequencies over which the signal has non-zero energy.

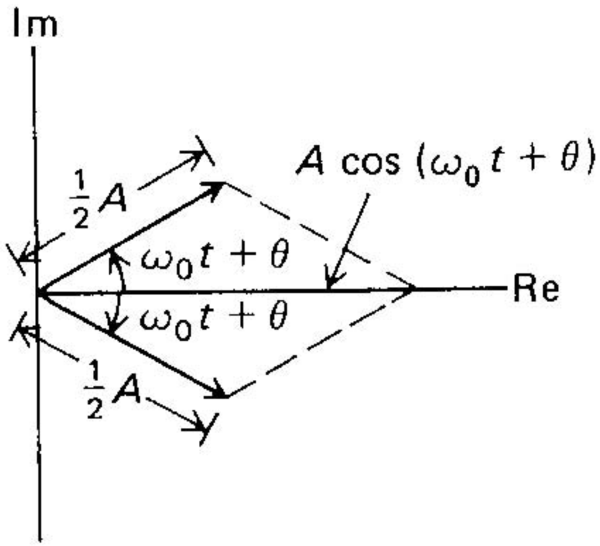
How is the signal bandwidth defined for a non-band limited signal?

- The null-to-null bandwidth is the range of frequencies between zeros in the magnitude spectrum.
- The 3-dB bandwidth is the range of frequencies where the magnitude spectrum falls no lower than $1/\sqrt{2}$ of its maximum value.
- The (noise) equivalent bandwidth is the width of a fictitious rectangular spectrum such that the power in the rectangular band is equal to the power associated with the actual spectrum over positive frequencies.

- What is a phasor?
  - It is a signal representation that allows us to determine the frequency content of a signal.
  - Example: $x(t) = A\cos(2\pi f_0 t + \theta)$
  - Remember: $e^{j\theta} = \cos\theta + j\sin\theta \quad \Rightarrow \quad \cos\theta = \mathrm{Re}\{e^{j\theta}\}$
  - Then: $x(t) = \mathrm{Re}\{Ae^{j(\theta + 2\pi f_0 t)}\}$
  - Alternatively, $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
  - Then:

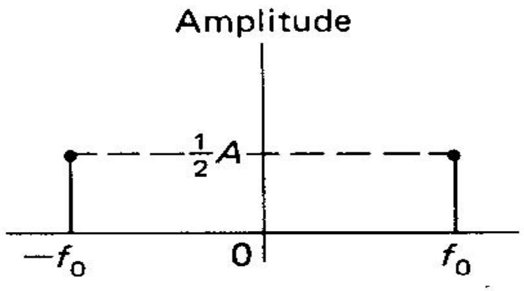


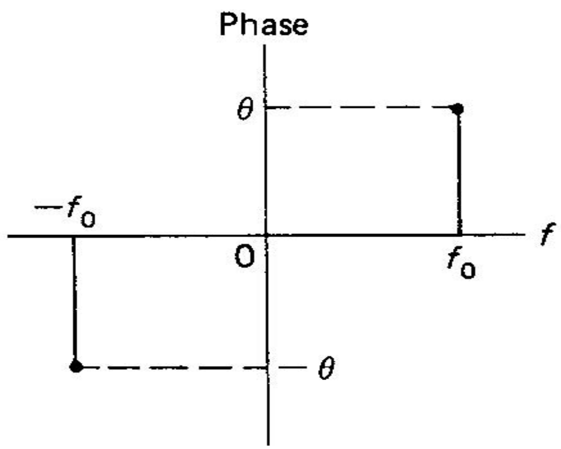
$$x(t) = A\cos(2\pi f_0 t + \theta) = \frac{A}{2}e^{j(\theta + 2\pi f_0 t)} + \frac{A}{2}e^{-j(\theta + 2\pi f_0 t)}$$



Two phasors rotating in opposite directions.
Each phasor has the same length $A/2$ but opposite phase.
At any time t, the signal $x(t)$ is given by the vector addition of these two rotating phasors.
The sum always falls on the real axis.

- This phasor consists only of components at $\pm f_0$, where $+f_0$ represents the anti-clockwise rotating vector, and $-f_0$ represents the clockwise rotating vector.
- A plot of the magnitude and phase of $\frac{A}{2}e^{\pm j\theta}$ at $\pm f_0$ gives sufficient information to characterize *x(t)*.





Complex envelope of bandpass signals

- The carrier frequency itself doesn't contain any information.
- Complex envelope $x_l(t) = A(t)e^{j\theta(t)}$
- The signal *x(t)* can be expressed as

$$x(t) = \mathrm{Re}\{x_l(t)e^{j2\pi f_0 t}\} = \mathrm{Re}\{A(t)e^{j2\pi f_0 t + \theta(t)}\} = A(t)\cos[2\pi f_0 t + \theta(t)]$$

- $x_l(t)$ is the equivalent lowpass signal.
- For linearly modulated signals, it is sometimes more convenient to deal with the equivalent low-pass signal.