A Closed-Form Expression For the Bandwidth of the Plenoptic Function Under Finite Field of View Constraints

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Motivation
Consider an unknown scene with a certain texture pasted to the surface. In order to render good quality new viewpoints of the scene, using Image-Based Rendering (IBR) techniques, it must be adequately sampled. Suppose the sampling device is a camera mounted to a robot.

Important Questions
• How should the camera travel relative to the scene?
• Where to sample along the path?
• Whether to zoom in or not?

Our Approach
• Approximate the scene with a planar facet model and bandlimited texture.
• Bandwidth analysis of the plenoptic function for such a model under the constraints of finite field of view and finite plane.
• Use analysis to determine a non-uniform distribution for the camera along a 1D path parallel to the scene.

The Plenoptic Function
Consider the 2D plenoptic function \( P(x, \theta) \), \( \psi(x, \theta) \), where:
- \( P(x, \theta) \) is the intensity of the light ray, travelling from the scene, at camera location \( x \) and pixel location \( \theta \).
- \( \psi(x, \theta) \) is the slant of the plane.
- \( \Delta \) is the pixel resolution.
- \( \Delta \) is the finite spacing leading to replicated spectra.
- Using analysis, non-uniform distribution for the camera along a 1D path parallel to the scene.

Slanted Plane Analysis
Applying both finite constraints to the plenoptic function of the slanted plane, lead to windowing in the EPI domain, shown below:

1. Finite Camera Spacing leading to replicated spectra.
2. Finite Field of View constraint on the plane.

Finite Camera Spacing leading to replicated spectra.
The plenoptic spectrum is derived by taking the Fourier transform of the finite EPI in Fig (14), mathematically this is shown by the integral:

\[
P_{\text{finite}}(\omega_0, \omega_v) = \int_{-\Delta}^{\Delta} \int_{-\Delta}^{\Delta} P(x, \theta) e^{-j \omega_0 x - j \omega_v \theta} dx d\theta
\]

The solution to this integral, when \( P(x, \theta) = e^{j \omega_0 x + j \omega_v \theta} \), is:

\[
|P_{\text{finite}}(\omega_0, \omega_v)| = |P(\omega_0, \omega_v)| \left( \frac{\sin \left( \frac{2\pi \omega_0 \Delta}{c} \right)}{\frac{2\pi \omega_0 \Delta}{c}} \right) \left( \frac{\sin \left( \frac{2\pi \omega_v \Delta}{c} \right)}{\frac{2\pi \omega_v \Delta}{c}} \right)
\]

The plenoptic spectrum is defined as:

\[
\Omega(\omega_0, \omega_v) = \left| P_{\text{finite}}(\omega_0, \omega_v) \right|
\]

The Essential Bandwidth
The plenoptic spectrum of the slanted plane is band-unlimited in both \( \omega_0 \) and \( \omega_v \), hence we cannot define an exact bandwidth region. However, we can define the Essential Bandwidth (E).

A finite region that contains 90% of the signal’s energy.

Essential Bandwidth Model
The essential bandwidth is parameterized using four parameters:
- \( \Omega_0 \) is the maximum value in \( \omega_0 \).
- \( \Omega_v \) is the maximum value in \( \omega_v \).
- \( \Omega_{vl} \) is the slant of region.
- \( A \) is the width of region in \( \omega_v \).

Applied to the slanted plane:
- Restrict the degrees of freedom so \( \Omega_0 \) is the only free parameter.
- \( \Omega_{vl} \) is the slant of region.
- \( A \) is the width of region in \( \omega_v \).

Approximate the decay along each line outside the region and equate the total energy to 10%

Fronto-Parallel Plane
A special case when \( \phi = 0 \), a finite fronto-parallel plane (FPP) at depth \( z \). The plenoptic spectrum is in:

\[
\Omega(\omega_0, \omega_v) = \left| P_{\text{finite}}(\omega_0, \omega_v) \right|
\]

where \( P(\omega_0, \omega_v) \) is the Fourier transform of \( g(x, z) \), bandlimited to \( \omega_v \).

- Finite Plane Width: \( \Omega_{vl} \) : Cutoff with Sinc function in spectral domain along the axis \( \omega_v \).
- Finite FOV: \( \Omega_0 \) : Cutoff with Sinc function in spectral domain parallel to the \( \omega_v \) axis.

Sampling and Reconstruction
The essential bandwidth of a Sinc function is the width of its main lobe, thus the essential bandwidth parameters for an FPP are:

- \( \Omega_{vl} = \omega_v \sqrt{\frac{A}{\Omega_0}} \)
- \( \Omega_0 = \omega_0 \sqrt{\frac{A}{\Omega_v}} \)
- \( \omega_v = \sqrt{\frac{A}{\Omega_v}} \)

Fig 10: Sampled EPI

The solution to this integral, when \( P(x, \theta) = e^{j \omega_0 x + j \omega_v \theta} \), is:

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\]

Simulation Results
A scene consisting of two identical FPPs at different depths:

- Uniform Sampling: Sample rate dictated by nearest plane, thus other plane is oversampled.
- Nonuniform: Sample rate varies depending on the plane being sampled.

References