IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE/ISE PART II: MEng, BEng and ACGI

COMMUNICATIONS 2

Wednesday, 15 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

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                          Second Marker(s) :   J.A. Barria

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EXAM QUESTIONS

1.   a)   i)   Explain the terms "noise", "external noise", and "internal noise". [3]

           ii)   Explain the terms "white noise", "Gaussian noise", and "additive white Gaussian noise". [3]

           iii)   Consider bandpass noise $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ which has the power spectral density shown in Fig. 1.1. Draw the power spectral density (PSD) of baseband noise $n_c(t)$ or $n_s(t)$ if the center frequency is chosen as: [4]

- $f_c = 7$ Hz
- $f_c = 10$ Hz
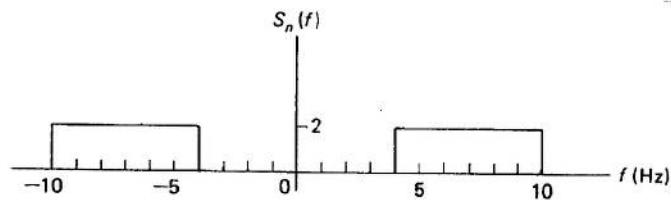
Figure 1.1 PSD of bandpass noise.

   b)   i)   Explain the advantages and disadvantages of synchronous detection and envelope detection respectively for standard amplitude modulation (AM). [5]

           ii)   With help of a diagram, explain the operation of coherent detection for frequency shift keying (FSK). [5]

   c)   i)   Name two primary resources in communications. [2]

           ii)   Write down the expression of the capacity of a Gaussian channel with bandwidth W. [2]

           iii)   What does the channel coding theorem say about the relation between transmission rate R and channel capacity C? [3]

           iv)   Consider a telephone line channel. If the signal to noise ratio (SNR) is 20 dB and the bandwidth available is 4 kHz, calculate the corresponding channel capacity. [3]

d) Consider an information source generating the random variable X with probability distribution

x_k	x_1	x_2	x_3	x_4
$P(X = x_k)$	0.4	0.2	0.25	0.15

i) Calculate the entropy of this source. [3]

ii) Construct a binary Huffman code. [5]

iii) Compute the average codeword length. [2]

2. a) Figure 2.1 shows the diagram of the FM receiver. The bandpass filter has band-width B_T, while the baseband low-pass filter has bandwith W. Let the FM signal be $s(t) = A\cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau]$ and assume the bandpass noise $w(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ has single-sided power spectral density N_0.
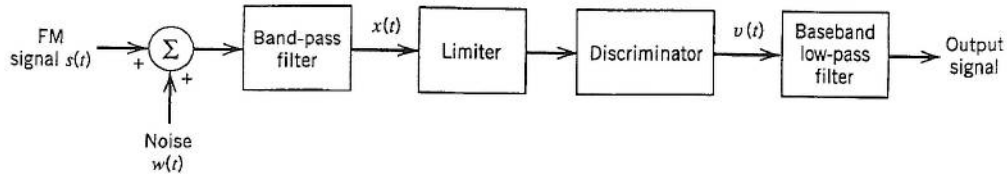

Figure 2.1 Block diagram of the FM receiver.

   i) Explain the function of each of the blocks in Figure 2.1.   [5]

   ii) Given the noise at the output of the discriminator

$$f_i(t) = \frac{1}{2\pi A}\frac{dn_s(t)}{dt},$$

   derive an expression for its power spectral density.   [5]

   iii) Sketch power spectral densities of $n_s(t)$, $f_i(t)$, and the noise at the output of the lowpass filter.   [5]

   b) Consider pre-emphasis and de-emphasis.

   i) Show that in order to achieve flat noise power spectral density at the output of the FM receiver, the ideal de-emphasis filter has a transfer function $H_{de}(f) = 1/f$ within the message bandwidth.   [5]

   ii) Discuss why an FM transmitter with a corresponding pre-emphasis filter $H_{pre}(f) = f$ is essentially phase modulation.   [5]

   c) The signal-to-noise ratio (SNR) improvement factor is defined as

$$I = \frac{\text{Noise power without pre/de-emphasis}}{\text{Noise power with pre/de-emphasis}}.$$

   Derive an expression of the improvement factor I for the scaled ideal de-emphasis filter $H_{de}(f) = f_0/f$, then compute the gain in dB for the parameter $W = 15\,\text{kHz}$ and $f_0 = 3\,\text{kHz}$.   [5]

3. a) A uniform quantizer for PCM has 2^n levels. The input signal is

$$m(t) = [A + A\cos(\omega_1 t)]\cos(\omega_2 t)$$

where $\omega_1 \neq \pm\omega_2$. Assume the dynamic range of the quantizer matches that of the input signal.

  i) Work out the signal power. [3]

  ii) Write down the probability density function of the quantization noise and the quantization noise power. [3]

  iii) Work out the SNR in dB at the output of the quantizer. [3]

  iv) Determine the minimum value of n such that the output SNR is no less than 60 dB. [3]

  v) What can be done to increase the output SNR? [3]

b) An (n, k) linear block code has the following parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad (3.1)$$

  i) What are the values of n and k? [2]

  ii) Give a systematic generator matrix G of this code. [3]

  iii) Compute the syndrome table for a single error. [5]

  iv) The vector $y = [1000001]$ is received. Find the syndrome and hence the most likely data bits. [5]

ANSWERS

EE2-4 COMMUNICATIONS 2

B — Bookwork
E — New examples
A — New applications

1. a) i) Noise refers to unwanted waves that disturb communications. [3B]

External noise: interference from nearby channels, human-made noise, natural noise for external.
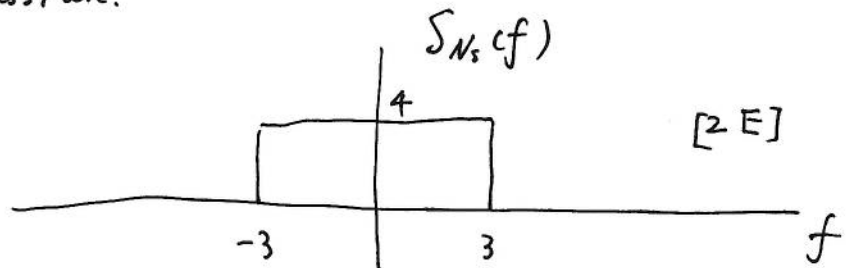
Internal noise: noise from within electronic devices, such as thermal noise.
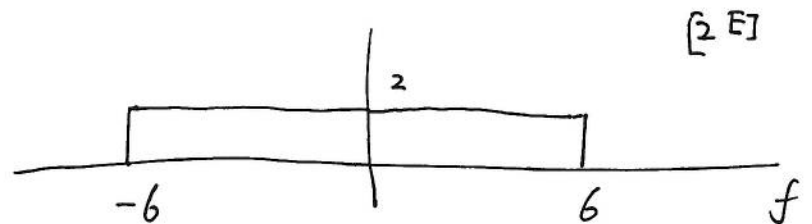
ii). White noise: the PSD is constant

Gaussian noise: the PDF is Gaussian. [3 B]

additive white Gaussian noise: noise is additive, white, and Gaussian.

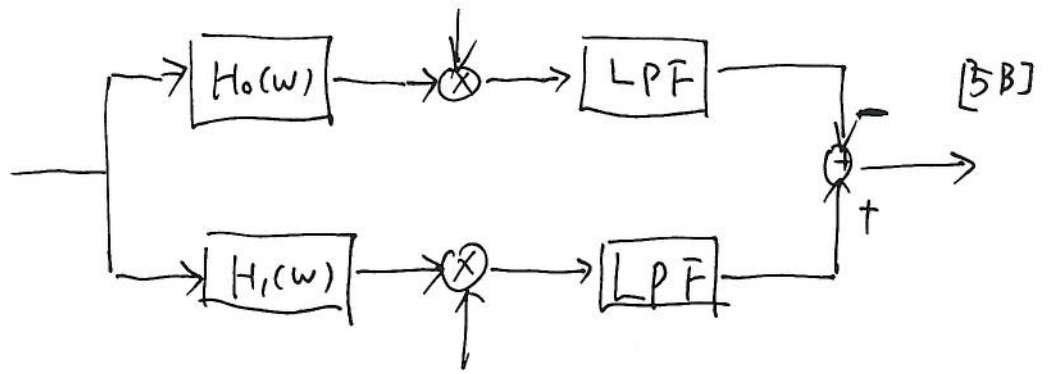iii). $f_c = 7 Hz$

[2 E]

$f_c = 10 Hz$

[2 E]

b) i) Synchronous detection: good noise performance, complex sync circuit. [2 B]

envelope detection: simple circuit, good performance at high SNR, but suffers from threshold effect. [3B]

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ii)

[5 B]

The BPFs $H_0(w)$ and $H_1(w)$ have central frequency f_0 and f_1, respectively.

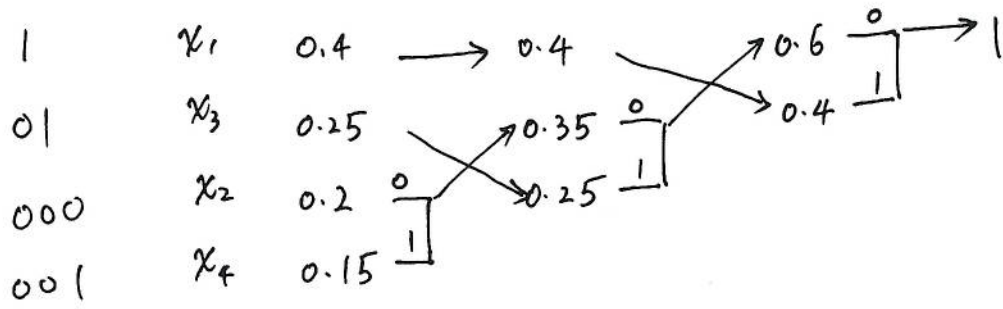c) i) power, bandwidth [2 B]

ii)
$$C = w \log_2(1 + SNR)$$
[2 B]

iii) Channel coding theorem: As long as $R \leq C$, we can achieve reliable communication over a noisy channel (i.e., with arbitrarily small probability of error); conversely, it is impossible to transmit messages without error if $R > C$. [3 B]

iv) $20 dB \Rightarrow SNR = 100$ [3 E]

$$C = 4k \times \log_2(1 + 100) = 26.6 \ kbps$$

d) i) $H(X) = -\sum p(x_k) \log p(x_k)$ [3 E]

$= -0.4 \times \log 0.4 - 0.2 \times \log 0.2 - 0.25 \times \log 0.25 - 0.15 \times \log 0.15$

$= 1.9$

ii)

1	x_1	0.4	$\longrightarrow$ 0.4	$\nearrow$ 0.6 $\overset{0}{}$ $\rightarrow$ 1	
01	x_3	0.25	$\nearrow$ 0.35 $\overset{0}{}$	$\searrow$ 0.4 $\overset{1}{}$	[5 E]
000	x_2	0.2 $\overset{0}{}$	$\searrow$ 0.25 $\overset{1}{}$		
001	x_4	0.15 $\overset{1}{}$			

iii) $L = 1 \times 0.4 + 2 \times 0.25 + 3 \times 0.35$ [2 E]

$\qquad = 1.95$

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2. a) i) The bandpass filter removes out-of-band noise. [5B]

   The Limiter results in a constant envelope.

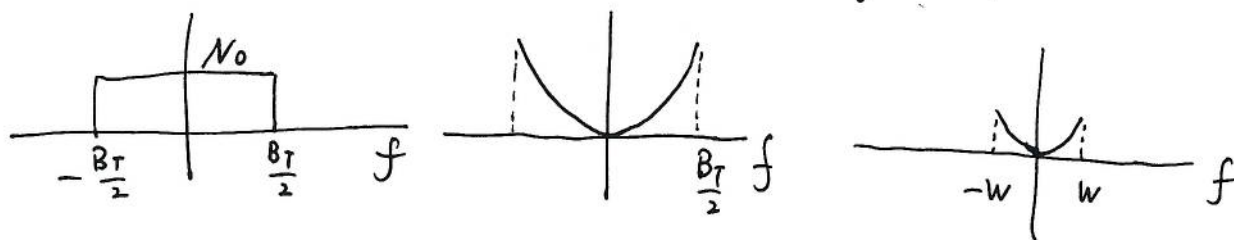   The discriminator outputs the deviation in the instantaneous frequency, i.e., it recover the message signal.

   Low-pass filter: has a bandwidth w. It passer the message and removes out-of-band noise.

   ii) This can be viewed as a linear system with transfer function $\frac{1}{2\pi A} j 2\pi f = j\frac{f}{A}$. [3B]

   Therefore, PSD for $f_i(t)$ is

   $$\frac{f^2}{A^2} N_0, \qquad |f| \le \frac{B_T}{2}.$$

   iii) [5B]

   b) i) It's clear that to equalize the PSD $\frac{f^2}{A^2} N_0$, we need a de-emphasis filter $H_{de}(f) = \frac{1}{f}$. [5 E]

   ii) $H_{pre}(f) = f$ is a differentiation circuit. Thus the signal becomes $s(t) = A \cos[2\pi f_c t + k_f m(t)]$, which is PM. [5 A]

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c) Without deemphasis, the noise power is

$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} N_0 \, df = \frac{2}{3} \frac{W^3 N_0}{A^2}.$$ [1 B]

with deemphasis, the noise power is

$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} N_0 \cdot \frac{f_0^2}{f^2} \, df$$

$$= \int_{-W}^{W} \frac{f_0^2 N_0}{A^2} \, df$$ [2 A]

$$= \frac{2 W f_0^2 N_0}{A^2}.$$

Then,

$$I = \frac{\dfrac{2 W^3 N_0}{3 A^2}}{\dfrac{2 W f_0^2 N_0}{A^2}} = \frac{W^2}{2 f_0^2}$$ [1 A]

When $W = 15 \text{ kHz}$, $f_0 = 2.1 \text{ kHz}$

$$I = \frac{15^2}{2 \cdot 2.1^2} = 25.5.$$ [1 A]

In dB,

$$I \, (dB) = 14 \, dB.$$

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3. a) i) $\quad P_S = \frac{A^2}{2} + \frac{A^2}{4} = \frac{3}{4} A^2$ [3 E]

ii) The quantization noise has a uniform PDF

$$f(q) = \frac{1}{\Delta}, \qquad |q| < \frac{\Delta}{2}$$

Power: $\quad P_N = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \cdot \frac{1}{\Delta} dx$ [3 E]

$$= \frac{2}{\Delta} \cdot \frac{1}{3} \left(\frac{\Delta}{2}\right)^3$$

$$= \frac{\Delta^2}{12}$$

The signal range is $[-2A, 2A]$.

$$\Delta = \frac{2A}{2^n} = \frac{A}{2^{n-2}}$$

iii) $\quad SNR = \frac{P_S}{P_N} = \frac{\frac{3}{4} A^2}{\frac{(A/2^{n-2})^2}{12}} = \frac{9}{16} \cdot 2^{2n}$ [3 E]

$$SNR(dB) = 6n - 2.5 \; dB$$

iv) $\quad n = 11$ so that $SNR \geq 60 dB$ [3 E]

v) $\quad$ increase n. [3 B]

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b) i) $n = 7$ $k = 4$ [2 E]

ii)   [3 E]

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

iii) $S = e \cdot H^T$ [5 A]

Syndrome table

S	e
1 1 1	1 0 0 0 0 0 0
1 1 0	0 1 0 0 0 0 0
1 0 1	0 0 1 0 0 0 0
0 1 1	0 0 0 1 0 0 0
1 0 0	0 0 0 0 1 0 0
0 1 0	0 0 0 0 0 1 0
0 0 1	0 0 0 0 0 0 1

iv)

$S = y \cdot H^T$ [5 A]

$= (1 1 0)$

$\Rightarrow e = (0 1 0 0 0 0 0)$

$\Rightarrow$ the second bit is wrong

$\Rightarrow$ sent code word $x = [1 1 0 0 0 0 1]$

$\Rightarrow$ data bits $= [1 1 0 0]$

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