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E2.4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE/ISE PART II: MEng, BEng and ACGI

COMMUNICATIONS 2

Friday, 11 June 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K.K. Leung, K.K. Leung
	Second Marker(s) :	J.A. Barria, J.A. Barria

Special Instructions for Invigilator: **None**

Information for Students: **None**

1. This is a compulsory question.

a. Noise.

i. What is white noise? [2]

ii. Give the power spectral density of white noise, covering both positive and negative frequencies. [2]

iii. Can white noise be physically realizable? Why? [2]

iv. Give the autocorrelation function for white noise. [3]

v. Based on the result in part iv, what can be said about any two different samples of white noise? [2]

b. Shannon capacity.

Consider a communication channel where W, S and $N_o/2$ denote the channel bandwidth, the received signal power and the power spectral density of white noise, respectively.

i. What is the Shannon capacity for the channel? [2]

ii. Now assume that the transmission power is doubled, while other parameters remain unchanged. What is the channel capacity? [2]

iii. Suppose that the channel bandwidth is now $2W$ and that the received signal power remains to be S. What is the new channel capacity? [3]

c. Frequency modulation.

Consider a frequency modulation (FM) transmission with additive white noise.

i. How does the power spectral density of the noise at the detector output depend on frequency? [2]

ii. What are the pre-emphasis and de-emphasis for the FM system? Why and how can they improve the output signal-to-noise ratio (SNR)? [5]

iii. Draw a block diagram to show an FM system using pre-emphasis and de-emphasis. [3]

d. Digital communications.

Consider a digital communication system where an analogue signal with a bandwidth of W Hz is sampled and converted into pulse-code-modulation (PCM) words. Each PCM word has m bits. Let V_{pp} and L be the peak-to-peak voltage (dynamic range) of the analog signal and the number of quantization levels, respectively. Further, we use Δ and e to denote the uniform interval (gap) between two adjacent quantization levels and the quantization error, respectively. The system is designed such that the maximum quantization error will not exceed a fraction, p, of the peak-to-peak voltage.

i. Express the number of quantization levels, L, in terms of m. [2]

ii. Express the maximum quantization error (i.e., the maximum value of e) in terms of Δ. [2]

iii. Derive the number of bits per PCM word, m, in terms of p. [4]

iv. What is the minimum transmission rate such that the signal can be recovered at the receiving end? Express your result in terms of p and W. [4]

2. Noise effect on amplitude modulation (AM) with double-sideband suppressed carrier (DSB-SC).

Consider a signal $s(t)$ of the AM with DSB-SC using the following notation:

A is the amplitude of the carrier,
f_c is the carrier frequency,
$m(t)$ is the message (information) signal,
$n(t)$ is the noise,
$n_c(t)$ is the in-phase component of the noise,
$n_s(t)$ is the quadrature phase component of the noise, and
N_o is the power spectral density for the noise in the baseband transmission.

a. Provide an expression for the AM-DSB-SC signal $s(t)$. [4]

b. Name a detection method suitable for recovering the message signal from $s(t)$. [2]

c. Draw a block diagram to depict the AM-DSB-SC transmission, the additive white noise channel and the receiver structure using the detection method identified in part b. [3]

d. Give an expression for the pre-detection signal as a function of the AM-DSB-SC signal, $n_c(t)$, and $n_s(t)$ of the noise. [4]

e. Derive an expression for the detector output for the AM-DSB-SC? What can be said from the expression about the relationship between the message signal and the noise? [5]

f. Let the signal power at the receiver output be denoted by $P = E\{m^2(t)\}$. Based on the expression obtained in part e, derive the signal-to-noise ratio (SNR) at the receiver output for the AM-DSB-SC. Explain your result. [4]

g. Using the result in part a, determine the transmission power of the AM-DSB-SC signal. [2]

h. Derive a relationship between the SNR of the receiver output for the AM-DSB-SC and that for the baseband transmission with the same transmitted power. [4]

i. Consider the figure of merit as the ratio of the SNR at the receiver output for a modulation scheme to that of the baseband transmission. Using this figure of merit, is the AM-DSB-SC transmission more efficient than the regular AM transmission? [2]

3. Noise representation.

Consider a narrowband noise $n(t)$ of bandwidth $2W$ centered on frequency f_c. The narrowband noise can be defined in terms of the in-phase and quadrature components, as well as in terms of the envelope and phase.

a. Give an expression of $n(t)$ in terms of the in-phase component $n_c(t)$, the quadrature component $n_s(t)$, and f_c. [4]

b. The narrowband noise can also be defined as $n(t) = r(t)\cos[2\pi f_c t + \Theta(t)]$ where $r(t)$ and $\Theta(t)$ denotes the envelope and phase, respectively. Express $r(t)$ and $\Theta(t)$ in terms of $n_c(t)$ and $n_s(t)$. [4]

c. Let N_c and N_s denote the random variables obtained by observing (at some fixed time) the random processes represented by the sample functions $n_c(t)$ and $n_s(t)$, respectively. Note that N_c and N_s are independent Gaussian random variables of zero mean and the same variance σ^2. Obtain the joint probability density function (pdf) $f_{N_c,N_s}(x, y)$ for N_c and N_s. [6]

d. Let R and Θ be the random variables obtained by observing (at some time t) the random process represented by the envelope $r(t)$ and phase $\Theta(t)$, respectively. Using the facts that $x = r\cos\theta$, $y = r\sin\theta$ and $dxdy = r\,dr\,d\theta$, and the joint pdf for N_c and N_s obtained in part c, derive the joint pdf $f_{R,\Theta}(r, \theta)$ for R and Θ. [6]

e. Is the joint pdf of R and Θ obtained in part d independent of the angle Θ ? What is the physical interpretation? [2]

f. Obtain the marginal pdf for the angle Θ. [4]

g. Obtain the marginal pdf for the envelope R. What is the common name of the pdf? [4]

4. a. Information theory.

A discrete source produces the symbols A and B with probabilities of ¾ and ¼, respectively. The symbols are grouped into blocks of two and encoded as follows:

Grouped symbols	Binary code
AA	1
AB	01
BA	001
BB	000

i. Compute the entropy of the new symbol blocks. [3]
ii. Obtain the average code length of the coding scheme. [2]
iii. Is the coding scheme optimal? Why? [2]
iv. Use the Huffman encoding procedure to generate the code words for the symbol blocks. [4]
v. Is the set of code words generated by the Huffman encoding algorithm in part iv unique? (That is, is it possible for the Huffman encoding algorithm to generate a different set of code words in part iv?) [2]
vi. Is the coding scheme obtained in part iv more efficient than the original one given in the question? Why? [3]

b. Error control code.

i. What is the simple parity bit? What is the error control capability for the simple parity bit? [2]
ii. What is the Hamming distance between the following code words: 01110011 and 10001011? [2]
iii. What is the relationship between the number of correctable errors and the minimum of Hamming distance between any two code words? Why? [4]

c. Baseband communications.

Consider a binary source alphabet where a symbol 0 is represented by 0 volt and a symbol 1 is represented by 1 volt. Assume these symbols are transmitted over a baseband channel having uniformly distributed noise with a probability density function:

$$p(x) = \begin{cases} \frac{1}{2} & for\ -1 \le x \le 1 \\ 0 & otherwise. \end{cases}$$

Assume that the single decision threshold T is in the range of 0 and 1 volt. If the symbols 0 and 1 are sent with probabilities p_0 and $1 - p_0$ respectively, derive an expression for the probability of error. [6]

Model Answers E2.4

2010 Communications 2

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1. a. noise

i. White noise is the noise that has uniform components at all frequencies

ii. $S(f) = \frac{N_0}{2}$ for all frequencies f

$-\infty \le f \le \infty$

iii. Physically, white noise cannot be realizable because it represents infinite power.

iv. $R(\tau) = \frac{N_0}{2} \delta(\tau)$

v. Any two different samples of white noise, no matter how closely together in time they are taken, are uncorrelated.

b. i Noise power $N = 2W \cdot \frac{N_0}{2} = W N_0$

$$C = W \log_2 (1 + SNR)$$

$$\Rightarrow \quad C = W \log_2 \left(1 + \frac{S}{N_0 W}\right)$$

ii. New capacity $C = W \log_2 \left(1 + \frac{2S}{N_0 W}\right)$

iii. New capacity

$$C = 2W \log_2 \left(1 + \frac{S}{2 N_0 W}\right)$$

1\. C. Frequency modulation

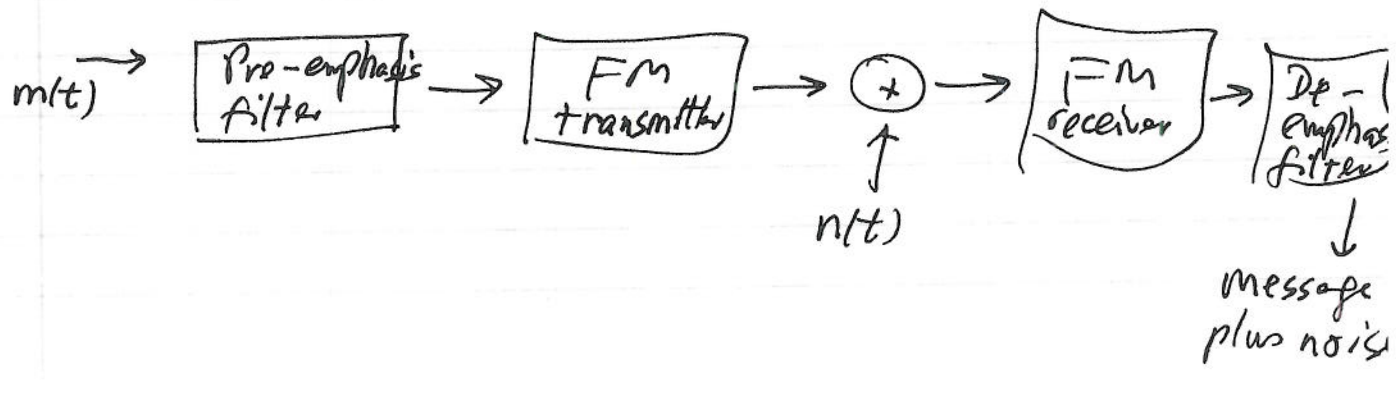
i. PSD of the noise at the detector output is directly proportional to the square of frequency.

ii. Pre-emphasis : to artificially emphasize (or amplify) the high frequency components of the message signal before modulation, and hence, before noise is introduced.

De-emphasis : to de-emphasize (or attenuate) the high frequency components at the receiver, and restore the original PSD of the message signal.

How it works: Since noise at the detector output is directly proportional to the square of frequency, de-emphasis will suppress the noise ~~efficiently~~ effectively. Since the pre-emphasis has already pre-amplified the high frequency components of the message signal, de-emphasis will have no impact on the message signal, but just suppresses noise.

iii)



1. d. Digital communications.

i. $L = 2^m$

ii. $e_{max} = \frac{\Delta}{2}$

iii. $e_{max} = \frac{\Delta}{2} = \frac{1}{2} \cdot \frac{V_{pp}}{L}$

Since it is required that $e_{max} \leq p\, V_{pp}$

then $\frac{1}{2} \cdot \frac{V_{pp}}{L} \leq p\, V_{pp}$

$$\Rightarrow \quad L \geq \frac{1}{2p}$$

As $L = 2^m$, $L \geq \frac{1}{2p} \Rightarrow 2^m \geq \frac{1}{2p}$

Therefore, $m \geq \log_2\left(\frac{1}{2p}\right)$ bits

iv. By the Nyquist sampling criterion, the minimum sampling rate is $2W$. Since each PCM word has m bits,

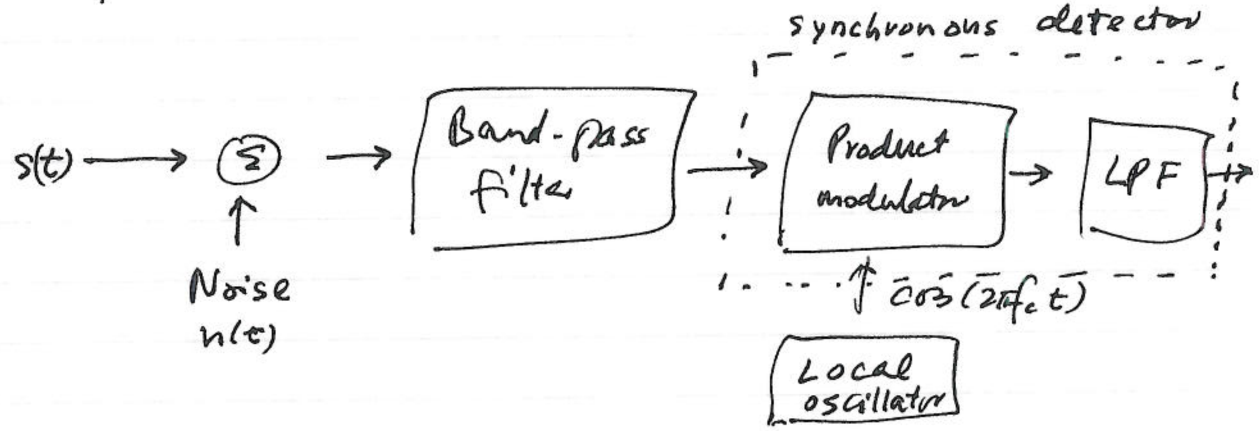
the minimum transmission rate is $2mW$ bits/sec

i.e, $$R \geq 2\underbrace{\left\lceil \log_2\left(\frac{1}{2p}\right)\right\rceil}_{\text{the ceiling operator}} W \quad \text{bits/sec}$$

2\. a. $s(t) = A\, m(t) \cos(2\pi f_c t)$

b. Synchronous detection or coherent detection

c.



d. $x(t) = s(t) + n(t)$

$$= A\, m(t) \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

$$x(t) = \left[A\, m(t) + n_c(t)\right] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

e.

$$v(t) = x(t) \cos(2\pi f_c t)$$

$$= \left[A\, m(t) + n_c(t)\right] \cos^2(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$= \frac{1}{2}\left[A\, m(t) + n_c(t)\right] + \frac{1}{2}\left[A\, m(t) + n_c(t)\right] \cdot \cos(4\pi f_c t) - \frac{1}{2} n_s(t) \sin(4\pi f_c t)$$

After the LPF,

$$y(t) = \frac{1}{2} A\, m(t) + \frac{1}{2} n_c(t)$$

The output message signal is unmutilated and the noise component is additive to the message, disregard of the input SNR.

2.f. Signal power at the receiver output

$$P_s = E\{\tfrac{1}{4}A^2 m^2(t)\} = \tfrac{1}{4}A^2 E\{m^2(t)\} = \tfrac{1}{4}A^2 P$$

From result in part e,

$$y(t) = \tfrac{1}{2} A m(t) + \tfrac{1}{2} n_c(t)$$

For the DSB-SC modulation, the band-pass filter has a bandwidth of $2W$ in order to accomodate the upper and lower sidebands of the modulated signal $s(t)$. Thus, the avg. power of the filtered noise $n(t)$ is $2WN_0$.

Since the avg. power of the (low-pass) in-phase noise component $n_c(t)$ is the same as that of the (band-pass) filtered noise $n(t)$. And since the expression of $y(t)$ above has a factor of $1/2$ for $n_c(t)$, the avg power of noise at the receiver output is $\left(\tfrac{1}{2}\right)^2 \cdot 2WN_0 = \tfrac{1}{2} WN_0$

Combining this with the signal power,

$$SNR_0 = \frac{A^2P/4}{WN_0/2} = \frac{A^2P}{2WN_0}$$

2. g. $s(t) = A\, m(t) \cos(2\pi f_c t)$

The transmitted power is

$$P_T = E\{A^2 m^2(t) \cos^2(2\pi f_c t)\}$$

$$\Rightarrow P_T = \frac{A^2 P}{2}$$

h. For base band transmission.

The noise power is WN_0

Therefore, using the result in part g,

the $SNR_{baseband} = \dfrac{A^2 P}{2 N_0 W}$.

Comparing baseband and DSB-SC, we see that

$$SNR_{baseband} = SNR_{DSB-SC}$$

i. Yes, DSB-SC is more efficient than the regular AM transmission according to the figure of merit.

For DSB-SC, the figure is 1

For AM, the figure is $\dfrac{P}{A^2+P} < 1$!

3 a. $$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

where $$n_c(t) = \sum_k a_k \cos(2\pi(f_k - f_c)t + \theta_k)$$

$$n_s(t) = \sum_k a_k \sin(2\pi(f_k - f_c)t + \theta_k)$$

b. $$n(t) = r(t)\cos(2\pi f_c t + \theta(t))$$

where $$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

c. $$f_{N_c}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$f_{N_s}(y) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

Since N_c & N_s are independent, the joint pdf is

$$f_{N_c, N_s}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

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3 d. $$x = r\cos\theta$$

$$y = r\sin\theta$$

$$dx\,dy = r\,dr\,d\theta$$

Subs. the above into

$$f_{N_c,N_s}(x,y)\,dx\,dy = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)dx\,dy$$

we get

$$f_{R,\Theta}(r,\theta)\,dr\,d\theta = \frac{1}{2\pi\sigma^2}\cdot\exp\left(-\frac{r^2}{2\sigma^2}\right)r\,dr\,d\theta$$

$$= \frac{r}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)dr\,d\theta$$

$$\Rightarrow f_{R,\Theta}(r,\theta) = \frac{r^2}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

e. yes, $f_{R,\Theta}(r,\theta)$ is independant of θ. That means, ~~$f_{R,\Theta}(r,\theta)$~~ θ is uniformly distributed & independent of R.

f. $$f_\Theta(\theta) = \begin{cases} 1/2\pi & \text{for } 0 \le \theta \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

g. Since R and Θ are independent,

$$f_{R,\Theta}(r,\theta) = f_R(r) \cdot f_\Theta(\theta)$$

Therefore, $f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$

which is commonly known as the Rayleigh distribution.

4. a. i. For the group of 2 symbols, the probs are:

$$AA: \frac{3}{4}\cdot\frac{3}{4} = 0.5625$$

$$AB: \frac{3}{4}\cdot\frac{1}{4} = 0.1875$$

$$BA: \frac{1}{4}\cdot\frac{3}{4} = 0.1875$$

$$BB: \frac{1}{4}\cdot\frac{1}{4} = 0.0625$$

The entropy is

$$H = -0.5625\log_2(0.5625) - 2\times 0.1875\log_2(0.1875) - 0.0625\log_2(0.0625)$$

$$= 1.62545 \quad \text{bits/2-symbol block}$$

or 0.8123 bits/symbol

ii.

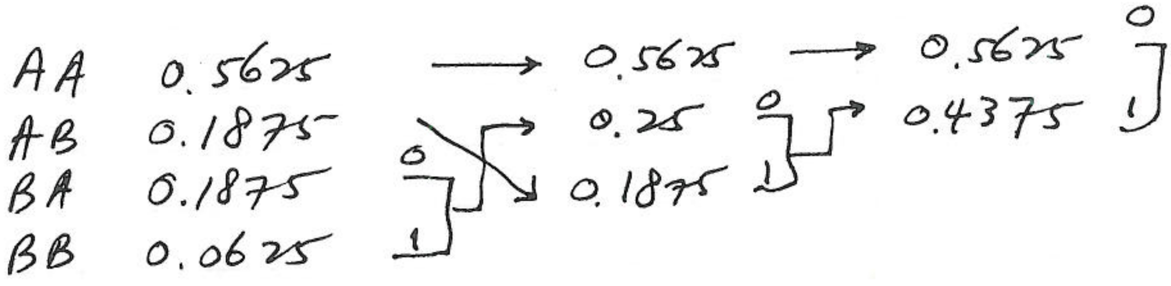
$$\bar{L} = 1\times 0.5625 + 2\times 0.1875 + 3\times 0.1875 + 3\times 0.0625$$

$$\bar{L} = 1.6875 \text{ bits/2-symbol block}$$

or $\bar{L} = 0.84375$ bits/symbol

iii. Since $\bar{L} > H$, the coding scheme is not optimal.

4. a. iv.



The coding scheme is:

AA	0
AB	11
BA	100
BB	101

v. The code words generated by the Huffman algorithm are not unique.

vi. By comparison, the cod words generated in part iv have the same average code length when compared with that for the initial scheme given in the question. Therefore, the new scheme and the initial one have the same efficiency.

4. b.

i. The simple parity bit is a single bit that corresponds to the sum of the other message bits (in modulo 2).
This can detect odd number of errors.

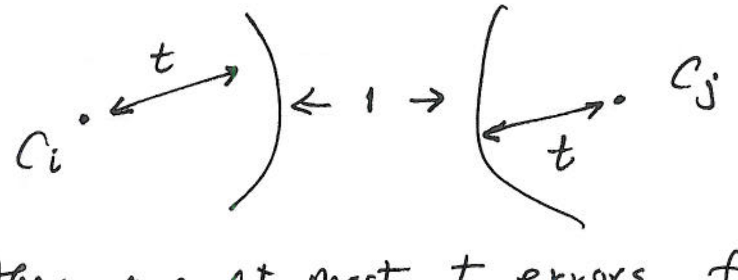
ii. The Hamming distance is 5.

iii. The number of correctable errors t is

$$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor$$ where $\lfloor x \rfloor$ is

the largest integer that is smaller or equal to x and d_{min} is the minimum Hamming distance.

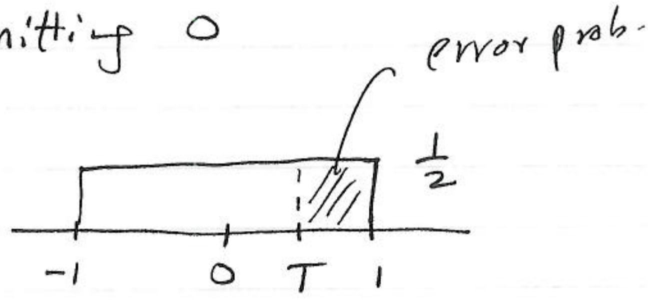
Consider two codewords c_i and c_j



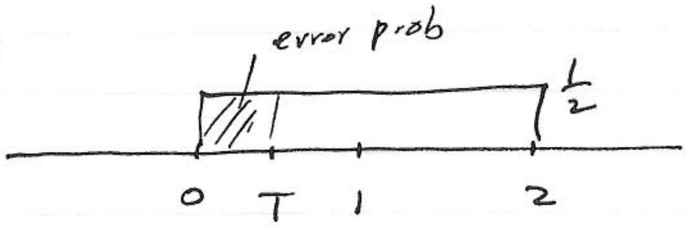
When there are at most t errors for both c_i and c_j, the resultant codewords still differ from each other (by one Hamming distance). Thus, the erroneous codewords can be corrected (or treated) as the "nearest" to the valid codewords.

4.c.

Transmitting 0



Transmitting 1



The prob of error when 0 is sent:

$$P_{e0} = \frac{1}{2}(1-T).$$

Similarly,

$$P_{e1} = \frac{T}{2}.$$

The overall error prob. P_e:

$$P_e = P_0 \cdot P_{e0} + (1-P_0) P_{e1}$$

$$\Rightarrow \quad P_e = P_0 \frac{1}{2}(1-T) + (1-P_0)\frac{T}{2}$$

$$= \frac{P_0}{2} - \frac{P_0 T}{2} + \frac{T}{2} - \frac{P_0 T}{2}$$

$$\Rightarrow \quad P_e = \frac{P_0}{2} - P_0 T + \frac{T}{2}$$

where P_0 is the prob. of sending 0.