Renewable Energy 34 (2009) 196–201

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Complex-valued prediction of wind profile using augmented complex statistics

D.P. Mandic [a], S. Javidi [a,*], S.L. Goh [b], A. Kuh [c], K. Aihara [d]

[a] Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BT, United Kingdom
[b] Shell Exploration and Production, Assen, The Netherlands
[c] Department of Electrical Engineering, University of Hawaii, Honolulu, USA
[d] Institute for Industrial Science, University of Tokyo, Tokyo, Japan

ARTICLE INFO

Article history:
Received 17 July 2007
Accepted 27 March 2008
Available online 14 July 2008

Keywords:
Wind forecasting
Complex representation
Augmented statistics

ABSTRACT

This paper presents a novel approach for the simultaneous modelling and forecasting of wind whereby the wind field is considered as a vector of its speed and direction components in the field of complex numbers $\mathbb{C}$. To account for the intermittency and coupling of wind speed and direction, we propose to use the recently introduced framework of augmented complex statistics. The augmented complex least mean square (ACLMS) algorithm is introduced and its usefulness in wind forecasting is analysed. Simulations over different wind regimes support the approach.

1. Introduction

The modelling and forecasting of wind has been recognised as a prerequisite towards the efficient operation of wind turbines (WTs) and the optimal distribution of energy coming from wind farms (WFs). Short-term wind prediction is crucial in the damage protection and vibration control of WTs, whereas medium- to long-term prediction serves as a basis for the integration of wind energy into the grid. Clearly, the two aspects of wind forecasting operate on different time scales, and due to the wind signal averaging for longer-term prediction, the approaches addressing short- and long-term wind forecasting are different in their nature. The power generated by wind turbines is difficult to forecast, due to the continuous fluctuation of both the wind speed and direction. Various field measurements have shown that the direction of wind as compared with wind speed has less influence on WT power output because each turbine is usually built to face into the wind when operating. Consequently, and especially at stronger winds, there is no significant difference in the power generated for different wind directions. However, the impact of wind direction on output power is more prominent at milder winds since they usually come from much wider directions [1]. The importance of wind direction is of further significance in spatial correlation studies which aim to assess the influence of WT position in a wind park [1].

Normally, studies on multivariate wind forecasts do not simultaneously model all of the wind parameters. While wind speed and direction are shown to influence the turbine power simultaneously, their separate forecasts introduce an error in both the wind dynamics and wind power forecasts. All this emphasises the need to process wind signal as a vector field defined by wind speed and its direction, amongst other factors.

In our recent work [2,3], we have introduced a novel framework for the analysis and modelling of vector fields, whereby the wind vector is represented as a complex-valued quantity, and both wind speed and direction are modelled simultaneously. Notice that this way we inherently introduce heterogeneous fusion of the wind speed and direction. The problem of building a data fusion model via the complex vector space is illustrated in Fig. 1, where the top diagram represents wind measurements as a vector of its speed $v(k)$ and direction $\theta(k)$ components, in the N–E coordinate system, whereas the bottom diagram illustrates the distribution of wind speeds over various directions. There is a clear inter-dependence between wind signal components, a fact that is not taken into account in the current approaches to wind forecasting. Although both the speed and direction are integral components of the wind signal, in practical applications, only the speed component is taken into account, hence introducing a systematic error in forecasts [4].

With respect to processing signals in the complex domain, the introduction of the complex-valued least mean square (CLMS) algorithm [5] has initiated much work in applying complex-valued algorithms to various adaptive linear and nonlinear predictors. In fact, it has been shown that it is not only advantageous but also natural to process some classes of real world data in $\mathbb{C}$ [6]. To design

* Corresponding author.
E-mail addresses: d.mandic@imperial.ac.uk (D.P. Mandic), soroush.javidi02@imperial.ac.uk (S. Javidi), vanessa.goh@shell.com (S.L. Goh), kuh@spectra.eng.hawaii.edu (A. Kuh), aihara@sat.t.u-tokyo.ac.jp (K. Aihara).

0960-1481/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.renene.2008.03.022

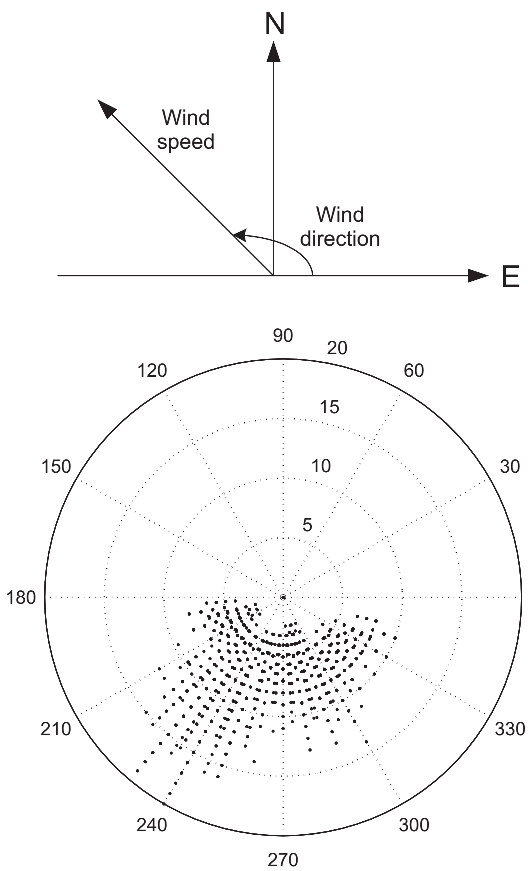

Fig. 1. Wind recordings: (top) a complex-valued representation and (bottom) wind lattice.

an algorithm suitable for forecasting in $\mathbb{C}$, we need a precise mathematical model that describes the evolution of system parameters. Hence, extensions of learning algorithms from $\mathbb{R}$ to $\mathbb{C}$ are not trivial and often involve some constraints, for instance, simplified models of complex statistics. This might prove sub-optimal for classes of signals with significant correlation between the real and imaginary parts, and we ought to seek alternative ways to include the full second-order statistical information available.

Our aim here is to bring together the concepts of linear adaptive forecasting in $\mathbb{C}$ with some new advances in complex statistics [7–10], in order to provide enhanced short-term wind forecasting. We have already addressed the theoretical background of this problem [11,12]; in this article our aim is to introduce the augmented complex statistics into linear sequential adaptive models and to focus on the application in wind forecasting, for various wind regimes and at various degrees of averaging. For generality, our proposed model is a linear sequential least mean square (LMS) type of predictor operating in the complex domain and enhanced with so-called "augmented complex statistics".

2. Analysis of wind characteristics and forecasting architecture

To illustrate the problems faced in wind modelling, consider data obtained from the Iowa (USA) Department of Transport.[1] The data were sampled at 1-, 3- and 6-h intervals. Table 1 shows the statistical properties of the wind data sets considered.

[1] Real-life wind measurement is publicly available from "http://mesonet.agron.iastate.edu/request/awos/1min.php".

Table 1
Statistical properties of the wind data sets

	Set		
	1 h	3 h	6 h
Cumulative samples	1200	1200	1200
Minimum speed (m/s)	0	0	0
Maximum speed (m/s)	13.0582	12.2865	11.4663
Mean speed (m/s)	3.2905	3.2905	3.2905
Standard deviation (m/s)	2.3387	2.2653	2.1520

Clearly, the statistics of wind change with the degree of averaging (or downsampling), for instance when averaging over 10 or more samples the resulting distribution becomes approximately Gaussian hence facilitating linear models.

Our recent work showed that wind signal components are only locally predictable and correlated [13], which is a strong indication that wind measurements could be treated as a complex-valued compact signal rather than two separate univariate variables [14]. These approaches were based on nonlinear sequential models of wind, which were used in order to facilitate its non-Gaussian nature. Here, we propose to use some recent advances in complex statistics, in order to be able to utilise enhanced linear models which are capable of using the full statistical information from such data.

2.1. Forecasting configuration

Forecasting of more than one step ahead can be achieved either in a direct or a recursive manner. The approach used in this paper is the recursive method [15], where at time instant k the predictor predicts all the intermediate values up to $(k+T)$ steps ahead by using the previously estimated values at $k+1, \ldots, k+N-1$ [16,17]. Fig. 2 shows the layout of a finite impulse response (FIR) filter, for which the output is given by

$$y(k) = \mathbf{x}^{\mathrm{T}}(k)\mathbf{w}(k) \tag{1}$$

where $\mathbf{x}(k) \triangleq [x(k), \ldots, x(k-M+1)]^{\mathrm{T}}$ denotes the complex input signal vector, $\mathbf{w}(k) \triangleq [w_1(k), \ldots, w_M(k)]^{\mathrm{T}}$ denotes the complex weight vector, M the number of tap inputs, and $(\cdot)^{\mathrm{T}}$ denotes the vector transpose operator. The error signal $e(k)$ required for adaptation of the adaptive weights is obtained as

$$e(k) = d(k) - y(k) = e^{\mathrm{r}}(k) + \mathrm{j}e^{\mathrm{i}}(k) \tag{2}$$

where $\mathrm{j} = \sqrt{-1}$ and the superscripts $(\cdot)^{\mathrm{r}}$ and $(\cdot)^{\mathrm{i}}$ denote, respectively, the real and imaginary parts of a complex number.

The weight adaptation in a general complex-valued stochastic gradient setting is given by

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\nabla_{\mathbf{w}}J(k)_{|\mathbf{w}=\mathbf{w}(k)} \tag{3}$$

where μ is the rate of adaptation and $J(k)$ is the cost function defined by [18]

$$J(k) = \frac{1}{2}|e(k)|^2 = \frac{1}{2}\left[e(k)e^*(k)\right] \tag{4}$$

where $(\cdot)^*$ denotes the complex conjugation operator, and symbol $|\cdot|$ denotes the absolute value of a complex number. Based on Eqs. (3) and (4) the weight update can be computed as [5]

$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k) - \mu\nabla_{\mathbf{w}}J(k) \\ &= \mathbf{w}(k) + \mu e(k)\mathbf{x}^*(k)\end{aligned} \tag{5}$$

which concludes the overview of the complex-valued gradient descent based least mean square (CLMS) algorithm.

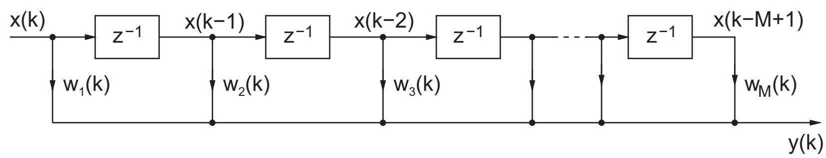

Fig. 2. Linear adaptive finite impulse response filter.

3. The augmented linear adaptive predictor

Consider two zero-mean complex-valued random vectors (RVs), $\mathbf{x}=\mathbf{x}^{\mathrm{r}}+\mathrm{j}\mathbf{x}^{\mathrm{i}}$ and $\mathbf{y}=\mathbf{y}^{\mathrm{r}}+\mathrm{j}\mathbf{y}^{\mathrm{i}}$, for which the four possible covariance matrices (real valued) are given by [7]

$$\begin{aligned}\mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{r}}} &= \mathrm{cov}[\mathbf{x}^{\mathrm{r}},\mathbf{y}^{\mathrm{r}}], \quad \mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{i}}} = \mathrm{cov}\left[\mathbf{x}^{\mathrm{r}},\mathbf{y}^{\mathrm{i}}\right]\\ \mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{r}}} &= \mathrm{cov}\left[\mathbf{x}^{\mathrm{i}},\mathbf{y}^{\mathrm{r}}\right], \quad \mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{i}}} = \mathrm{cov}\left[\mathbf{x}^{\mathrm{i}},\mathbf{y}^{\mathrm{i}}\right]\end{aligned} \tag{6}$$

These real-valued matrices have an equivalent compact, complex-valued, representation given by

$$\mathbf{P}_{\mathbf{xy}} = E\left[\mathbf{xy}^{\mathrm{H}}\right], \quad \mathbf{P}^{\xi}_{\mathbf{xy}} = E\left[\mathbf{xy}^{\mathrm{T}}\right] \tag{7}$$

where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose. We can solve Eq. (7) for $\mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{r}}}$, $\mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{i}}}$, $\mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{r}}}$ and $\mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{i}}}$ to obtain

$$\begin{aligned}\mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{r}}} &= \frac{1}{2}\mathcal{R}\left(\mathbf{P}_{\mathbf{xy}}+\mathbf{P}^{\xi}_{\mathbf{xy}}\right), \quad \mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{i}}} = \frac{1}{2}\mathcal{R}\left(\mathbf{P}_{\mathbf{xy}}-\mathbf{P}^{\xi}_{\mathbf{xy}}\right)\\ \mathbf{P}_{\mathbf{x}^{\mathrm{i}}\mathbf{y}^{\mathrm{r}}} &= \frac{1}{2}\mathcal{I}\left(\mathbf{P}_{\mathbf{xy}}+\mathbf{P}^{\xi}_{\mathbf{xy}}\right), \quad \mathbf{P}_{\mathbf{x}^{\mathrm{r}}\mathbf{y}^{\mathrm{i}}} = \frac{1}{2}\mathcal{I}\left(-\mathbf{P}_{\mathbf{xy}}+\mathbf{P}^{\xi}_{\mathbf{xy}}\right)\end{aligned} \tag{8}$$

where the symbols $\mathcal{R}$ and $\mathcal{I}$ denote, respectively, the real and imaginary parts of a complex vector or matrix. In the literature, nearly always only $\mathbf{P}_{\mathbf{xy}}$ is considered, and is referred to as the *covariance matrix*, whereas $\mathbf{P}_{\mathbf{xy}}{}^{\xi}$ is called the *pseudo-covariance* matrix.[2] By using the augmented states in the complex LMS setting, similarly to the general augmented state estimation problem outlined in Ref. [11], we aim at exploiting the full statistical information from the complex representation of wind.

3.1. Augmented complex LMS (ACLMS) algorithm

In most practical applications, it is assumed that the theory of complex-valued random variables (RVs) is not fundamentally different from the theory of real RVs, and consequently, the definition of the covariance matrix of an RV $\mathbf{x}$ is modified from $E[\mathbf{xx}^{\mathrm{T}}]$ in $\mathbb{R}$ into $E[\mathbf{xx}^{\mathrm{H}}]$ in $\mathbb{C}$ [18]. This assumption, however, is not fully justified; instead some recent research [8,9] shows that the covariance matrix $E[\mathbf{xx}^{\mathrm{H}}]$ will *not* completely describe the second-order statistical behavior of $\mathbf{x}$. Our aim is therefore to derive a complex-valued LMS algorithm for FIR filters operating in $\mathbb{C}$, which is based on the augmented complex statistics and therefore has the potential to fully utilise the available information within the data. To achieve this, extending the standard derivation from Eqs. (1) to (3), we consider both the complex RV $\mathbf{x}$ and its conjugate $\mathbf{x}^*$ to produce a $(2n\times 1)$ vector $\mathbf{x}^{\mathrm{a}}=[\mathbf{x},\mathbf{x}^*]^{\mathrm{T}}$. Following the approaches from Refs. [11,12], we shall now derive the ACLMS algorithm, whereby the overall ‘augmented’ input to the FIR predictor $\mathbf{x}^{\mathrm{a}}(k)$ becomes

$$\mathbf{x}^{\mathrm{a}}(k) = \left[\mathbf{x}(k),\mathbf{x}^*(k)\right]^{\mathrm{T}} \tag{9}$$

The corresponding augmented complex-valued weights of the filters are given as

$$\mathbf{w}^{\mathrm{a}}(k) \triangleq \left[w_1(k),\ldots,w_M(k),w_1^*(k),\ldots,w_M^*(k)\right]^{\mathrm{T}} \tag{10}$$

The output of the filter resulted from the augmented multiplication of the augmented vectors is given by

$$y(k) = \{\mathbf{x}^{\mathrm{a}}\}^{\mathrm{T}}(k)\mathbf{w}^{\mathrm{a}}(k) \tag{11}$$

The error signal $e(k)$ required for adaptation of adaptive weights is obtained as

$$e(k) = d(k)-y(k) = e^{\mathrm{r}}(k)+\mathrm{j}e^{\mathrm{i}}(k) \tag{12}$$

The ACLMS weight update equation can now be rewritten in the same form as Eq. (5) to give

$$\mathbf{w}^{\mathrm{a}}(k) = \mathbf{w}^{\mathrm{a}}(k)+\mu e^{\mathrm{a}}(k)(\mathbf{x}^{\mathrm{a}})^*(k) \tag{13}$$

This completes the derivation of the augmented complex LMS (ACLMS) algorithm.

4. Simulation results

To illustrate the benefits of the proposed approach, two sets of experiments were conducted. In the first set of simulations, the coarsely sampled wind data from Iowa weather stations explained above were used, whereas in the second experiment a finely sampled set of 24-h data recorded at the Institute of Industrial Science, University of Tokyo, were analysed.

4.1. Simulations using coarsely sampled Iowa data

For the first data set, average wind data for 1 h, 3 h and 6 h, denoted, respectively, by W-1, W-3 and W-6, were used as inputs (Table 1). For training purposes, the recordings were standardised to zero mean and unit variance. Initial weight values of the predictor were chosen randomly. The adaptive predictor was trained with 1200 data points taken from the complex wind measurements, as illustrated in Fig. 1.

Performance measure: The measurement used to assess the performance was the prediction gain R_{p} given by [19]

$$R_{\mathrm{p}} \triangleq 10\log_{10}\left(\frac{\sigma_x^2}{\hat{\sigma}_{\mathrm{e}}^2}\right)\ [\mathrm{dB}] \tag{14}$$

where σ_x^2 denotes the variance of the input signal $\{x(k)\}$, whereas $\hat{\sigma}_{\mathrm{e}}^2$ denotes the estimated variance of the forward prediction error $\{e(k)\}$. For rigour, one more performance index was used to measure the forecasting performance, whereby the error mean B and the coefficient of multiple determination r were used [17]

$$B = \frac{1}{T}\sum_{\alpha=1}^{T}\left|x_\alpha-\hat{x}_\alpha\right| \tag{15}$$

[2] A complex RV $\mathbf{x}$ is called *proper* if its pseudo-covariance $\mathbf{P}_{\mathbf{xx}}{}^{\xi}$ vanishes [7]. For convenience, in many applications, complex-valued random vectors (RVs) are treated as proper. However, $\mathbf{P}_{\mathbf{xx}}{}^{\xi}$ may not necessarily vanish, in this case it is called *improper*.

$$r^2 = 1 - \frac{\sum_{\alpha=1}^{T} \left| x_\alpha - \widehat{x}_\alpha \right|^2}{\sum_{\alpha=1}^{T} \left| x_\alpha - \overline{x} \right|^2} \tag{16}$$

where T is the number of samples to be forecasted, x_α is the actual signal value, $\widehat{x}_\alpha$ is the forecasted value, and $\overline{x}$ is the mean of the data. The error mean B is used to measure whether the predictor is biased whereas the coefficient of multiple determination can have several possible value ranges.

$$r^2 = \begin{cases} 1, & \text{if } \forall_\alpha\, \widehat{x}_\alpha = x_\alpha \\ 0 < r^2 < 1, & \text{if } \widehat{x}_\alpha \text{ is a better forecast than } \overline{x} \\ 0, & \text{if generally } \widehat{x}_\alpha = \overline{x} \\ r^2 < 0, & \text{if } \widehat{x}_\alpha \text{ is a worse forecast than } \overline{x} \end{cases}$$

The coefficient of multiple determination r for which the values are close to unity indicates perfect prediction.

To quantify the benefits of the proposed approach, Table 2 shows the results of six steps ahead prediction for wind data averaged at 1-, 3- and 6-h interval. The performance measures (r^2) shown in Table 2 indicate small improvement in the performance when the average interval of wind measurements increases, this is because wind averaged at longer intervals approaches the Gaussian distribution, for which standard complex-valued sequential algorithms are well suited.

In all cases the predictors based on augmented statistics significantly improved performance over the standard complex filter, both in terms of bias and prediction accuracy.

4.2. Performance for the finely sampled 24-h urban wind data

The data set was taken using an ultrasonic anemometer and sampled at 50 Hz. This was then passed through a moving average filter to reduce the effects of high frequency noise; and then resampled at 1 Hz. The window size w_F of the moving average filter varied according to

$$w_F = \{1, 2, 10, 20, 60, 300\}$$

where the window size is given in seconds.

The data comprised of north–south (V_N) and east–west (V_E) direction readings of wind speed, which was used to create the complex wind signal V as follows:

Table 2
Performance measures for 1-, 3- and 6-h average of wind data for six steps ahead prediction

	Wind					
	1 h		3 h		6 h	
Algorithms	CLMS	ACLMS	CLMS	ACLMS	CLMS	ACLMS
Bias	0.088934	0.073271	0.074041	0.018205	0.1107	0.062436
r	0.1149	0.3081	0.14988	0.32099	0.1258	0.4657

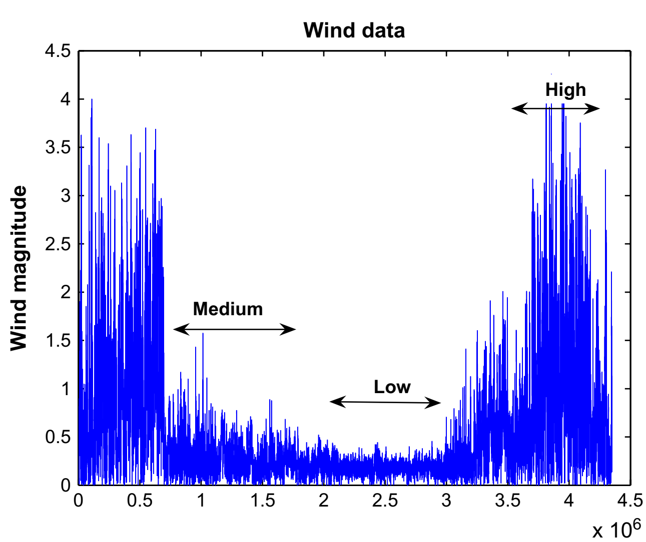

Fig. 3. Complex wind signal magnitude. Three wind speed regions have been identified as low, medium and high.

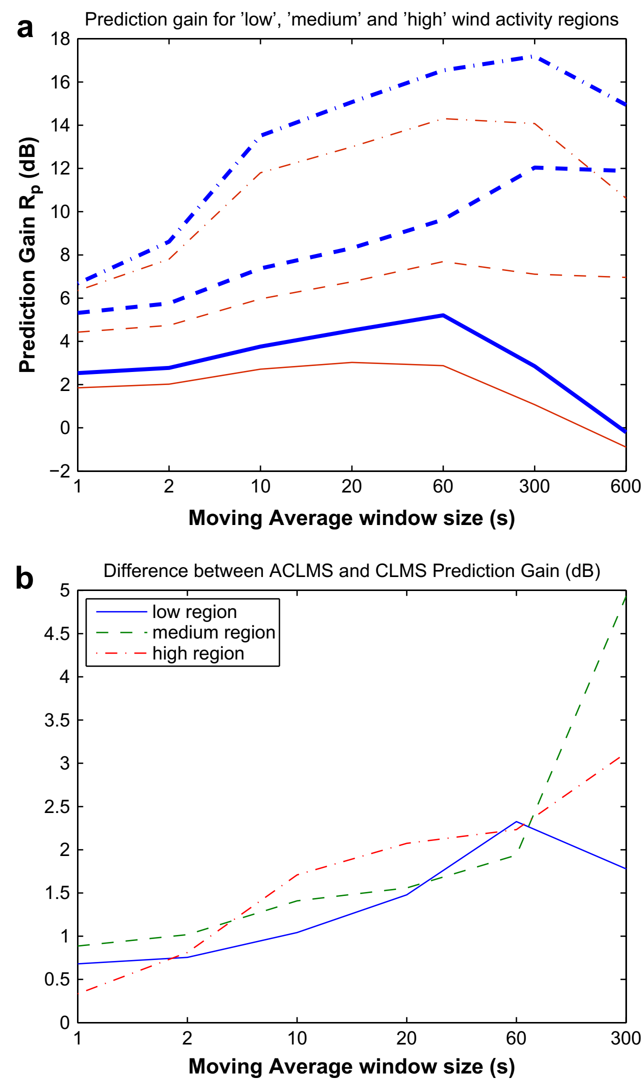

Fig. 4. (a) Prediction gain of the ACLMS (thick line) and CLMS (thin line) algorithms in the *low* (solid), *medium* (dashed) and *high* (dot–dash) regions. (b) Difference between the prediction of the ACLMS and CLMS algorithms for the three regions.

Table 3
Prediction gain (dB) for the low, medium and high regions and according to window size

Window size w_F (s)	Low		Medium		High	
	ACLMS	CLMS	ACLMS	CLMS	ACLMS	CLMS
1	2.53	1.85	5.32	4.43	6.68	6.35
2	2.77	2.02	5.76	4.74	8.62	7.80
10	3.76	2.72	7.37	5.96	13.51	11.80
20	4.51	3.03	8.32	6.76	15.07	13.00
60	5.21	2.88	9.63	7.69	16.53	14.30
300	2.85	1.07	12.04	7.11	17.20	14.07

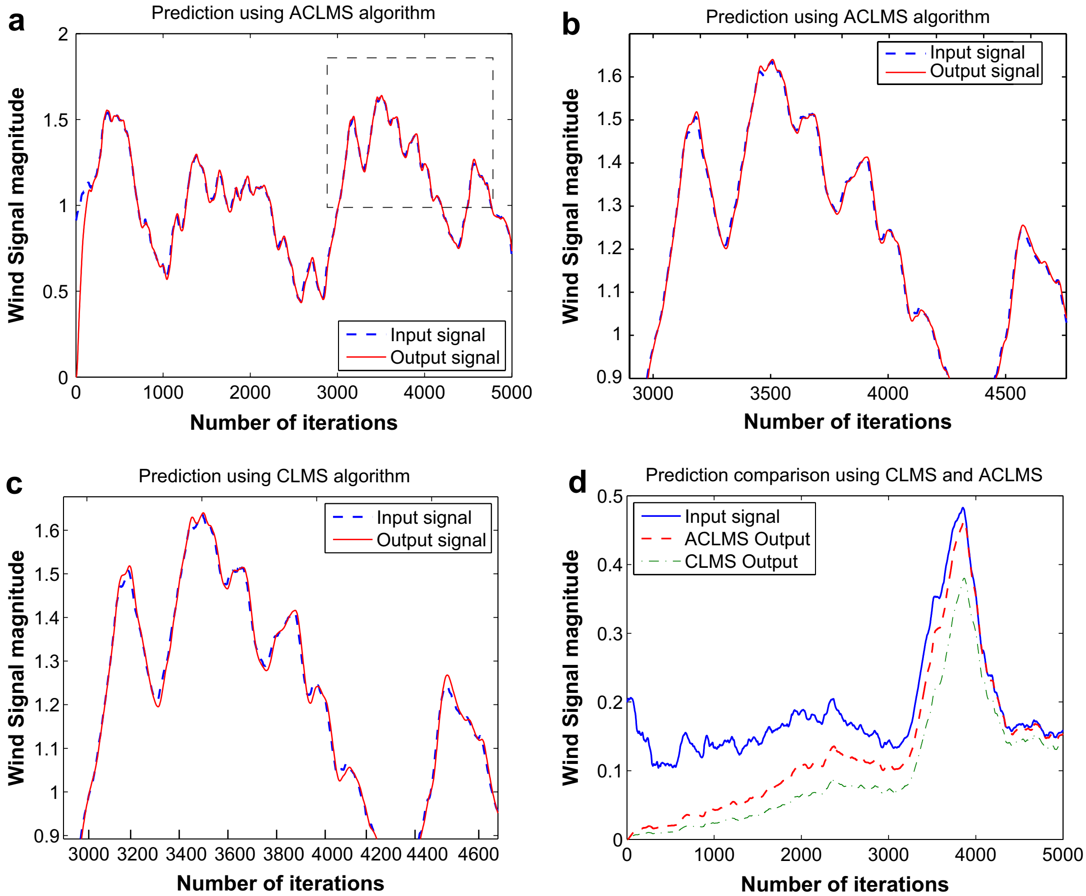

Fig. 5. (a) Input and predicted signal of the *high* region using the ACLMS algorithm, (b) zoomed area of boxed portion of (a), (c) zoomed portion of (a) using the CLMS algorithm, and (d) input and predicted signal of the *medium* region, comparing the performance of the ACLMS and CLMS after 5000 iterations.

$$\begin{aligned} v &= \sqrt{V_E^2 + V_N^2} \\ \varphi &= \arctan\left(\frac{V_N}{V_E}\right) \\ V &= v\,e^{i\varphi} \end{aligned} \tag{17}$$

The complex wind data obtained are shown in Fig. 3. Throughout the day, the wind strength changed, as labelled on the figure by *high*, *medium* or *low*. To analyse the effect of wind speed, 5000 samples were taken from each region to train the adaptive predictor using one step ahead prediction. Simulation results are shown in Fig. 4(a) and summarised in Table 3.

It is evident that the ACLMS algorithm has provided better predictions compared to the CLMS algorithm in all the three considered regions. It is also seen that the best prediction was obtained from the *high* region where the wind speed was strongest, having a maximum prediction gain of 17.20 dB; this clearly indicated the benefits of using augmented statistics for data with large dynamics.

The effect of the moving average window size can be seen in Fig. 4(b) where the difference between the ACLMS and CLMS prediction gain is given. The overall trend shows a direct proportionality between the window size and the difference between the prediction gain of the two algorithms, i.e. as the window size increases, the ACLMS algorithm outperforms the CLMS algorithm by larger amounts, with the exception of the *low* region.

Fig. 5(b) and (c) shows a more detailed graph of the *high* region input and predicted signal of the boxed portion of Fig. 5(a) using, respectively, the ACLMS and CLMS algorithms. Fig. 5(d) also compares the performance of the two algorithms with data taken from the *medium* region. It is seen that after 5000 iterations, the ACLMS algorithm has outperformed CLMS.

5. Conclusions and discussions

This paper has introduced a novel approach for estimation of the wind signal in the complex domain taking into account the strong correlation between the two wind components, speed and direction. Recent advances in complex statistics have been utilised to provide a mathematical model for enhancing the performance of linear predictors for complex data, the so-called augmented complex LMS algorithm, by taking the pseudo-covariance matrix as well as the covariance matrix.

Two sets of data were used to compare the performance of the proposed method. At long sampling intervals due to averaging, the data exhibit near Gaussian statistical properties, making the increase of performance between the two algorithms relatively small.

However, it is seen that at shorter sampling intervals there is a distinct increase in the performance of the ACLMS algorithm, which is due to the different treatment of the underlying covariance and pseudo-covariance matrices.

Acknowledgement

We would like to acknowledge Gill Instruments Ltd who have provided ultrasonic anemometers used for testing of our algorithms.

References

[1] Li S, Wunsch DC, O'Hair EA, Giesselmann M. Using NNs to estimate wind turbine power generation. IEEE Transaction on Energy Conversion 2001;16(3): 276–82.

[2] Goh SL, Popovic DH, Mandic DP. Complex-valued estimation of wind profile and wind power. In: Proceedings of the 12th IEEE Mediterranean electrotechnical conference, MELECON 2004, vol. 3, 2004; p. 1037–40.
[3] Goh SL, Chen M, Popovic DH, Aihara K, Obradovic D, Mandic DP. Complex-valued forecasting of wind profile. Renewable Energy 2006;31:1733–50.
[4] Alexiadis MC, Dokopoulos PS, Sahsamanoglou HS, Manousaridis IM. Short-term forecasting of wind speed and related electrical power. Solar Energy 1998;63(1):61–8.
[5] Widrow B, McCool J, Ball M. The complex LMS algorithm. Proceedings of the IEEE 1975;63:712–20.
[6] Mandic DP, Javidi S, Souretis G, Goh VSL. Why a complex valued solution for a eal domain problem. Proceedings of the IEEE workshop on machine learning for signal processing 2007:284–389.
[7] Neeser F, Massey J. Proper complex random processes with applications to information theory. IEEE Transactions on Information Theory 1992;39(4):1293–302.
[8] Picinbono B. On circularity. IEEE Transactions on Signal Processing 1998; 42(12):3473–82.
[9] Picinbono B. Second-order complex random vectors and normal distributions. IEEE Transactions on Signal Processing 1996;44(10):2637–40.
[10] Schreier P, Scharf L. Second-order analysis of improper complex random vectors and processes. IEEE Transactions on Signal Processing 2003;51(3):714–25.
[11] Goh SL, Mandic DP. An augmented extended Kalman filter algorithm for complex-valued recurrent neural networks. Neural Computation 2007;19(4): 1293–302.
[12] Goh SL, Mandic DP. An augmented CRTRL for complex-valued recurrent neural networks. Neural Networks 2007;20(10):1061–6.
[13] Tregidgo D. M.Eng. thesis. Imperial College London; 2004.
[14] Gautama T, Mandic DP, Hulle MMV. A non-parametric test for detecting the complex-valued nature of time series. International Journal of Knowledge-based and Intelligent Engineering Systems 2004;8(2):99–106.
[15] Wan E. Time series prediction using a neural network with embedded tapped delay-lines. In: Predicting the future and understanding the past. SFI studies in the science of complexity; 1993. p. 195–217.
[16] Atiya AF, El-Shoura SM, Shaheen SI, El-Sherif MS. A comparison between neural-network forecasting techniques – case study: river flow forecasting. IEEE Transactions on Neural Networks 1999;10(2):402–9.
[17] Drossu R, Obradovic Z. Rapid design of neural networks for time series prediction. IEEE Computational Science and Engineering 1996;3(2):78–89 [see also Computing in Science and Engineering].
[18] Haykin S. Neural networks: a comprehensive foundation. Prentice Hall; 1994.
[19] Haykin S, Li L. Nonlinear adaptive prediction of nonstationary signals. IEEE Transactions on Signal Processing 1995;43(2):526–35.