Error Performance Bounds for Routing Algorithms in Wireless Cooperative Networks

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***Abstract*—Cooperative diversity has recently emerged as a promising approach to improving reception reliability by realizing spatial diversity gains for nodes with single antenna. We consider here cooperative ad-hoc wireless networks where communications between two nodes can be assisted by a single relay using two time slots. This paper continues our investigation of PHY techniques and cross-layer routing algorithms in such networks. Specifically, we investigate here the optimal relay location for cooperative link in networks with infinite node density. By using this result, we analyze the upper-bound error performance for routing algorithms in the infinitely dense networks. Furthermore, we study the performance bounds for regularly dense networks with linear topology. Theoretical analysis shows that the proposed routing algorithm performs close to the optimal error performance.**

I. Introduction

In this paper, we continue our investigation of PHY techniques and cross-layer routing algorithms in cooperative networks where communications between two nodes can be assisted by a single relay using two time slots. Specifically, we investigate here the optimal relay location for cooperative link in networks with infinite node density. By using this result, we analyze the upper-bound error performance for routing algorithms in the infinitely dense networks. Furthermore, we study the performance bounds for regularly dense networks with linear topology. Theoretical analysis shows that the proposed routing algorithm performs close to the optimal error performance.

II. System Model

We consider a cooperative network in where a source node communicates with a destination node with the help of one relay. Cooperative transmission from the source to the destination is carried out using two time slots as follows. In the first time slot, the source broadcasts its data to the relay and the destination. In the second time slot, only the relay is allowed to send the data received from the source to the destination according to the decode-and-forward mechanism. It is shown in [1] that the full second-order of diversity can be achieved from such CL strategy. Such cooperative communications also provide significant improvement to reception reliability.

We employ a propagation model to consider path loss, shadow fading and Rayleigh fading. The wireless link a_{ij} between the nodes i and j is modelled as $\mathrm{a}_{ij} = h_{ij}/d_{ij}^{k/2}$, where d_{ij}, the distance between the nodes i and j, represents the large-scale behavior of the channel gain, k is the path-loss exponent and h_{ij} captures the channel fading characteristics due to the rich scattering environment. In addition, the channel fading parameter h_{ij} is assumed to be independent and identically distributed (i.i.d), complex Gaussian variable with zero mean and unit variance.

For direct transmission, the outage event for spectral efficiency R is given by $I_{s,d} < R$ and corresponds to

$$|\mathrm{a}_{s,d}|^2 < \frac{2^R - 1}{\rho}\,, \tag{1}$$

For Rayleigh fading, $|\mathrm{a}_{s,d}|^2$ is exponentially distributed with parameter $d_{s,d}^k$ and the outage probability is given by

$$P_D^{out} = d_{s,d}^k\left(\frac{2^R - 1}{\rho}\right). \tag{2}$$

where R is the data rate in bit/s/Hz which is defined by the quality of service (QoS) requirement and $\rho = E_b/N_0$ is the transmission power to noise ratio (SNR).

For cooperative transmission, the outage performance of the selected decode-and-forward relaying transmission is

$$P_C^{out} = \frac{1}{2} d_{s,d}^k (d_{s,r}^k + d_{r,d}^k) \frac{(2^{2R} - 1)^2}{\rho^2}\,. \tag{3}$$

where the notation s, d and r denotes the source, destination and relay nodes, respectively. It is also assumed that the source and relay node have the same transmission power to noise ratios.

A. Optimal Relay Location for CL

It is clear that relay selection is crucial for the performance of cooperative transmission. This is so because a good quality relay yields strong multi-user diversity gain, thus potentially enhancing the system performances (i.e., BER, transmission power and data rate). To derive of error performance bounds, we assume here that the network under consideration has infinite node density. We develop a theoretical analysis to provide new insights into the optimal relay location that minimizes the bit error rate (BER).

For the assumed infinitely dense two-dimensional network, which allows the selection of relay node at any location, the optimization problem is to find the optimal relay that minimizes the end-to-end BER of cooperative link with the constraint on the distances among the associated nodes, as formulated as follows

$$\begin{cases} \min & \mathrm{BER} = \frac{(2^{2R}-1)^2}{2\rho^2} D^k (x^k + y^k) \\ \text{s.t.} & x + y = D \end{cases} \tag{4}$$

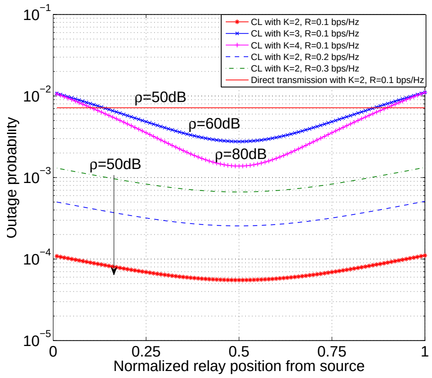

Fig. 1. Relay optimization in linear network

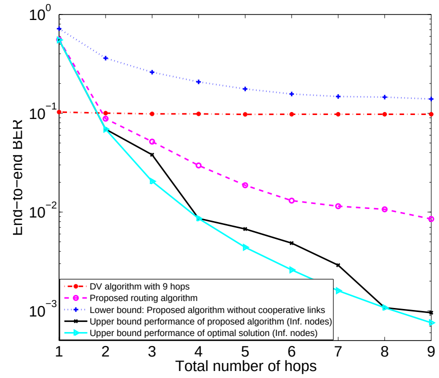

Fig. 2. BER versus the number of hops in the route for R=0.1 bps/Hz

where x, y, and D are the source, destination and relay nodes, respectively.

After solving this problem, we have $x = y = D/2$. Clearly, in order to achieve the best BER performance in this linear network, the relay node is always chosen at the middle between each pair of source and destination nodes,which is also confirmed by Figure 1.

B. Interference Effect on Optimal Relay Location

With consideration of interference, the outage probablity is

$$P_C^{out} \approx \Big(\frac{d_{s,r}^k d_{s,d}^k |\mathrm{a}_{i,r}|^2}{2|\mathrm{a}_{i,d}|^2} + (1 - d_{s,r}^k \gamma |\mathrm{a}_{i,r}|^2)\frac{d_{s,d}^k d_{r,d}^k}{2}\Big)(\gamma|\mathrm{a}_{i,d}|^2)^2 . \quad (5)$$

where $\gamma = 2^{2R} - 1$.

Different to the scenario without interference, the outage probability is no longer a function of transmission power and optimal relay location could be variable by the assumption of different interference environment.

III. Routing Performance Evaluation

In this section, we analyze our proposed routing algorithm [2] and compare with the optimal routing solution in two special scenarios: networks with infinite node density and those with finite node density in linear topology. Furthermore, we derive the upper and lower bounds for the end-to-end BER.

A. Performance Evaluation in Infinitely Dense Network

Theorem1: For infinitely dense network where node exists at any location, the upper bound BER for the proposed routing is proportional to $1/A^{2k-1}$, where A, being perfect power of 2, is the largest integer that smaller than the total number of hops N and k is the pass loss exponent. □

Motivated by such conclusions above, we can find an upper bound on the performance of our proposed routing algorithm in 2D networks as well as an upper bound on the optimal solution, which are shown in Figure 2. We observe that the proposed algorithm exhibits performance close to optimal, especially when the hop number N satisfies $log_2(N) =$ integer.

B. Performance Evaluation in Regular Network

Following the ideas above, in this subsection, we compare the minimum end-to-end BER achieved by our proposed algorithm with that of the optimal routing solution for a regular linear network scenario, where nodes are located at equal distance from each other on a straight line. We assume that this distance between two adjunct nodes is D and the total number of nodes is n.

Theorem2: For a regular linear network with n nodes,

$$g = \begin{cases} 0, & \text{if } \log_2(n-1) or \log_2(n) = \text{integer} \\ \frac{11}{4}, & \text{if } \log_2(\frac{n-1}{3}) = \text{integer} \\ \frac{33}{2(n-1)}, & \text{otherwise for an odd number nodes} \end{cases}$$

In general, Theorem 2 tells us that the proposed routing algorithm can have a BER close optimal. For example, for the first case where $n-1$ or n is perfect power of 2, the proposed algorithm yields the exactly same BER as the optimal route. The gap ratio can be close to zero for the third case where the number of nodes is large enough.

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