On the Application of Network Coding with Diversity to Opportunistic Scheduling

Zhiguo Ding[1], Kin K. Leung[2] and Mugen Peng[3]
[1]Department of Communication Systems, Lancaster, UK
[2]Electrical Engineering Department, Imperial College, London, UK
[3]Key Lab. of Universal Wireless Comm. for Ministry of Education, BUPT, Beijing, China.

***Abstract*—In this correspondence, we study the application of network coding to opportunistic scheduling for wireless uplink channels. An important observation is that existing scheduling protocols occasionally have to allow the users which do not have the largest channel gain to be scheduled, which can maintain the fairness but reduce the throughput. The key idea proposed in this paper is to always schedule the user with best channel gain for transmission, and meantime the use of network coding encourages the scheduled users to help the ones which have not been served previously. Analytical and numerical results have been developed to show that the proposed network coding schedulers can achieve better tradeoff of fairness and system throughput than comparable schemes.**

I. Introduction

Opportunistic scheduling has been recognized as a bandwidth efficient technique to utilize multi-user diversity and increase the data throughput of wireless systems. However, such an opportunistic scheme could be unfair to certain users since users experiencing long-term deep fading could be never served. To balance the tradeoff between fairness and system throughput, many efficient opportunistic scheduling protocols have been developed, however, it is inevitable for these protocols to schedule certain users which do not have the largest instantaneous channel gain for transmission [1]. Therefore, a penalty of system throughput will be caused since unideal users are scheduled for transmission. The question we are seeking to answer in this paper is whether it is possible to always schedule the best user and at the same time maintain the fairness among users.

Network coding, a concept originally developed for routing optimization, has received a lot of attention because of its superior ability to utilize the broadcasting nature of radio propagation [2]. In [3]–[5], physical layer network coding protocols have been developed, where the performance gain of ergodic sumrate and robustness can be achieved simultaneously. In this paper, we study the application of network coding to opportunistic scheduling for wireless uplink channels. Different to existing scheduling protocols, the user which has the largest instantaneous channel gain will be always scheduled, which can ensure large system throughput. And the key idea of the proposed network coding schedulers is to ask the scheduled users which have been served previously to help the ones never been served, which can balance the tradeoff between fairness and system throughput. Furthermore, the relay selection strategy is proposed to be combined with the network coding scheduler and the impact of relay selection on the delay and throughput is also discussed. Analytical results have been developed to show that the proposed scheduler without relay selection only requires the delay at the same order as the round robin scheme ($\mathcal{O}(n)$) where n is the total user number. And the system throughput achieved by the proposed scheduler with relay selection is larger than that achieved by the pure opportunistic scheme for constant n and is growing as $\mathcal{O}(n\log n)$. Monte-Carlo simulation results have also been provided to demonstrate the performance of the proposed network coding schedulers.

II. Description for Opportunistic Network-coding Assisted Scheduling Protocol

Consider a multiple access communication scenario where n sources deliver their messages to the same destination. Each node is constrained by the half duplex assumption, and TDMA is used in this paper due to its simplicity, where T denotes the duration for each time slot. Denote t_m as a time by which the destination has received at least one package from m sources. Without loss of generality, we only focus on the transmission strategy between t_m and t_{m+1}. Define two node subsets, $\Omega_{m,1}$ containing the nodes each of which has successfully delivered at least one package to the destination, and $\Omega_{m,2}$ as the complimentary node subset of $\Omega_{m,1}$.

During the first time slot $[t_m, t_m+T]$, a source which has the largest channel gain, denoted as S_{t_m}, will be scheduled, the same as the pure opportunistic scheduler. Provided S_{t_m} scheduled previously $S_{t_m}\in\Omega_{m,1}$, the network coded transmission protocol will be introduced. A source randomly chosen from the subset $\Omega_{m,2}$, denoted as $S_{t_m,co}$, will broadcast its message at the same time as S_{t_m} is transmitting. As a result, both the destination and all other nodes receive a mixture of the messages from the two nodes[1].

During the second time slot, another source which has the largest channel gain, denoted as S_{t_m+T}, will be scheduled to transmit. If $S_{t_m+T}\in\Omega_{m,2}$, it means that a new node which has not delivered any messages to the destination previously will be scheduled and hence $t_{m+1}=t_m+1$. Provided that $S_{t_m+T}\in\Omega_{m,1}$ and $S_{t_m+T}\neq S_{t_m}$, the source S_{t_m+T} will forward the mixture of the messages received from S_{t_m} and

[1]As shown in [5], the optimal number for the source nodes to participate in cooperation is two, which is the reason that only two sources are transmitting simultaneously.

$S_{t_m,co}$. Otherwise, traditional direct transmission will be used for the second time slot. After the second time slot, a similar transmission strategy will be carried on until one of the two following events happen. One type of the events is that at one time slot, one of the sources from $\Omega_{m,2}$ has the largest instantaneous channel gain and can be scheduled to deliver its message to the destination. The other is that a node from $\Omega_{m,1}$ but other than S_{t_m} has been scheduled to transmit and the second stage of network coding transmission is accomplished.

A. A Simple Example for the Proposed Opportunistic Scheduling Protocol

Consider an uplink scenario with 4 users, denoted as U_n, where $1 \leq n \leq 4$. For the purpose of illustration, we assume that the users which have the best channel condition for the first 4 time slots are U_1, U_3, U_1 and U_3 respectively. As shown in Fig. 1, during the first time slot, U_1 will be scheduled to transmit its own message $s_{1,1}$ which will be detected at the receiver. The transmission strategy at the second time slot is similar to the first time slot. At the third time slot, U_1 is again scheduled to transmit a new message $s_{1,2}$. At the same time, U_2, the one which has not be scheduled so far, will also transmit a message $s_{2,1}$. The destination has received a mixture of the two messages, and it cannot separate them. At the fourth time slot, U_3 has been scheduled for the second time and it will transmit its observation received at the previous time slot, a different combination of the two messages ($s_{1,2}$ and $s_{2,1}$) compared with the mixture observed at the destination during the previous time slot. At the end of the fourth time slot, the destination can detect the two messages, $s_{1,2}$ and $s_{2,1}$. So in summary, all users can be served in time, which improves the fairness performance. And at each time slot, a user which has the best instantaneous channel condition is always scheduled to transmit, which is important to support a large data rate.

B. Delay Performance of the Proposed Opportunistic Scheduling Protocol

Denote D_m as the delay for the destination to receive a package from a new source in $\Omega_{m,2}$ after m sources have been served, $D_m = t_{m+1} - t_m$. Hence the total delay to ensure that the destination has received at least one package from all sources, denoted as D, can be expressed as $D = \sum_{m=0}^{n-1} D_m$. According to the linearity of expectations, the averaged total delay can be expressed as

$$\bar{D} = \mathcal{E}\{D\} = \sum_{m=0}^{n-1} \mathcal{E}\{D_m\}, \tag{1}$$

where $\mathcal{E}\{\cdot\}$ denotes the operation of expectation. According to the proposed scheduling protocol, the event of $D_m = 1$ implies that one of the $n-m$ sources in $\Omega_{m,2}$ has been scheduled to transmit, which means $P(D_m = 1) = P(S_{t_m} \in \Omega_{m,1}) = \frac{n-m}{n}$. Obviously we have used the assumption that there is no transmission failure. With the probability $1 - P(D_m = 1)$, the network coding strategy will be used, where two nodes, $S_{t_m} \in \Omega_{m,1}$ and $S_{t_m,co} \in \Omega_{m,2}$, will transmit simultaneously. At the second time slot, three types of transmission strategies will be used according to the fact which node will has the largest instantaneous channel gain. Provided that $S_{t_m+T} \in \Omega_{m,2}$, a source new to the existing m ones in $\Omega_{m,1}$ is scheduled and hence $D_m = 2$. Provided that $S_{t_m} \in \Omega_{m,1}$, $S_{t_m+T} \in \Omega_{m,1}$ and $S_{t_m+T} \neq S_{t_m}$, a two-stage network coding transmission has completed, which also results in $D_m = 2$. The third type is due to the fact that a node has the largest channel gain for the two successive time slots, $S_{t_m} = S_{t_m+T}$, and hence $D_m > 2$. so we can have the probability $P(D_m = 2) = \frac{m}{n}\left(\frac{n-m}{n} + \frac{m-1}{n}\right)$. Following the same steps, for $k \geq 2$, the probability for $D_m = k$ can be expressed as

$$P(D_m = k) = \frac{m}{n}\left(\frac{1}{n}\right)^{k-2}\left(\frac{n-m}{n} + \frac{m-1}{n}\right), \tag{2}$$

for $k \geq 2$. Note that for $n \to \infty$ and $k > 2$, we can observe $P(D_m = k)/P(D_m = 2) \to 0$, which means that two time slots are typically enough to ensure a new source scheduled for large n. Recall that for the coupon collector's problem, it could cost a lot of times in order to get a different coupon, specially in the case that most coupons have been collected. However, by using network coding, short delay can be expected for all D_m and hence D_m is no longer a proportional function of m as the coupon collector problem. The delay D_m can be obtained as

$$\begin{aligned} \mathcal{E}\{D_m\} &= \sum_{k=1}^{\infty} k P(D_m = k) \qquad (3) \\ &= \frac{n-m}{n} + \sum_{k=2}^{\infty} k \frac{m}{n}\left(\frac{1}{n}\right)^{k-2} \frac{n-1}{n}. \end{aligned}$$

Accordingly the expectation of the total delay can be calculated as

$$\begin{aligned} \mathcal{E}\{D\} &= \sum_{m=0}^{n-1} \mathcal{E}\{D_m\} \qquad (4) \\ &= \sum_{m=0}^{n-1}\left(\frac{n-m}{n} + \sum_{k=2}^{\infty} k \frac{m}{n}\left(\frac{1}{n}\right)^{k-2} \frac{n-1}{n}\right). \end{aligned}$$

Conditioned on $n \to \infty$, the expectation of the delay can be written as

$$\mathcal{E}\{D\} \approx \sum_{m=0}^{n-1}\left(\frac{n-m}{n} + 2\frac{m}{n} + \mathcal{O}(\frac{1}{n})\right). \tag{5}$$

By using the fact that $\sum_{m=1}^{n-1} m = \frac{n}{2}(n-1)$, the expectation of the delay can be shown as

$$\begin{aligned} \mathcal{E}\{D\} &\approx \sum_{m=1}^{n} \frac{m}{n} + \sum_{m=0}^{n-1} 2\frac{m}{n} + \mathcal{O}(1) \qquad (6) \\ &\approx \frac{3}{2}(n-1) + \mathcal{O}(1) \to \mathcal{O}(n). \end{aligned}$$

which demonstrates that the proposed scheduling protocol can achieve the same delay as the round-robin scheme. On the other hand, it is well known that the pure opportunistic scheduling scheme requires the delay with the order of $\mathcal{O}(n \ln n)$, which could be significantly larger than the delay achieved by the proposed scheduler.

	time slot 1	time slot 2	time slot 3	time slot 4
Primary user	U_1	U_3	U_1	U_3
Primary user's tx. message	$s_{1,1}$	$s_{3,1}$	$s_{1,2}$	mixture of $s_{1,2}$ and $s_{2,1}$
Coop. user			U_2	
Coop. user's tx. message			$s_{2,1}$	
Messages detectable at des.	$s_{1,1}$	$s_{3,1}$		$s_{1,2}$ and $s_{2,1}$

Fig. 1. A simple example for the proposed opportunistic scheduling scheme with network coding

III. A Tradeoff of Fairness and System Throughput

An ideal scheduler should achieve not only the delay at the same order as the round robin scheduler, but also the system throughput at the same order as the pure opportunistic one simultaneously. Recall from [4], the sumrate achieved by the two-stage network coded transmission can be expressed as

$$\mathcal{I} = \frac{1}{2}\log\left[1+\rho\frac{\beta_1^2|h_{RD}|^2}{\beta_1^2+|h_{RD}|^2}+\rho\beta_0^2 + \frac{\rho^2|h_{R,D}|^2}{\beta_1^2+|h_{R,D}|^2}|h_{1,D}h_{2,R}-h_{2,D}h_{1,R}|^2\right]. \quad (7)$$

where $\beta_0 = \sqrt{|h_{1D}|^2+|h_{2D}|^2}$, $\beta_1 = \sqrt{|h_{1R}|^2+|h_{2R}|^2+1/\rho}$, h_{mn} denotes the coefficient of the channel from the node m to node n and ρ is denoted as signal-to-noise ratio (SNR). Here we employ a propagation model which includes path loss and Rayleigh fading [6], and hence we can have $h_{ij} = \frac{g_{ij}}{d_{ij}^{\alpha/2}}$, where d_{ij} is the distance between the two nodes, g_{ij} captures Raleigh fading and α is the path loss factor. Provided that the distance between the users the base station is much bigger than the distance between the users, the expression of sumrate can be simplified as

$$\mathcal{I} \approx \frac{1}{2}\log\left[1+\rho|h_{RD}|^2+\rho\beta_0^2 + \frac{\rho^2|h_{R,D}|^2}{\beta_1^2}|h_{1,D}h_{2,R}-h_{2,D}h_{1,R}|^2\right]. \quad (8)$$

A closed form expression for the ergodic sumrate shown in (8) is difficult to obtain, and simulation results provided in the next section show that the proposed scheduler could result in some loss of throughput compared with the pure opportunistic scheme. The reason is that the proposed scheduler can only ensure that the relay used at the second stage has a good connection to the base station, but the quality of the connection between the relay and the two sources is not promised. Such a loss of system throughput can be avoided by introducing relay selection, where an optimization problem can be formulated as

$$\underset{h_{1R},h_{2R}}{\arg\max}\,\mathcal{I} \approx \underset{h_{1R},h_{2R}}{\arg\max}\log\left[\frac{1}{\beta_1^2}|h_{1,D}h_{2,R}-h_{2,D}h_{1,R}|^2\right],$$

where h_{RD} is treating as a constant since it already has the largest value relay-destination channel. By utilizing *Cauchy-Schwarz inequality* [7], we observe that

$$|h_{1D}h_{2R}-h_{2D}h_{1R}|^2 \le (|h_{1D}|^2+|h_{2D}|^2)(|h_{1R}|^2+|h_{2R}|^2) \quad (9)$$

where the equality holds when $\frac{h_{1D}}{h_{2R}^*} = \frac{-h_{2D}}{h_{1R}^*}$. Assume that n is large enough to find a relay satisfying such a linear relationship, and hence the sumrate can be maximize as

$$\begin{aligned}\mathcal{I}_{max} &= \frac{1}{2}\log\left[1+\rho|h_{RD}|^2+\rho\beta_0^2+\rho^2|h_{R,D}|^2\beta_0^2\right] \quad (10)\\ &= \frac{1}{2}\log\left(1+\rho|h_{RD}|^2\right)\left(1+\rho\beta_0^2\right) > \mathcal{I}_{oppt},\end{aligned}$$

where $\mathcal{I}_{oppt} = \log\left(1+\rho|h_{RD}|^2\right)\left(1+\rho|h_{1D}|^2\right)$ is the sumrate achieved by the pure opportunistic scheduling. By using the extreme value theory [8], the asymptotic property of the sumrate achieved by the network coding transmission with relay selection can be shown as

$$\mathcal{I}_{max} \approx \log\rho\log n, \quad (11)$$

which is the same order achieved by the pure opportunistic scheduling.

While the use of relay selection can increase system throughput, it could results in some penalty for the delay performance. Accordingly we propose a new scheduling protocol, named as network coding scheduling with group size θ, in order to achieve a better fairness-throughput tradeoff. The difference between the two schemes with and without grouping only exists for the second stage of network coding transmission. Define the relaying pool as $\Omega_{m,pool} = \{\Omega_{m,1} - S_{t_m}\}$. Provided that the first stage of network coding has accomplished previously, for the scheme without grouping, the second stage will be completed whenever one node from $\Omega_{m,pool}$ is scheduled. For the new scheme, all nodes from $\Omega_{m,pool}$ are first ordered according to their achievable sumrate. Define a best-relay subset as $\Omega_{m,best}$ which contains the $\min\{\theta, m-1\}$ best relays from $\Omega_{m,pool}$. Hence the second stage of the scheduling scheme with grouping is completed whenever one node from $\Omega_{m,best}$ is scheduled.

A. Delay analysis for the network coding scheduler with group size θ

Again the delay D_m is focused here. Since the first stage of the network coding scheduler with grouping is the same as previously, hence the probability to schedule a new source within the first time slot is also the same as before, $P(D_m = 1) = \frac{n-m}{n}$. However, at the second time slot, the cooperative transmission can be completed only if a new node from $\Omega_{m,2}$ is scheduled or the second stage of network coding is accomplished due to a node from $\Omega_{m,best}$ scheduled. Hence the probability $P(D_m = 2)$ can be expressed as

$$P(D_m = 2) = \frac{m}{n}\left(\frac{n-m}{n}+\frac{d}{n}\right), \quad (12)$$

where $d = \min\{\theta, m-1\}$. Similarly we can have a general expression for the probability $P(D_m = k)$ as

$$P(D_m = k) = \frac{m}{n}\left(\frac{m-d}{n}\right)^{k-2}\left(\frac{n-m}{n}+\frac{d}{n}\right). \quad (13)$$

Comparing (13) and (2), we can have an important observation that $P(D_m = k)$ with node selection is no longer approaching to zero for $k > 2$ and $n \to \infty$. The expectation for the delay D_m can be expressed as

$$\mathcal{E}\{D_m\} = \frac{n-m}{n} + \sum_{k=2}^{\infty} k\frac{m}{n}\left(\frac{m-d}{n}\right)^{k-2}\left(\frac{n-m}{n}+\frac{d}{n}\right).$$

Accordingly the expectation for the total delay can be obtained as

$$\mathcal{E}\{D\} = \sum_{m=0}^{n-1}\left(\frac{n-m}{n} + \frac{m}{n}\left(\frac{n-m+d}{n}\right)\sum_{k=2}^{\infty} k\left(\frac{m-d}{n}\right)^{k-2}\right).$$

Define $\alpha = \frac{m-d}{n}$, and the following infinity summary can be expressed as

$$\sum_{k=2}^{\infty} k\alpha^{k-2} = \frac{1}{\alpha}\left(\frac{d}{d\alpha}\alpha\sum_{l=0}^{\infty}\alpha^l - 1\right) \quad (14)$$

Provided that $|\alpha| < 1$, we can have $\sum_{l=0}^{\infty}\alpha^l = \frac{1}{1-\alpha}$ according to Eq(0.231) in [9]. Hence we can have

$$\sum_{k=2}^{\infty} k\alpha^{k-2} = \frac{1}{\alpha}\left(\frac{d}{d\alpha}\frac{\alpha}{1-\alpha} - 1\right) = \frac{1}{\alpha}\left(\frac{1}{(1-\alpha)^2} - 1\right).$$

Since $\frac{n-m}{n} \leq 1$, the expectation of total delay can be simplified as

$$\begin{aligned}\mathcal{E}\{D\} &= \sum_{m=0}^{n-1}\left(\frac{n-m}{n} + \frac{1}{\alpha}\frac{m}{n}(1-\alpha)\left(\frac{1}{(1-\alpha)^2} - 1\right)\right) \\ &= \sum_{m=0}^{n-1}\left(\frac{n+d}{n-m+d}\right). \quad (15)\end{aligned}$$

Dependent on the value of θ, the delay can have different relationship with n.

1) When θ is a constant: In such a case, the total delay can be expressed as the addition of two factors

$$\mathcal{E}\{D\} = \sum_{m=0}^{\theta-1}\left(\frac{n+m-1}{n}\right) + \sum_{m=\theta}^{n-1}\left(\frac{n+\theta}{n-m+\theta}\right)(16)$$

It can be easily obtained that $\sum_{m=0}^{\theta-1}\left(\frac{n+m}{n}\right) = \frac{n-1}{n}\theta + \frac{\theta(\theta-1)}{2n}$. On the other hand,

$$\sum_{m=\theta}^{n-1}\left(\frac{n+\theta}{n-m+\theta}\right) = (n+\theta)H_n - (n+\theta)H_\theta,$$

where H_n denotes the Harmonic number, $H_n = \sum_{k=1}^{n}\frac{1}{k}$. Conditioned on $n \geq 1$ and $d \geq 1$, and utilizing the approximation of Harmonic number, we can obtain

$$\mathcal{E}\{D\} \approx \theta + \frac{\theta(\theta-1)}{2n} + (n+\theta)\ln\frac{n+\theta}{\theta}. \quad (17)$$

Since θ is a constant we have

$$\mathcal{E}\{D\} \approx (n+\theta)\ln\frac{n+\theta}{\theta} + \mathcal{O}(1) \to \mathcal{O}(n\ln n), (18)$$

which is the same order as the one achieved by the pure opportunistic scheduling.

2) When $\theta = \lfloor\frac{m-1}{P}\rfloor$: In such a case, the group size will be linearly increasing with the number of available relays, where P is a constant and $P \geq 2$. For the case that $P = 1$, relay selection is not used, and it can be easily calculated from (15) that the total delay is approximated as $\mathcal{O}(n)$ consistent to the results developed in the previous section. Without loss generality, assume n is an multiple times of P. Rewrite the ergodic sumrate in (15) as

$$\begin{aligned}\mathcal{E}\{D\} &= \sum_{p=0}^{P-1}\sum_{k=0}^{\frac{n}{P}-1}\left(\frac{n+d}{n-(Pk+p)+d}\right) \quad (19)\\ &\leq \sum_{p=0}^{P-1}\sum_{k=0}^{\frac{n}{P}-1}\left(\frac{n+k-1}{n-(Pk+p)+k-1}\right).\end{aligned}$$

Note that the inside summary can be upper bounded as

$$\begin{aligned}&\sum_{k=0}^{\frac{n}{P}-1}\left(\frac{n+k}{n-(Pk+p)+k}\right) \leq \left(\frac{P(n-1)-1}{P-1}\right) \quad (20)\\ &\times\sum_{k=0}^{\frac{n}{P}-1}\left(\frac{1}{n-1-(P-1)k-p}\right) - \frac{1}{P-1}\frac{n}{P}.\end{aligned}$$

Hence the expectation of the total delay can be upper bounded as

$$\begin{aligned}\mathcal{E}\{D\} &\leq \sum_{p=0}^{P-1}\frac{P(n-1)-1}{P-1}\sum_{k=0}^{\frac{n}{P}-1}\left(\frac{1}{n-1-(P-1)k-p}\right) \\ &= \left(\frac{P}{P-1}(n-1) - \frac{1}{P-1}\right)\sum_{l=\frac{n}{P}-1}^{n-1}\frac{1}{l}.\end{aligned}$$

Finally the asymptotic property of the delay can be shown as

$$\begin{aligned}\mathcal{E}\{D\} &\leq \left(\frac{P}{P-1}(n-1) - \frac{1}{P-1}\right)\left(H_{n-1} - H_{\frac{n}{P}-2}\right) \\ &\approx \frac{P}{P-1}n\ln P \to \mathcal{O}(n),\end{aligned}$$

which means that the proposed scheduler can achieve the delay at the same order as the round robin.

IV. Numerical Results

In this section, the performance of the proposed network coding scheduling protocols is evaluated by using Monte-Carlo simulations. The performance of the round robin and pure opportunistic schemes is also shown for comparison. In Fig.2 the averaged delay required for all schemes is shown as a function of the number of users, and in Fig.3 the achievable throughput is shown as a function of the user number with fixed SNR. As can be seen from Fig.2-3, the round robin scheme can achieve the smallest relay, but the worst system throughput among the studied schemes. The proposed network coding scheduler without relay selection can achieve the delay performance close to the round robin scheme, but there is a performance loss for the system throughput compared with the pure opportunistic scheme. For the proposed scheduler with relay selection, the performance loss of system throughput can be avoided, but the required delay is slightly increased. In particular, if the group size is set as $\theta = 1$, then only the best relay will be used for data forwarding. The achievable sumrate can be even larger than the one achieved by the pure opportunistic scheduler, but the required delay becomes as worse as the opportunistic scheme. If the group size is set as $\theta = \lfloor \frac{m-1}{P} \rfloor$ and $P = 2$, the proposed scheduler can still yield a signiŒcant delay performance gain over the pure opportunistic scheme, and at the same time only results in slight loss of system throughput. In practice, we can adjust the value of θ adaptively according to the quality of service and hence yield a better tradeoff between the delay and system throughput.

V. Conclusion

In this paper, we have studied the application of network coding to opportunistic scheduling for wireless uplink channels. The key idea proposed in this paper is to always schedule the user with best channel gain, and meantime the use of network coding encourage the scheduled users to help the ones which have not been served previously. Analytical and numerical results have been developed to show that the proposed network coding schedulers can achieve better tradeoff of fairness and system throughput than comparable schemes.

VI. acknowledgement

Z. Ding's work was supported by the UK EPSRC research project under grant number EP/F062079/1. K. K. Leung's work was supported by US Army Research laboratory and the UK Ministry of Defence and was accomplished under Agreement Number W911NF-06-3-0001. M. Peng's work was supported in part by China National Natural Science Foundation (grant No. 60602058), the National Advanced Technologies R&D Programs (863 grant No. 2009AA01Z244)

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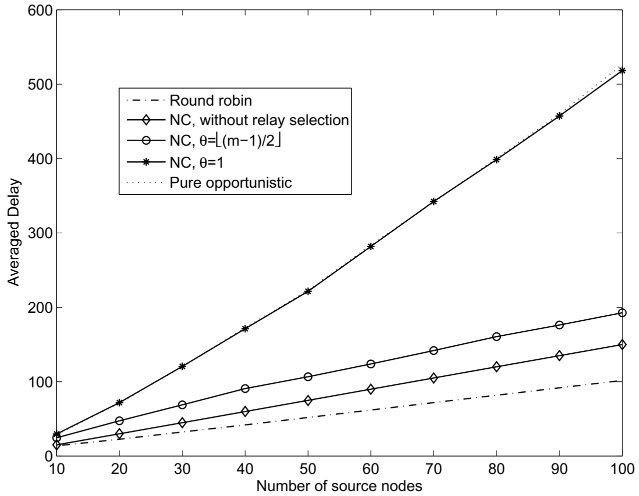

Fig. 2. The averaged delay vs the number of source nodes.

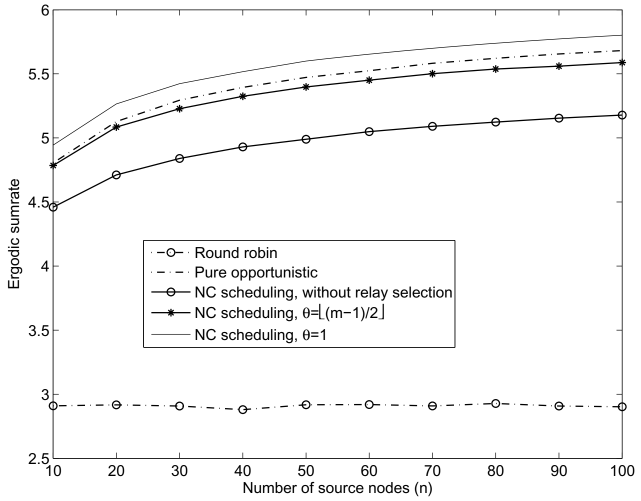

Fig. 3. Ergodic sum rate vs the number of source nodes. The SNR is Œxed at $\rho = 10$dB.