Imperial College London

FRESH - An algorithm for resolution enhancement of piecewise smooth signals and images

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Problem Statement

Real Scene Digital Image

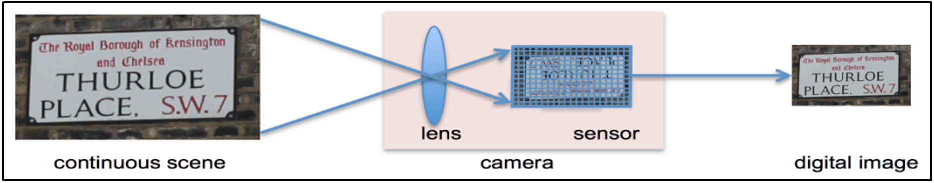
- A visual scene is turned into a **digital** image by a camera
- Can we overcome the limitation of the camera and, given the pixels, obtain a sharper image with increased resolution?
- The problem of enhancing the resolution of a single image is known as Single-Image Super-Resolution

Single Image Super Resolution: Example

Low-res input
128x128 pixels

Final result
512x512 pixels

Sampling and Resolution Enhancement

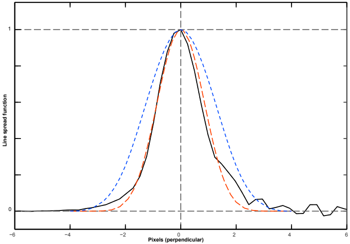


$$x(t) \longrightarrow \boxed{h(t) = \tilde{\varphi}(-t/T)} \xrightarrow{T} y_n$$

- Sampling and Resolution Enhancement are heavily connected through wavelet multi-resolution analysis
- The acquisition process can be modelled as low-pass filtering followed by sampling
- In a camera the low-pass filtering is due to the lenses and is modelled with the point spread function

Point Spread Function and Splines

(a)Original (2014 × 3039)



(b) Point Spread function

- In a camera the low-pass filtering is due to the lenses and is modelled with the point spread function
- The point spread function in a camera behaves like a spline function

Acquisition Process and Wavelet Decomposition

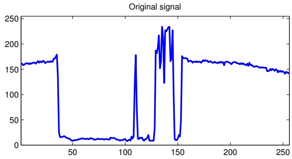
- The acquisition process remove the fine details of the image
- Since the low-pass filter is a spline, the acquisition process can be interpreted as a process that removes the wavelet coefficients at fine scales
- **Key insight**: Exploit the dependency across scale of the wavelet coefficients to retrieve the lost details.

(a) The high-resolution image 'Peppers'

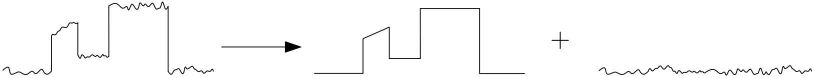
(b) Low-pass and high-pass subbands of a 2-level 2D wavelet transform of (a)

(c) We only have access to the low-pass subband of the 2-level 2D wavelet transform in (b)

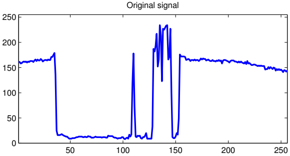
Modelling of Dependencies Across Scales



- We model lines of images as piecewise regular functions defined as the combination of a piecewise polynomial signal and a globally smooth function that lies in shift-invariant subspace:



Modelling of Dependencies Across Scales



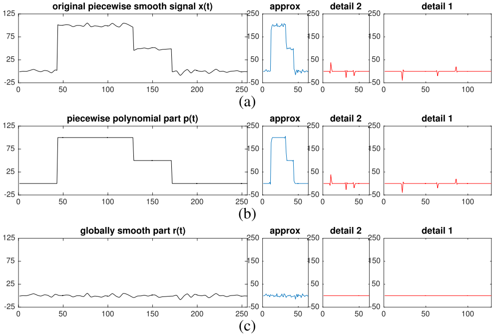
- We model lines of images as piecewise regular functions defined as the combination of a piecewise polynomial signal and a globally smooth function that lies in shift-invariant subspace:

$$x(t) = p(t) + r(t) = p(t) + \sum_{n} y_n \varphi(t/T - n)$$

Note that we assume: $\langle \varphi(t), \tilde{\varphi}(t-n) \rangle = \delta_n$

Modelling of Dependencies Across Scales

In the wavelet domain, the detail coefficients are only due to the piecewise polynomial signal

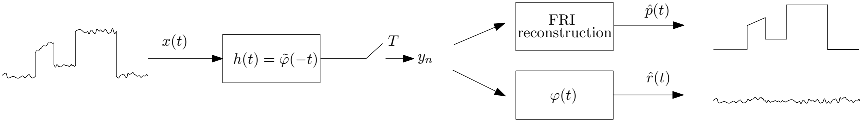


$$
\begin{aligned}
x(t) &= p(t) + r(t)\\
&= \underbrace{\sum_{n=-\infty}^{\infty} y^{p}_{J,n}\varphi_{J,n}(t) + \sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty} d^{p}_{m,n}\psi_{m,n}(t)}_{p(t)}\\
&\quad + \underbrace{\sum_{n=-\infty}^{\infty} y^{r}_{J,n}\varphi_{J,n}(t)}_{r(t)}\\
&= \sum_{n=-\infty}^{\infty} (\underbrace{y^{p}_{J,n}+y^{r}_{J,n}}_{y_{J,n}})\,\varphi_{J,n}(t) + \sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty} d^{p}_{m,n}\psi_{m,n}(t).
\end{aligned}
$$

Reconstruction of Piecewise Smooth Signals

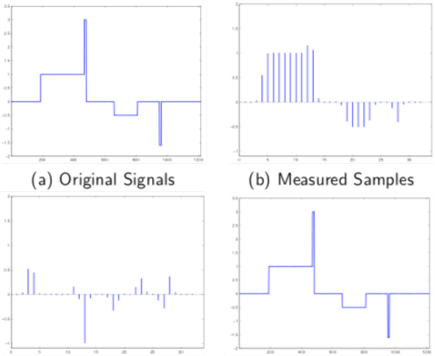
Key Insight:

- The residual can be recovered using traditional linear reconstruction methods
- Piecewise polynomial signals are continuous sparse signals and can be recovered using sparse sampling theory (i.e., finite rate of innovation theory [DragottiVB:07, UriguenBD:13])



Note that we assume: $\langle \varphi(t), \tilde{\varphi}(t-n) \rangle = \delta_n$

Exact Reconstruction of Piecewise Polynomial Signals



(a) Original Signals

(b) Measured Samples

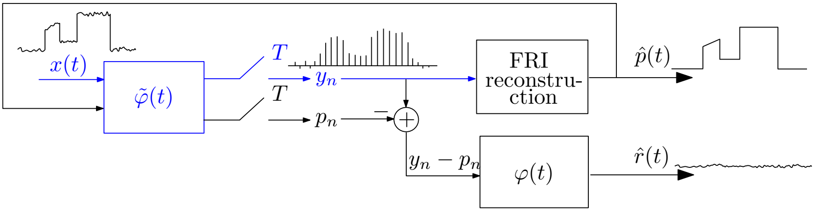
(c) Finite Difference

(d) Reconstructed Signal

Piecewise polynomial signals are continuous sparse signals and can be recovered using sparse sampling theory (i.e., finite rate of innovation theory [DragottiVB:07, UriguenBD:13])

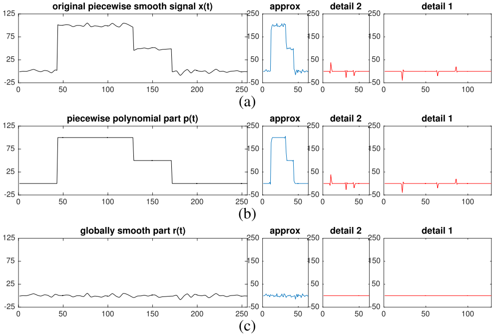
Reconstruction of Piecewise Smooth Signals

- remove the contribution of the reconstructed polynomial part $\hat{p}(t)$ from the samples y_n.
- reconstruct the residual $\hat{r}(t)$ by classical linear reconstruction.



Modelling of Dependencies Across Scales

In the wavelet domain, the detail coefficients are only due to the piecewise polynomial signal



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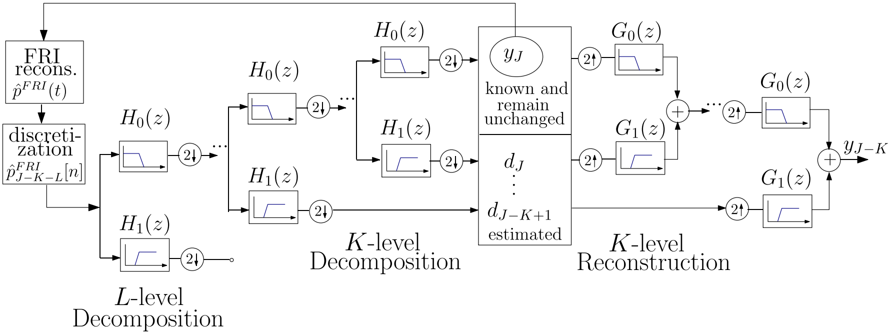
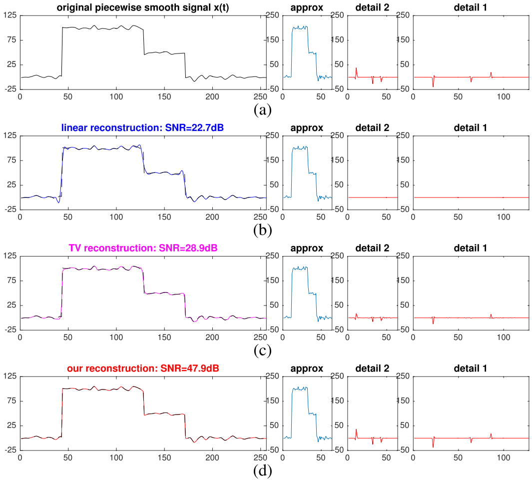
&\quad + \underbrace{\sum_{n=-\infty}^{\infty} y^r_{J,n}\varphi_{J,n}(t)}_{r(t)} \\
&= \sum_{n=-\infty}^{\infty} (\underbrace{y^p_{J,n}+y^r_{J,n}}_{y_{J,n}})\,\varphi_{J,n}(t) + \sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty} d^p_{m,n}\psi_{m,n}(t).
\end{aligned}
$$

Resolution Enhancement of Piecewise Smooth Signals

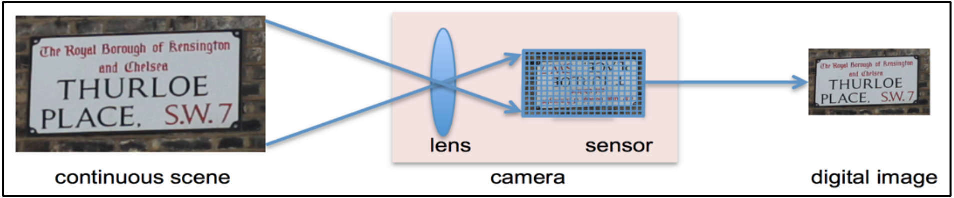


Use iterated filter banks to merge the samples with the details provided by the piecewise polynomial signal

Numerical Results



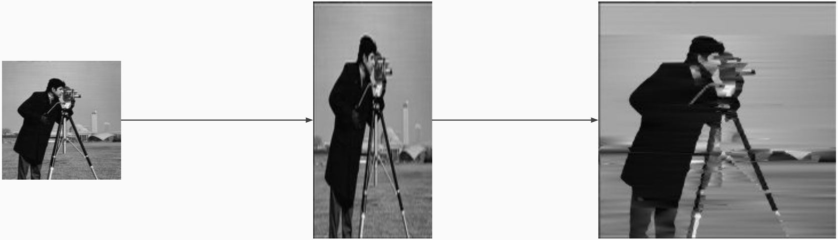
FRESH: FRI-based Single-Image Super-Resolution



- Algorithm capable of **increasing the resolution of digital images up to 4X**.
- Based on applying the 1-D resolution enhancement algorithm along several directions of the image
- The upsampled images are merged using wavelet theory
- Self-learning further improves performance
- **Accurately retrieve fine details** lost during the acquisition process.

[WeiD:TIP16]

FRESH: FRI-Based Single-Image Super-Resolution



Input image

128x128 pixels

Linear upsampling along columns

256x128 pixels

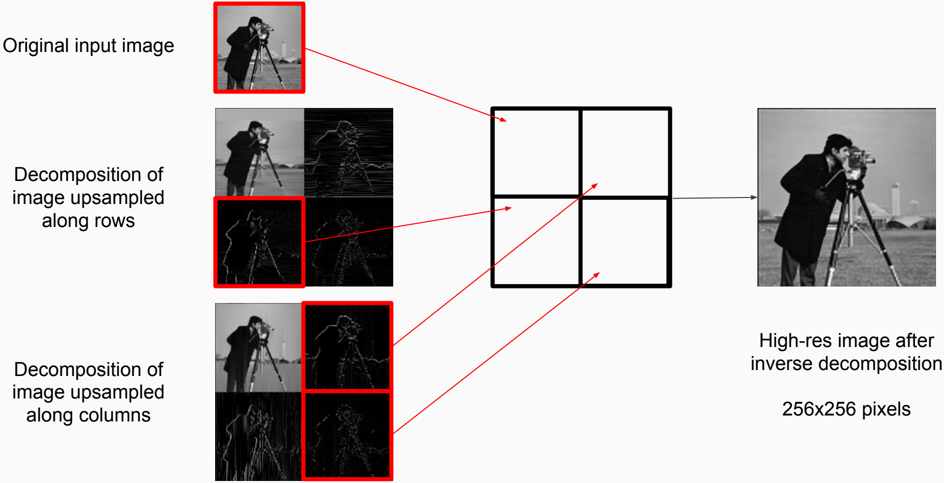
FRI upsampling along rows

256x256 pixels

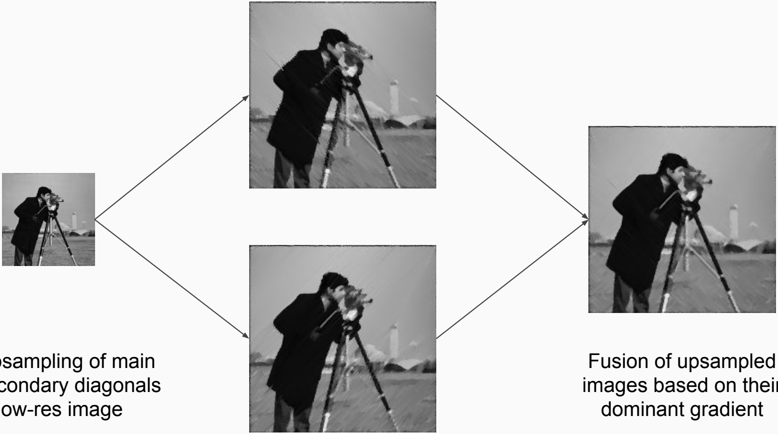
FRESH: FRI-Based Single-Image Super-Resolution

Input image	Linear upsampling along rows	FRI upsampling along columns
128x128 pixels	128x256 pixels	256x256 pixels

FRESH: FRI-based Single-Image Super-Resolution



FRESH: FRI-based Single-Image Super-Resolution



FRI upsampling of main and secondary diagonals of low-res image

Fusion of upsampled images based on their dominant gradient

FRESH Results: Lena

Low-res input
128x128 pixels

Final result
512x512 pixels

FRESH: Numerical Comparisons

(a)

Original

(b)

Linear (25.9dB)

(c)

A+ (27.3dB)

(d)

FRESH (27.7dB)

FRESH Results: Real Data

Low-res input
64 x64 pixels

Final result
256x256 pixels

Conclusions

FRESH: A new Single Image Super-Resolution Algorithm

- based on combining the multiresolution property of wavelets with sampling theory and based on the power of modelling structures across scales
- does not require the use of external dictionaries for learning
- competitive against state-of-the-art methods (all based on heavy learning strategies)

References

On Single-Image Image Super-Resolution

- X. Wei and P.L. Dragotti, FRESH -FRI-based single image super-resolution algorithm, IEEE Trans on Image Processing, Vol.25(8), pp. 3723-3735, August 2016.

On Sparse Sampling

- J. Uriguen, T. Blu, and P.L. Dragotti ‘FRI Sampling with Arbitrary Kernels’, IEEE Trans. on Signal Processing, November 2013

- P.L. Dragotti, M. Vetterli and T. Blu, ‘Sampling Moments and Reconstructing Signals of Finite Rate of Innovation: Shannon meets Strang-Fix’, IEEE Trans. on Signal Processing, vol.55 (5), pp.1741-1757, May   2007.