FRESH - An algorithm for resolution enhancement of piecewise smooth signals and images

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Problem Statement

- A visual scene is turned into a **digital** image by a camera
- Can we overcome the limitation of the camera and, given the pixels, obtain a sharper image with increased resolution?
- The problem of enhancing the resolution of a single image is known as **Single-Image Super-Resolution**
Single Image Super Resolution: Example

Low-res input
128x128 pixels

Final result
512x512 pixels
Sampling and Resolution Enhancement

- Sampling and Resolution Enhancement are heavily connected through wavelet multi-resolution analysis.
- The acquisition process can be modelled as low-pass filtering followed by sampling.
- In a camera the low-pass filtering is due to the lenses and is modelled with the point spread function.

\[ x(t) \xrightarrow{h(t)} y_n \]

where

\[ h(t) = \tilde{\varphi}(-t/T) \]
Point Spread Function and Splines

- In a camera the low-pass filtering is due to the lenses and is modelled with the point spread function.
- The point spread function in a camera behaves like a spline function.
Acquisition Process and Wavelet Decomposition

- The acquisition process removes the fine details of the image.
- Since the low-pass filter is a spline, the acquisition process can be interpreted as a process that removes the wavelet coefficients at fine scales.
- **Key insight:** Exploit the dependency across scale of the wavelet coefficients to retrieve the lost details.

![Peppers Image](a) The high-resolution image 'Peppers'

![Wavelet Transform](b) Low-pass and high-pass subbands of a 2-level 2D wavelet transform of (a)

![Low-Pass Subband](c) We only have access to the low-pass subband of the 2-level 2D wavelet transform in (b)
We model lines of images as piecewise regular functions defined as the combination of a piecewise polynomial signal and a globally smooth function that lies in shift-invariant subspace:
Modelling of Dependencies Across Scales

- We model lines of images as piecewise regular functions defined as the combination of a piecewise polynomial signal and a globally smooth function that lies in shift-invariant subspace:

\[ x(t) = p(t) + r(t) = p(t) + \sum_{n} y_n \varphi(t/T - n) \]

**Note that** we assume: \( \langle \varphi(t), \tilde{\varphi}(t - n) \rangle = \delta_n \)
Modelling of Dependencies Across Scales

In the wavelet domain, the detail coefficients are only due to the piecewise polynomial signal

\[ x(t) = p(t) + r(t) \]

\[ = \sum_{n=-\infty}^{\infty} y_{J,n}^p \varphi_{J,n}(t) + \sum_{m=-\infty}^{J} \sum_{n=-\infty}^{\infty} d_{m,n}^p \psi_{m,n}(t) \]

\[ + \sum_{n=-\infty}^{\infty} y_{J,n}^r \varphi_{J,n}(t) \]

\[ = \sum_{n=-\infty}^{\infty} (y_{J,n}^p + y_{J,n}^r) \varphi_{J,n}(t) + \sum_{m=-\infty}^{J} \sum_{n=-\infty}^{\infty} d_{m,n}^p \psi_{m,n}(t). \]
Reconstruction of Piecewise Smooth Signals

**Key Insight:**
- The residual can be recovered using traditional linear reconstruction methods.
- Piecewise polynomial signals are continuous sparse signals and can be recovered using sparse sampling theory (i.e., finite rate of innovation theory [DragottiVB:07, UriguenBD:13]).

\[ h(t) = \hat{\varphi}(-t), \quad y_n = \hat{r}(t) \]

**Note that** we assume: \[ \langle \varphi(t), \hat{\varphi}(t - n) \rangle = \delta_n \]
Exact Reconstruction of Piecewise Polynomial Signals

Piecewise polynomial signals are continuous sparse signals and can be recovered using sparse sampling theory (i.e., finite rate of innovation theory [DragottiVB:07, UriguenBD:13])
Reconstruction of Piecewise Smooth Signals

- remove the contribution of the reconstructed polynomial part $\hat{p}(t)$ from the samples $y_n$.
- reconstruct the residual $\hat{r}(t)$ by classical linear reconstruction.
Modelling of Dependencies Across Scales

In the wavelet domain, the detail coefficients are only due to the piecewise polynomial signal

\[
x(t) = p(t) + r(t)
\]

\[
= \sum_{n=-\infty}^{\infty} y_{J,n}^p \phi_{J,n}(t) + \sum_{m=-\infty}^{J} \sum_{n=-\infty}^{\infty} d_{m,n}^p \psi_{m,n}(t)
\]

\[
+ \sum_{n=-\infty}^{\infty} y_{J,n}^r \phi_{J,n}(t)
\]

\[
= \sum_{n=-\infty}^{\infty} (y_{J,n}^p + y_{J,n}^r) \phi_{J,n}(t) + \sum_{m=-\infty}^{J} \sum_{n=-\infty}^{\infty} d_{m,n}^p \psi_{m,n}(t).
\]
Resolution Enhancement of Piecewise Smooth Signals

Use iterated filter banks to merge the samples with the details provided by the piecewise polynomial signal.
Numerical Results

Fig. 15. Our method is able to accurately recover a piecewise smooth signal from its approximation coefficients. (a) The original high-resolution piecewise smooth signal and its wavelet decomposition. (b) The linear reconstruction (22.7dB) and its wavelet decomposition. (c) TV reconstruction (28.9dB) and its wavelet decomposition. (d) Our reconstruction (47.9dB) and its wavelet decomposition.

TABLE II

Comparisons of Upsampling Results (Factor 4) Given by Different Methods in Terms of PSNR.

<table>
<thead>
<tr>
<th>Sampling Kernel</th>
<th>Biorthogonal 4.4</th>
<th>Linear Spline</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>22.7</td>
<td>28.9</td>
</tr>
<tr>
<td>TV</td>
<td>28.9</td>
<td></td>
</tr>
<tr>
<td>Our</td>
<td>47.9</td>
<td></td>
</tr>
</tbody>
</table>

2) Upsampling of Images Taken With a Camera:
Finally, we show that the proposed algorithm is also able to upsample the images taken with a real camera, where the blurring due to lens is not exactly a scaling function as assumed previously but can still be modeled as a spline. We demonstrate in Fig. 18 that the algorithm achieves visually good performance for upsampling factor of 4. In the following result, the original photographs are taken with Canon 400D, and its point spread function is modeled by the fifth order spline. The upsampling is performed only on the luminance component of the input image and the chrominance component are simply upscaled by bicubic interpolation.

C. Computation Complexity and Discussions
Upsampling an image of size $N \times N$ to $2^K N \times 2^K N$ with the basic algorithm proposed in Sec. IV-A requires number

- Algorithm capable of increasing the resolution of digital images up to 4X.
- Based on applying the 1-D resolution enhancement algorithm along several directions of the image.
- The upsampled images are merged using wavelet theory.
- Self-learning further improves performance.
- Accurately retrieve fine details lost during the acquisition process.

[WeiD:TIP16]
FRESH: FRI-Based Single-Image Super-Resolution

- **Input image**: 128x128 pixels
- **Linear upsampling along columns**: 256x128 pixels
- **FRI upsampling along rows**: 256x256 pixels
FRESH: FRI-Based Single-Image Super-Resolution

Input image
128x128 pixels

Linear upsampling along rows
128x256 pixels

FRI upsampling along columns
256x256 pixels

Original input image

Decomposition of image upsampled along rows

Decomposition of image upsampled along columns

High-res image after inverse decomposition
256x256 pixels

FRI upsampling of main and secondary diagonals of low-res image

Fusion of upsampled images based on their dominant gradient
FRESH Results: Lena

Low-res input
128x128 pixels

Final result
512x512 pixels
FRESH: Numerical Comparisons

(a) Original
(b) Linear (25.9dB)
(c) A+ (27.3dB)
(d) FRESH (27.7dB)
FRESH Results: Real Data

Low-res input
64 x 64 pixels

Final result
256 x 256 pixels
Conclusions

FRESH: A new Single Image Super-Resolution Algorithm

- based on combining the multiresolution property of wavelets with sampling theory and based on the power of modelling structures across scales
- does not require the use of external dictionaries for learning
- competitive against state-of-the-art methods (all based on heavy learning strategies)
References

On Single-Image Image Super-Resolution


On Sparse Sampling
