COMMUNICATIONS I

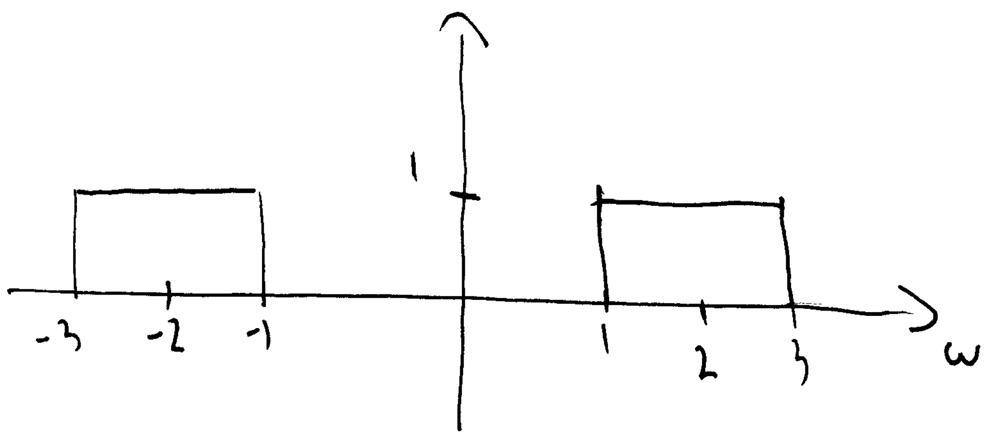
SOLUTIONS TO PROBLEM SHEET EIGHT

(a) $$m_1(t) = \frac{2}{\pi}\,\mathrm{SINC}(t)\cos(2t)$$

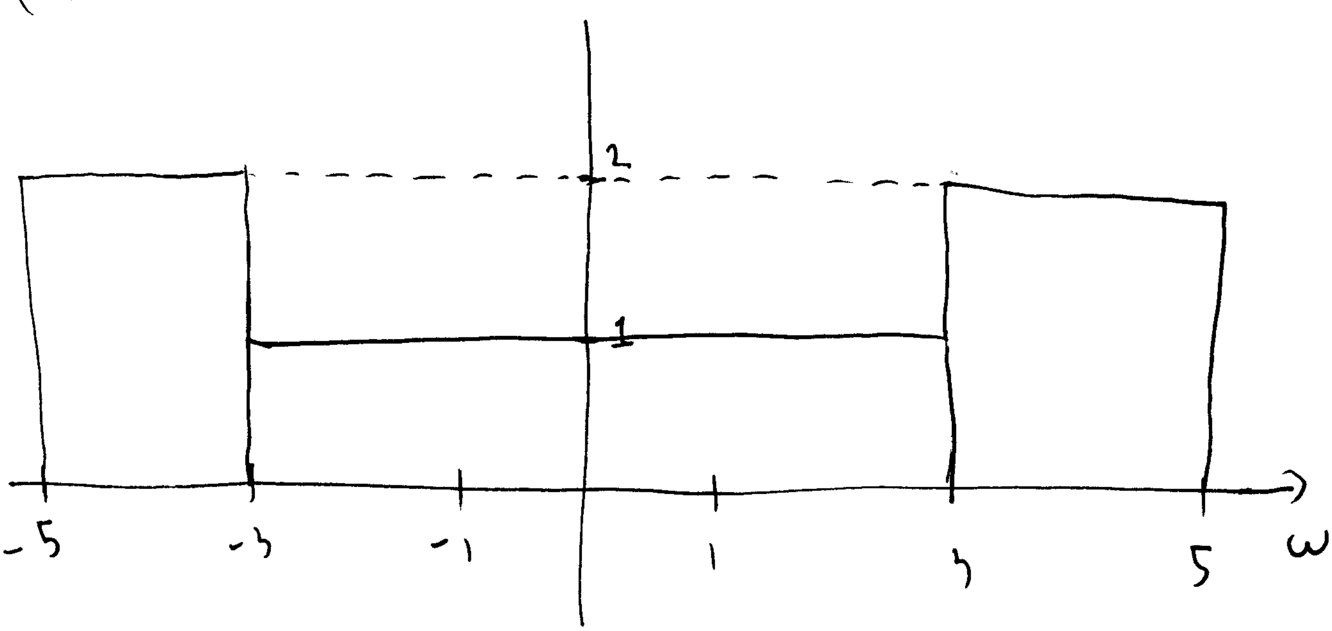
$$\frac{2}{\pi}\,\mathrm{SINC}(t) \Longleftrightarrow 2\,\mathrm{RECT}\left(\frac{\omega}{2}\right)$$

THEREFORE

$$\frac{2}{\pi}\,\mathrm{SINC}(t)\cos 2t \Longleftrightarrow \mathrm{RECT}\left(\frac{\omega-2}{2}\right)+\mathrm{RECT}\left(\frac{\omega+2}{2}\right)$$



(b)



(c) $$B_{FM} = 2(\Delta f + B) = 2\left(\frac{K_f \cdot X_p}{2\pi} + B\right)$$

$$B = \frac{5}{2\pi} \text{ Hz} \qquad X_p = X(0) = \frac{1}{\pi} + \frac{2}{\pi} + \frac{4}{\pi} = \frac{7}{\pi}$$

THUS

$$B_{FM} = 2\left(\frac{10\pi \cdot \frac{7}{\pi}}{2\pi} + \frac{5}{2\pi}\right) = \frac{75}{\pi} \text{ Hz}$$

(D) $$x(t) = A e^{-10t} \cdot u(t)$$

$$X(\omega) = A \int_{-\infty}^{\infty} e^{-10t} u(t) e^{-j\omega t} dt = \frac{A}{10 + j\omega}$$

USE PARSEVAL'S THEOREM TO FIND THE BAND WIDTH OF $x(t)$:

$$E_x = \frac{A^2}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{A^2}{20}$$

CALL $W = 2\pi B$, W MUST BE SUCH THAT

$$A^2 \frac{0.95}{20} = \frac{A^2}{2\pi} \int_{-W}^{W} |X(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-W}^{W} \frac{d\omega}{\omega^2 + 100} =$$

$$= \frac{1}{10\pi} \tan^{-1} \frac{W}{10} \Rightarrow W = 127.6 \text{ RAD/s}$$

AND $B = 20.2\,\text{Hz}$

THUS

$$B_{FM} = 2\left(\frac{A k_f}{2\pi} + B\right) = 2\left(\frac{10\pi A}{2\pi} + 20.2\right) = 50.4\,\text{Hz}$$

$$\Rightarrow \quad A = 1$$