

# Communications I

## Solutions to problem sheet seven

1. (a)

$$\phi_{PM}(t) = A \cos[\omega_c t + k_p m(t)] = 10 \cos[10000t + k_p m(t)]$$

We know that  $\phi_{PM}(t) = 10 \cos(13000t)$  with  $k_p = 1000$ . Therefore,  $m(t) = 3t$ .

(b)

$$\phi_{FM}(t) = A \cos[\omega_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha]$$

Since  $k_f = 1000$ ,  $3t = \int_0^t m(\alpha) d\alpha$ . Therefore,  $m(t) = 3$ .

2. (a) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P = \frac{A^2}{2} = \frac{100^2}{2} = 5000$$

(b) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with  $k_p = 4$  and message signal  $m(t) = \sin(2000\pi t)$  and it is an FM signal with  $k_f = 8000\pi$  and message signal  $m(t) = \cos(2000\pi t)$

(c) The instantaneous frequency is:

$$f_i = f_c + \frac{1}{2\pi} \frac{d}{dt} \theta(t) = f_c + \frac{4}{2\pi} \cos(2000\pi t) 2000\pi = f_c + 4000 \cos(2000\pi t)$$

Hence,  $\Delta f = 4000$ .

(d) Baseband signal bandwidth  $B = 1000\text{Hz}$ , therefore  $B_{EM} = 2(\Delta f + B) = 2(4000 + 1000) = 10\text{kHz}$

3. (a) Since

$$\text{sinc}(400\pi t) \iff \frac{1}{400} \text{rect}\left(\frac{\omega}{800\pi}\right),$$

the bandwidth of the message signal is  $B = 400\pi/(2\pi) = 200\text{Hz}$  and the resulting modulation index

$$\beta_f = \frac{k_f \max\{|m(t)|\}}{2\pi B} = \frac{k_f 10}{2\pi B} = 6 \Rightarrow k_f = 240\pi.$$

Hence, the modulated signal is

$$\begin{aligned} u(t) &= A \cos(2\pi f_c t + k_f \int_{-\infty}^t m(\tau) d\tau) \\ &= 100 \cos(2\pi f_c t + 2400\pi \int_{-\infty}^t \text{sinc}(400\tau) d\tau) \end{aligned}$$

(b) The maximum deviation of the modulated signal is

$$\Delta f_{max} = \beta_f B = 6 \times 200 = 1200\text{Hz}$$

(c) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by

$$B_{FM} = 2(\beta_f + 1)B = 2(6 + 1)200 = 2800\text{Hz}$$