COMMUNICATIONS I

SOLUTIONS TO PROBLEM SHEET FOUR

1.

TRIGONOMETRIC IDENTITIES USED

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

b) $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$

$$R_g(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos\omega_0 t \cdot \cos(\omega_0 t + \omega_0\tau)\,dt = \left(\begin{matrix}\text{USE IDENTITY}\\ \text{a)}\end{matrix}\right)$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos^2\omega_0 t\,\cos\omega_0\tau\,dt - \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sin\omega_0\tau\cdot\cos\omega_0 t\cdot\sin\omega_0 t\,dt$$

$$= \cos\omega_0\tau \cdot \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos^2\omega_0 t\,dt = \frac{\cos\omega_0\tau}{2}$$

$= 1/2$

$= 0$

USE IDENTITY b)

THUS

$$R_g(\tau) = \frac{\cos\omega_0\tau}{2} \Longleftrightarrow S_g(\omega) = \frac{\pi}{2}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

2.

$$|H(\omega)|^2 = \frac{1}{1+\omega^2}$$

a) $$P_g = \frac{1}{2\pi}\int_{-1}^{1} d\omega = \frac{1}{\pi}$$

$$P_y = \frac{1}{2\pi}\int_{-1}^{1} \frac{1}{\omega^2+1}\, d\omega = \frac{1}{4}$$

b) $$P_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[\delta(\omega+1) + \delta(\omega-1)\right] d\omega = \frac{1}{\pi}$$

$$P_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega^2+1}\left(\delta(\omega+1) + \delta(\omega-1)\right) d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\delta(\omega+1)}{1+\omega^2} d\omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\delta(\omega-1)}{\omega^2+1} d\omega = \frac{1}{2\pi}$$