EE1 and ISE1 Communications I

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Lecture twelve

Lecture Aims

- Angle Modulation
  - Phase and Frequency modulation
  - Concept of instantaneous frequency
  - Examples of phase and frequency modulation
  - Power of angle-modulated signals

Angle modulation

Consider a modulating signal $m(t)$ and a carrier $v_c(t) = A\cos(\omega_c t + \theta_c)$.

The carrier has three parameters that could be modulated: the amplitude A (AM) the frequency ω_c (FM) and the phase θ_c (PM).

The latter two methods are closely related since both modulate the argument of the cosine.

Instantaneous Frequency

- By definition a sinusoidal signal has a constant frequency and phase: $A\cos(\omega_c t + \theta_c)$
- Consider a generalized sinusoid with phase $\theta(t)$: $\phi(t) = A\cos\theta(t)$
- We define the instantaneous frequency ω_i as:

$$\omega_i(t) = \frac{d\theta}{dt}$$

- Hence, the phase is

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)d\alpha.$$

Phase modulation

We can transmit the information of $m(t)$ by varying the angle θ of the carrier. In **phase modulation (PM)** the angle $\theta(t)$ is varied linearly with $m(t)$:

$$\theta(t) = \omega_c t + k_p m(t)$$

where k_p is a constant and ω_c is the carrier frequency. Therefore, the resulting PM wave is

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

The instantaneous frequency in this case is given by

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

Frequency modulation

In PM the instantaneous frequency ω_i varies linearly with the **derivative** of $m(t)$. In **frequency modulation (FM)**, ω_i is varied linearly with $m(t)$. Thus

$$\omega_i(t) = \omega_c + k_f m(t).$$

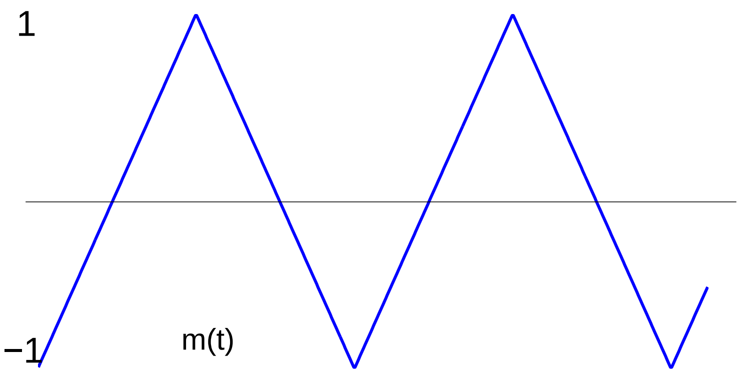
where k_f is a constant. The angle $\theta(t)$ is now

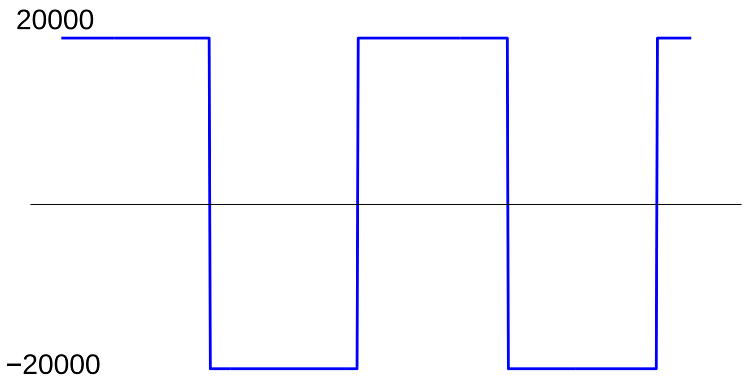
$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)] d\alpha = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha.$$

The resulting FM wave is

$$\phi_{FM}(t) = A \cos \left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha \right]$$

Example





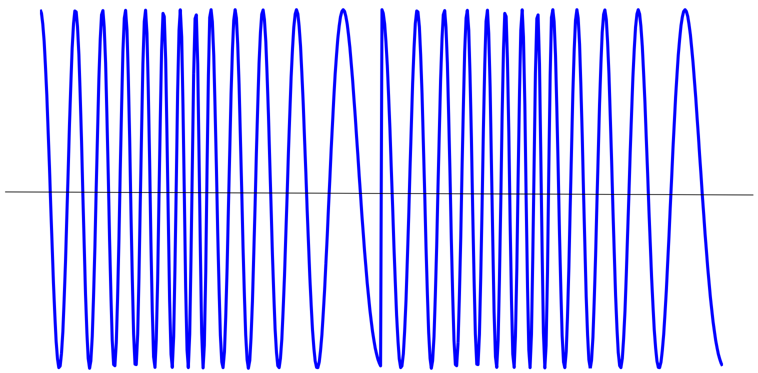
Sketch FM and PM signals if the modulating signal is the one above (on the left). The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and the carrier frequency $f_c = 100MH_z$.

FM example

- Instantaneous angular frequency $\omega_i = \omega_c + k_f m(t)$
- Instantaneous frequency $f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

$$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min} = 99.9 MHz$$

$$(f_i)_{max} = 10^8 + 10^5 [m(t)]_{max} = 100.1 MHz$$



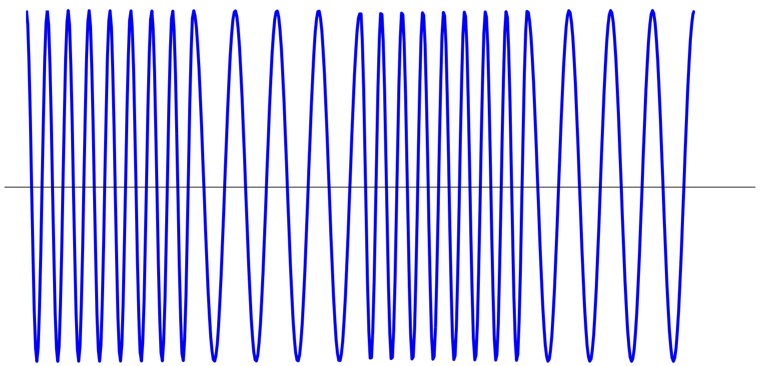
$\phi_{FM}(t)$

PM example

- Instantaneous frequency $f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t) = 10^8 + 5\dot{m}(t)$

$$(f_i)_{min} = 10^8 + 5[\dot{m}(t)]_{min} = 10^8 - 10^5 = 99.9MHz$$

$$(f_i)_{max} = 10^8 + 5[\dot{m}(t)]_{max} = 10^8 + 10^5 = 100.1MHz$$



Power of an Angle-Modulated wave

- General angle modulated waveform

$$\phi(t) = A \cos \theta(t)$$

- Instantaneous phase and frequency vary with the time, but amplitude A remains constant.
- Thus, the power of angle–modulated waves is always $\frac{A^2}{2}$.

Conclusions

Examined

- Instantaneous frequency
- PM and FM modulations
- Examples of PM and FM signals